وزارة التربية والتعليم العالي
المديريّـة العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: الإجتماع والإقتصاد

دورة الـعام ٢٠١٨ العاديّة
الاثنين ١١ حـزيران ٢٠١٨
مكيفة / احتياجات خاصة

**ملاحظة:** - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه ( دون الالتزام بترتيب المسائل الواردة في المسابقة).

# مسابقة في مادة الرياضيات

## المدة: ساعتان

## باللغة الإنكليزية

**الاسم : ..............................................**

**الرقم : ...............................................**

**I- (4 points)**

A restaurant distributes brochures each month for advertisement.

The table below shows the number of distributed brochures ($y_i$) in thousands and the monthly cost of distribution ($x_i$) in hundred thousands LL.

| Cost of distribution ($x_i$) in hundred thousands LL | 1 | 3.5 | 2 | 5 | 1.5 | 2.4 |
|---|---|---|---|---|---|---|
| Number of distributed brochures ($y_i$) in thousands | 1.2 | 6.4 | 2.6 | 7.2 | 2.1 | 3.2 |

1) Find the coordinates of the center of gravity $G(\bar{x}, \bar{y})$.
2) - **Represent** the scatter plot $(x_i, y_i)$ in a rectangular system
   - **Plot** G.
3) -**Write** an equation of the regression line $(D_{y/x})$
   - **draw** this line in the preceding system.
4) **Find** the correlation coefficient r
   **interpret** the value found.
5) The above model remains valid in the year 2018.

| Month | January 2018 | February 2018 | March 2018 | April 2018 | May 2018 | June 2018 |
|---|---|---|---|---|---|---|
| Cost of distribution ($x_i$) in hundred thousands LL | 1 | 3.5 | 2 | 5 | 1.5 | 2.4 |
| Number of distributed brochures ($y_i$) in thousands | 1.2 | 6.4 | 2.6 | 7.2 | 2.1 | 3.2 |

The restaurant manager receives an advertisement offer for the month of July 2018.

This offer indicates: "**4 000 distributed brochures for only 250 000 LL**".

**Justify** that this offer is better than the model of the regression line $(D_{y/x})$.

**II- (4 points)**

**Part A**

One student is randomly selected from the third secondary students of this school.

Consider the following events:

E: "The selected student is in the ES section",

G: "The selected student is in the GS section",

L: "The selected student is in the LS section",

S: "The selected student succeeded in the official exam".

Those students are distributed as follows:

| | E | G | L | Total |
|---|---|---|---|---|
| S | 12 % | 8 % | | 60 % |
| $\overline{S}$ | | | | |
| Total | 50 % | 10 % | 40 % | 100 % |

1) a- **Calculate** the probabilities $P(E \cap S)$ and $P(G \cap S)$.

   b- **Prove** that $P(L \cap S) = 0.22$ .

2) The selected student succeeded in the official exam.

   **Calculate** the probability that this student is in the LS section.

**Part B**

There are 50 students in the third secondary classes in this school in 2017. A computer software selects randomly and simultaneously the names of three students from the 50 students.

| | E | G | L | Total |
|---|---|---|---|---|
| S | 6 | 4 | | |
| $\overline{S}$ | | | | 20 |
| Total | 25 | 5 | 20 | 50 |

1) **Verify** that 30 students of this school succeeded in the official exam.

2) Let X be the random variable equal to the number of students who succeeded in the official exam among the three selected names of the students.

   a- **Calculate** $P(X = 1)$.

   b- **Calculate** the probability of selecting at least one name of a student who succeeded in the official exam.

**III- (4 points)**

At the beginning of the year 2015, Nabil deposits a capital of 60 million LL in a bank, at an annual interest rate of 6% , compounded annually.

At the beginning of every year, and after compounding the interest, Nabil deposits an additional amount of 3 000 000 LL in the same account.

For all natural numbers n, denote by $S_n$ the amount, in millions LL, that Nabil has in his account at the end of the year (2015 + n).

Thus, $S_0 = 60$ and $S_{n+1} = 1.06S_n + 3$ for all natural numbers n.

1) **Calculate** the amount of money in Nabil's account at the end of the year 2016.
2) Let $(V_n)$ be the sequence defined as $V_n = S_n + 50$ for all natural numbers n.
   a- **Show** that $(V_n)$ is a geometric sequence whose common ratio and first term $V_0$ are to be determined.
   b- **Show** that $S_n = 110 \times (1.06)^n - 50$ for all natural numbers n.
   c- **Show** that the sequence $(S_n)$ is strictly increasing.
3) **Calculate** the amount of money in Nabil's account at the end of the year 2020.
4) **Calculate** n so that $S_n \geq 90$.

**IV-(8 points)**

Consider the function f defined over the interval I = [1 , + ∞[ as $f(x) = (10x - 10)\, e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

**Part A**

1) **Determine** $\lim\limits_{x \to +\infty} f(x)$
   **-deduce** an asymptote to (C).
2) **-Show that** $f'(x) = 10(-x+2)e^{-x}$
   **-set** up the table of variations of f.
3) **Draw** (C).
4) The function F defined, over I, as $F(x) = -10\,x\,e^{-x}$ is an antiderivative of f.
   **Calculate** the area of the region bounded by the curve (C), the x-axis and the two lines with equations x = 2 and x = 4.

**Part B**

A company produces a certain type of objects.

The demand function f and the supply function g, defined over J = [2 , 10], are respectively modeled as $f(x) = (10x - 10)\, e^{-x}$ and $g(x) = e^{x-4}$, where f(x) and g(x) are expressed in thousands of objects and the unit price x is expressed in millions of LL.

*(The unit price is the price of 1000 objects)*

1) **Calculate** the number of demanded objects for a unit price of 3 000 000 LL.
2) **Find** the unit price for a supply of 1 000 objects.
3) The equation f(x) = g(x) has, over J, a unique solution $\alpha$.

   Suppose that $\alpha = 3.635$.

   a- **-Give** an economical interpretation of $\alpha$

   -**calculate** the corresponding number of objects.

   b- **Calculate,** in LL, the revenue corresponding to the value of $\alpha$ given above.
4) Denote by E(x) the elasticity of the demand with respect to the unit price x.

   a- **Show** that $E(x) = \dfrac{x^2 - 2x}{x - 1}$.

   b- For an increase of 1% on the unit price $x_0$ in millions LL, the demand will decrease by 1.5%.

   **Calculate** $x_0$.