وزارة التربية والتعليم العالي
المديريّـة العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة المتوسطة

دورة العام ٢٠١٨ العادية
السبت ٢ حزيران ٢٠١٨

عدد المسائل: خمس　　مسابقة في مادة الرياضيات　　الاسم:
المدة: ساعتان　　الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I - (2 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of the question and give, **with justification**, its corresponding answer.

N^o	**Questions**	**Proposed answers**		
		a	**b**	**c**
1	$\frac{1}{3}-\frac{1}{3}\times\frac{6}{7}=$	0	$\frac{1}{21}$	$\frac{6}{7}$
2	$\left(3+\sqrt{5}\right)^2-14=$	$9+\sqrt{5}$	0	$6\sqrt{5}$
3	The five grades of a student over 20 are: 10 ; 12 ; 13 ; 16 and 19. The average grade is:	13	14	14.5
4	$\begin{array}{\|c\|c\|}\hline x & \sqrt{2} \\ \hline \sqrt{2} & 4 \\ \hline\end{array}$ The table above is a table of proportionality for $x=$	4	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$

II - (3.5 points)

Given $A(x)=(3x-2)^2-(2x-1)(3x-2)$ and $B(x)=9x^2-4$.

1) a. Verify that $A(x)=(3x-2)(x-1)$.

b. Solve the equation $A(x)=0$.

2) Factorize $B(x)$.

3) Let $F(x)=\dfrac{(3x-2)(3x+2)}{A(x)}$.

a. For what values of x, is F(x) defined?
b. Simplify F(x).
c. Does the equation $F(x)=-12$ admit a solution? Justify.

III - (3.5 points)

1) Solve the following system: $\begin{cases}2x+5y=50\,000\\2x+3y=38\,000\end{cases}$

2) In a museum, 2 adults and 5 kids buy tickets and pay 50 000 LL;
4 adults and 6 kids pay 76 000 LL.

a. Prove that the previous information is modeled by the system given in question **1)**.

b. Find the price of the ticket of an adult and that of a kid.

3) For a group of 30 kids and 4 adults, the director of the museum decided to offer a reduction of 25% on the total amount paid for the tickets. Calculate then the amount paid.

IV - (5.5 points)

In an orthonormal system of axes x'Ox and y'Oy, consider the points $A(-1\,;0)$ and $B(1\,;4)$.

Let (d) be the line with equation $y=2x+2$.

1) **a.** Verify that A and B are two points on line (d).

b. Plot the points A and B then draw (d).

2) Let I be the point of intersection of (d) with y'Oy.

a. Calculate the coordinates of I.

b. Verify that I is the midpoint of [AB].

3) Let (d') be the perpendicular bisector of [AB].Verify that the equation of (d') is $y=-\frac{1}{2}x+2$.

4) Consider the point $M(4\,;0)$. Show that triangle MAB is isosceles of vertex M.

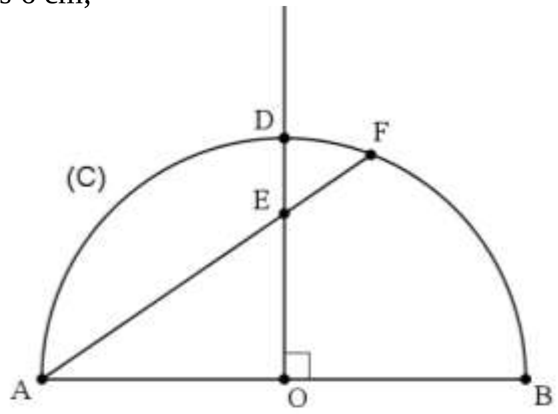
5) Let K be the translate of B under the translation with vector $\overrightarrow{MA}$.

Prove that quadrilateral MBKA is a rhombus.

V- (5.5 points)

In the adjacent figure:

- (C) is a semicircle of diameter [AB], with center O and radius 6 cm;
- The perpendicular bisector of [AB] intersects (C) at D;
- E is a point on segment [OD] so that $OE=4$ cm;
- (AE) intersects (C) at F.



1) Reproduce the figure.

2) Verify that $AE=2\sqrt{13}$ cm.

3) **a.** Prove that AFB is a right triangle at F.

b. Prove that the two triangles AOE and AFB are similar.

c. Deduce the value of $AE\times AF$.

4) The line (BF) intersects line (OD) at K and the line (BE) intersects line (AK) at I.

a. Prove that line (BE) is perpendicular to line (AK).

b. Deduce that I is a point on (C).

5) The tangent to (C) at A intersects (BE) at S.

a. Show that E is the midpoint of [BS].

b. Verify that $BS=4\sqrt{13}$ cm.

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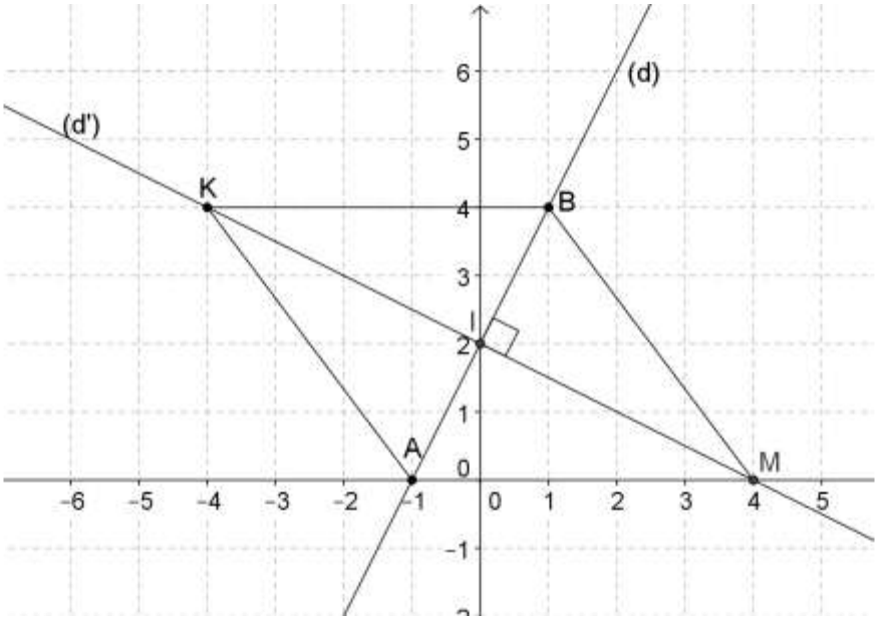
دورة العام ٢٠١٨ العادية
السبت ٢ حزيران ٢٠١٨

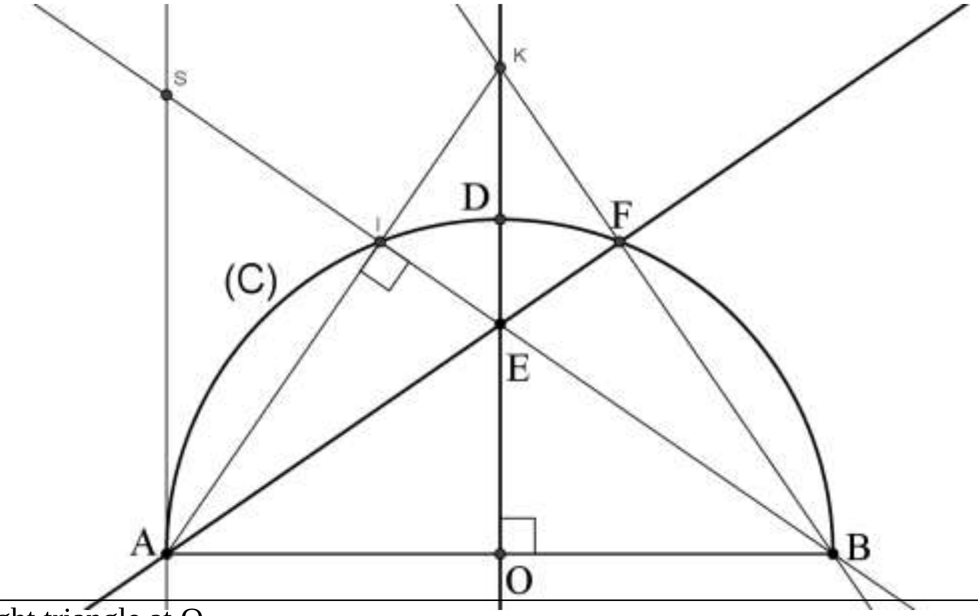
عدد المسائل: خمس

مسابقة في مادة الرياضيات

مشروع اسس التصحيح - انكليزي

Part	Answer Key	Notes
Question I		
1	$\frac{1}{3}-\frac{1}{3}\times\frac{6}{7}=\frac{1}{3}-\frac{2}{7}=\frac{7-6}{21}=\frac{1}{21}$ (b) **0.25 + 0.25**	**0.5**
2	$(3+\sqrt{5})^2-14=9+6\sqrt{5}+5-14=6\sqrt{5}$ (c) **0.25 + 0.25**	**0.5**
3	The average grade $=\frac{10+12+13+16+19}{5}=14$ (b) **0.25 + 0.25**	**0.5**
4	$\frac{x}{\sqrt{2}}=\frac{\sqrt{2}}{4}$ so : $4x=2$; $x=\frac{1}{2}$ (b) **0.25 + 0.25**	**0.5**
Question II		
1	$A(x)=(3x-2)^2-(2x-1)(3x-2)$ $A(x)=(3x-2)[(3x-2)-(2x-1)]$ **0.25** $A(x)=(3x-2)(3x-2-2x+1)$ **0.25** $A(x)=(3x-2)(x-1)$ **0.25**	**0.75**
2a	$A(x)=0$ $(3x-2)(x-1)=0$ gives $3x-2=0$ or $x-1=0$ **0.25** $x=\frac{2}{3}$ or $x=1$ **0.25**	**0.5**
2b	$B(x)=9x^2-4=(3x-2)(3x+2)$	**0.5**
3a	$F(x)=\frac{B(x)}{A(x)}=\frac{(3x-2)(3x+2)}{(3x-2)(x-1)}$; F(x) is defined when $x\neq\frac{2}{3}$ and $x\neq1$ **0.25 + 0.25**	**0.5**
3b	$F(x)=\frac{3x+2}{x-1}$	**0.25**
3c	$F(x)=-12$ **0.25 + 0.5 + 0.25** $\frac{3x+2}{x-1}=-12$ gives $x=\frac{10}{15}=\frac{2}{3}$ (rejected). $F(x)=-12$ does not admit a solution.	**1**
Question III		
1	$\begin{cases}2x+5y=50\,000\\2x+3y=38\,000\end{cases}$ $x=10\ 000$; $y=6\ 000$	**1**
2a	Let x be the price of the ticket of an adult **0.25** And y the price of the ticket of a kid **0.25** $\begin{cases}2x+5y=50\,000\\ \div2\ \ 4x+6y=76\,000\end{cases}$ implies $\begin{cases}2x+5y=50\,000\\2x+3y=38\,000\end{cases}$ **0.25 + 0.25**	**1**
2b	Using question 1) ; $x=10\ 000$; $y=6\ 000$. The price of the ticket of an adult is 10 000LL and that of a kid is 6 000 LL. **0.25 + 0.25**	**0.5**
3	$30\times6\,000+4\times10\,000=220\,000$ LL **0.5** $220\,000\times0.75=165\,000$ LL **0.5** After the reduction, the amount paid is 165 000 LL.	**1**

	Question IV	
1a	$y_A = 2x_A + 2$ and $y_B = 2x_B + 2$ $4 = 2(1) + \mathbf{2}$ $0 = 2(-1) + 2$ **0.25 + 0.25** $4 = 4$ $0 = 0$	**0.5**
1b	 **0.25** **0.25** **0.25**	**0.75**
2a	$I \in y'oy$ then $x_I = 0$ **0.25** $I \in (d)$ then $y_I = 2x_I + 2 = 2(0) + 2 = 2$ **0.5** So $I(0; 2)$	**0.75**
2b	$x_I = \frac{x_A+x_B}{2}$ $0 = \frac{-1+1}{2}$ **0.25** $0 = 0$ $y_I = \frac{y_A + y_B}{2}$ $2 = \frac{0+4}{2}$ **0.25** $2 = 2$ then I is the midpoint of [AB].	**0.5**
3	(d’) is the perpendicular bisector of [AB] then (d’) is perpendicular at the midpoint of [AB]. $(d') \perp (d)$ then $a_{(d)} \times a_{(d')} = -1$ so $a_{(d\prime)} = -\frac{1}{2}$ **0.25 + 0.25** $I \in (d')$ then $y_I = -\frac{1}{2}x_I + b$ so b = 2. $(d'): y = -\frac{1}{2}x + 2$ **0.25 + 0.25**	**1**
4	$M \in (d')$ since $y_M = -\frac{1}{2}x_M + 2$ so MA =MB since any point on the perpendicular bisector is equidistant from the extremities of this segment **0.5 + 0.5** Or MA = 5 and MB = 5….	**1**
5	$\overrightarrow{BK} = \overrightarrow{MA}$ (definition of translation) so MBKA is a parallelogram. **0.5** Moreover: MA = MB then MBKA is a rhombus. **0.5**	**1**

Question V		
1		**0.5**
2	OEA is a right triangle at O. **0.25** $AE^2 = OA^2 + OE^2 = 36 + 16 = 52$ (Pythagorean) **0.25** $AE = \sqrt{52} = 2\sqrt{13}$ cm **0.25**	**0.75**
3a	AFB is a right triangle at F since it is inscribed in a semicircle of diameter [AB]	**0.5**
3b	In the two triangles AOE and AFB: $\widehat{AOE} = \widehat{AFB} = 90^0$ **0.5** $\widehat{OAE} = \widehat{FAB}$ (common angle) **0.25** Therefore they are similar.	**0.75**
3c	$\frac{OA}{AF} = \frac{AE}{AB} = \frac{OE}{FB}$ (ratio of similitude) **0.25** $AE \times AF = AO \times AB = 6 \times 12 = 72$ **0.25**	**0.5**
4a	In triangle AKB we have: [AF] is the first height**.** **0.25** [KO] is the second height. **0.25** [AF] and [KO] intersect at E, the orthocenter of this triangle. [BS] passes through E, so it is the third height. Therefore: (BE) ⊥ (AK) **0.25**	**0.75**
4b	AIB is a right triangle at I so it is inscribed in the semicircle of diameter [AB]. Hence I is a point on (C).	**0.5**
5a	In triangle ASB, we have: O is the midpoint of [AB] **0.25** (OE) // (AS) (Two lines perpendicular to the same line) **0.25** Hence E is the midpoint of [BS] (Converse of midsegment theorem) **0.25**	**0.75**
5b	$AE = \frac{BS}{2}$ (Median relative to the hypotenuse) **0.25** Then: BS = 2 AE = $4\sqrt{13}$ cm. **0.25**	**0.5**