وزارة التربية والتعليم العالي
المديريّـة العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: الإجتماع والإقتصاد

دورة الـعام ٢٠١٨ العاديّة
الاثنين ١١ حـزيران ٢٠١٨

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

A restaurant distributes brochures each month for advertisement.
The table below shows the number of distributed brochures (y_i) in thousands and the monthly cost of distribution (x_i) in hundred thousands LL.

Month	January 2018	February 2018	March 2018	April 2018	May 2018	June 2018
Cost of distribution (x_i) in hundred thousands LL	1	3.5	2	5	1.5	2.4
Number of distributed brochures (y_i) in thousands	1.2	6.4	2.6	7.2	2.1	3.2

1) Find the coordinates of the center of gravity $G(\bar{x}, \bar{y})$.
2) Represent the scatter plot (x_i, y_i) in a rectangular system and plot G.
3) Write an equation of the regression line $(D_{y/x})$ and draw this line in the preceding system.
4) Find the correlation coefficient r and interpret its value.
5) The above model remains valid in the year 2018.
   The restaurant manager receives an advertisement offer for the month of July 2018.
   The offer indicates: "4 000 distributed brochures for only 250 000 LL".
   Is it more advantageous for the manager to take this offer or to remain using the given model? Justify.

II- (4 points)

In 2017, the students of the third secondary classes of a certain school are distributed as follows:

- 50% of the students are in the ES section of which 60% succeeded in the official exam.
- 10% of the students are in the GS section of which 80% succeeded in the official exam.
- 40% of the students are in the LS section.
- 60% of the students succeeded in the official exam.

Part A

One student is randomly selected from the third secondary students of this school.
Consider the following events:
E: "The selected student is in the ES section", G: "The selected student is in the GS section",
L: "The selected student is in the LS section", S: "The selected student succeeded in the official exam".

1) a- Calculate the probabilities $P(E \cap S)$ and $P(G \cap S)$.
   b- Prove that $P(L \cap S) = 0.22$.
2) The selected student succeeded in the official exam. Calculate the probability that this student is in the LS section.

Part B

There are 50 students in the third secondary classes in this school in 2017. A computer software selects randomly and simultaneously the names of three students from the 50 students.

1) Verify that 30 students of this school succeeded in the official exam.
2) Let X be the random variable equal to the number of students who succeeded in the official exam among the three selected names of the students.
   a- Calculate $P(X = 1)$.
   b- Calculate the probability of selecting at least one name of a student who succeeded in the official exam.

III- (4 points)

At the beginning of the year 2015, Nabil deposits a capital of 60 million LL in a bank, at an annual interest rate of 6% , compounded annually.

At the beginning of every year, and after compounding the interest, Nabil deposits an additional amount of 3 000 000 LL in the same account.

For all natural numbers n, denote by S_n the amount, in millions LL, that Nabil has in his account at the end of the year (2015 + n).

Thus, $S_0 = 60$ and $S_{n+1} = 1.06S_n + 3$ for all natural numbers n.

1) Calculate the amount of money in Nabil's account at the end of the year 2016.
2) Let (V_n) be the sequence defined as $V_n = S_n + 50$ for all natural numbers n.
   a- Show that (V_n) is a geometric sequence whose common ratio and first term V_0 are to be determined.
   b- Show that $S_n = 110 \times (1.06)^n - 50$ for all natural numbers n.
   c- Show that the sequence (S_n) is strictly increasing.
3) Calculate the amount of money in Nabil's account at the end of the year 2020.
4) Nabil wants to buy a piece of land that costs 90 million LL.
   In which year would Nabil be able, for the first time, to buy this piece of land? Justify.

IV-(8 points)

Consider the function f defined over the interval I = [1 , + ∞[as $f(x) = (10x - 10)\, e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

1) Determine $\lim_{x \to +\infty} f(x)$ and deduce an asymptote to (C).
2) Show that $f'(x) = 10(-x+2)e^{-x}$ and set up the table of variations of f.
3) Draw (C).
4) The function F defined, over I, as $F(x) = -10x\,e^{-x}$ is an antiderivative of f. Calculate the area of the region bounded by the curve (C), the x-axis and the two lines with equations x = 2 and x = 4.

Part B

A company produces a certain type of objects.

The demand function f and the supply function g, defined over J = [2 , 10], are respectively modeled as $f(x) = (10x - 10)\, e^{-x}$ and $g(x) = e^{x-4}$, where f(x) and g(x) are expressed in thousands of objects and the unit price x is expressed in millions of LL. *(The unit price is the price of 1000 objects)*

1) Calculate the number of demanded objects for a unit price of 3 000 000 LL.
2) Find the unit price for a supply of 1 000 objects.
3) The equation f(x) = g(x) has, over J, a unique solution α.
   Suppose that $\alpha = 3.635$.
   a- Give an economical interpretation of α and calculate the corresponding number of objects.
   b- Calculate, in LL, the revenue corresponding to the value of α given above.
4) Denote by E(x) the elasticity of the demand with respect to the unit price x.
   a- Show that $E(x) = \dfrac{x^2 - 2x}{x - 1}$.
   b- For an increase of 1% on the unit price x_0 in millions LL, the demand will decrease by 1.5%.
   Calculate x_0.

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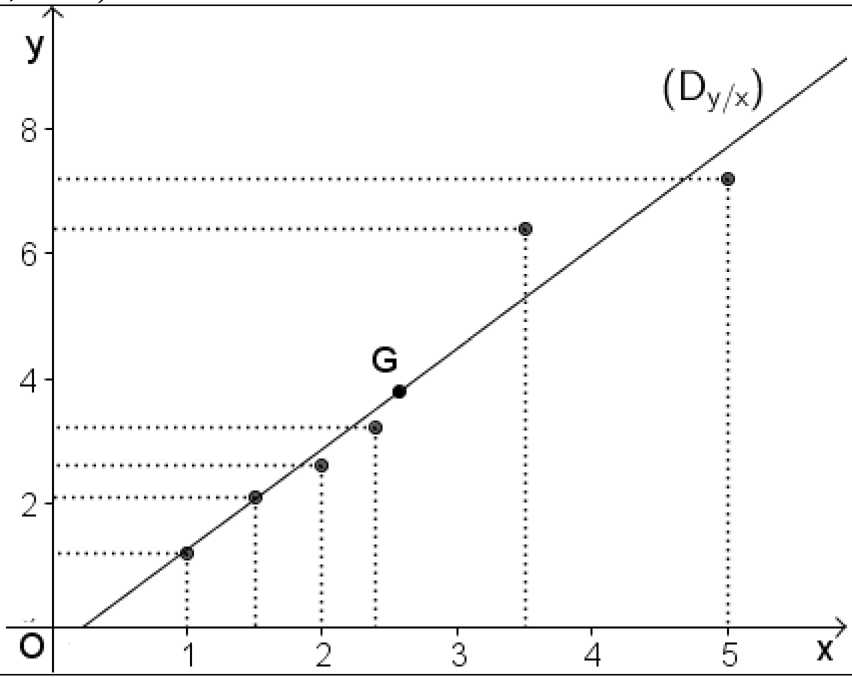
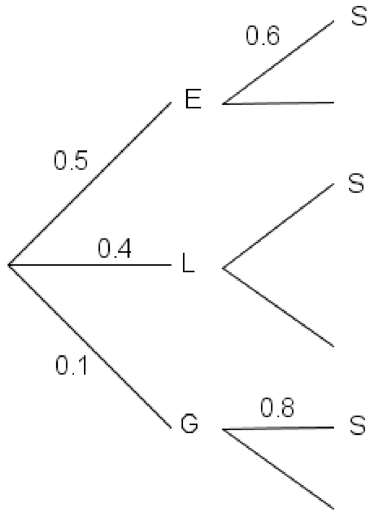
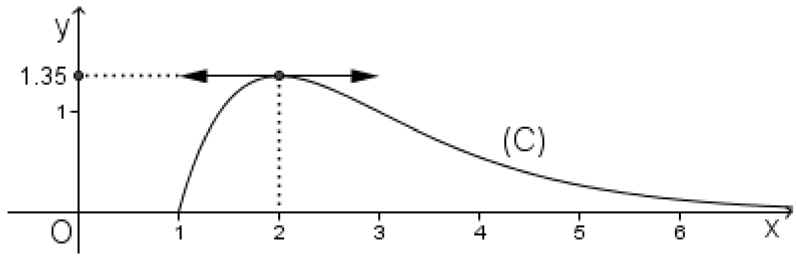
دورة العام ٢٠١٨ العاديّة
الإثنين ١١ حزيران ٢٠١٧

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	4 pts
1	G(2.566 ; 3.783)	1
2		1.5
3	y = 1.618x – 0.369	1.5
4	r = 0.971 strong positive linear correlation.	1
5	x = 2.5 then y = 3.676 (thousands of brochures) 3 676 brochures<4 000 brochures. The offer is more advantageous for the manager **Or** y = 4 then x = 2.70024 (in hundred thousands) but 270 024 LL > 250 000 LL, then the offer is more advantageous for the manager	2
Q.II	**Answers**	**4 pts**
		
A.1.a	P(E∩S) = 0.5 × 0.6 = 0.3 P(G∩S) = 0.1 × 0.8 = 0.08	2
A.1.b	P(L∩S) = P(S) – P(E∩S) – P(G∩S) = 0.6 – 0.3 – 0.08 = 0.22	1.5
A.2	P(L/S) = $\frac{P(L\cap S)}{P(S)} = \frac{0.22}{0.6} = \frac{11}{30} = 0.366$	1
B.1	50 × 0.6 = 30	0.5
B.2.a	P(X = 1) = $\frac{C_{30}^{1}\times C_{20}^{2}}{C_{50}^{3}} = \frac{57}{196} = 0.2908$	1
B.2.b	P = 1 – P(X = 0) = 1 – $\frac{C_{20}^{3}}{C_{50}^{3}} = \frac{923}{980} = 0.9418$	1

Q.III	Answers	4 pts
1	$S_1 = 1.06S_0 + 3 = 1.06(60) + 3 = 66.6$. The amount of money in Nabil's account at the end of the year 2016 was 66 600 000 LL.	**1**
2.a	$V_{n+1} = S_{n+1} + 50 = 1.06S_n + 53$ $\frac{V_{n+1}}{V_n} = \frac{1.06S_n + 53}{S_n + 50} = \frac{1.06(S_n + 50)}{S_n + 50} = 1.06$ Then (V_n) is a geometric sequence of common ratio is 1.06 and first term $V_0 = 110$.	**1.5**
2.b	$V_n = 110(1.06)^n$ then $S_n = V_n - 50 = 110 \times (1.06)^n - 50$	**1.5**
2.c	$S_{n+1} - S_n = 110 \times (1.06)^{n+1} - 110 \times (1.06)^n = 110(1.06)^n(0.06) > 0$. Then the sequence (S_n) is strictly increasing.	**1**
3	$S_5 = 110 \times (1.06)^5 - 50 = 97.204813$ The amount of money in Nabil's account at the end of the year 2020 will be 97 204 813LL.	**1**
4	$S_n > 90$ then $110 \times (1.06)^n - 50 > 90$ then $110 \times (1.06)^n > 140$ then $(1.06)^n > \frac{14}{11}$ then $n > 4.1$ so $n = 5$ then Nabil will be able, for the first time, to buy this piece of land in 2020 **OR** $S_4 = 88.8724 < 90 < S_5$ and (S_n) is strictly increasing so $n = 5$ then Nabil will be able, for the first time, to buy this piece of land in 2020	**1**

Q.IV	Answers	8 pts
A.1	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (10x - 10)e^{-x} = \lim_{x\to+\infty} (10xe^{-x} - 10e^{-x}) = 0$. Then x'x is a horizontal asymptote to (C) at $+\infty$.	**2**
A.2	$f'(x) = 10(-x + 2)e^{-x}$ x: 1 ; 2 ; $+\infty$ f '(x): + ; 0 ; − f(x): 0 ↗ 1.35 ↘ 0	**2.5**
A.3	 	**1.5**
A.4	Area $= \int_2^4 f(x)dx = [F(x)]_2^4 = -10(4e^{-4} - 2e^{-2}) = 1.97 \text{ (unit)}^2$.	**1**
B.1	$x = 3$, $f(3) = 0.995$ then 995 objects.	**1**
B.2	$g(x) = 1$; $e^{x-4} = 1$ then $x = 4$ hence 4 000 000 LL.	**1**
B.3.a	α is the equilibrium price in million LL then 3 635 000 LL. $f(\alpha) = f(3.635) = 0.695$ then 695 objects.	**2**
B.3.b	$R(\alpha) = \alpha f(\alpha) = 2{,}526325$ in million LL then 2 526 325 LL.	**1**
B.4.a	$E(x) = -x\frac{f'(x)}{f(x)} = \frac{x^2-2x}{x-1}$	**1**
B.4.b	$E(x) = 1.5$ then $x^2 - 2x = 1.5(x - 1)$ then $x^2 - 3.5x + 1.5 = 0$ so $x = 3$ acc or $x = 0.5 < 2$ rejected.	**1**