عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the point E(–2 ; 0 ; 1) and the line (d) defined as $x = m - 1$, $y = 2m$, $z = m + 2$ where $m \in \mathbb{R}$.

1) a- Verify that E is not on (d).
   b- Show that $x - z + 3 = 0$ is an equation of plane (P) determined by E and (d).
2) Consider in the plane (P) the circle (C) with center I(–3 ; –1 ; 0) and radius $\sqrt{3}$.
   a- Show that line (d) is tangent to circle (C) at point F(–2 ; –2 ; 1).
   b- Verify that E is on (C) and determine the coordinates of point A on (d) so that (AE) is tangent to (C).
3) Denote by (Δ) the line perpendicular to (P) at I.
   a- Write a system of parametric equations of (Δ).
   b- Calculate the coordinates of point M on (Δ) so that the volume of tetrahedron MIEF is equal to 2 cubic units. $\left(x_M \neq 0\right)$

II- (3 points)

Consider a fair cubic die numbered from 1 to 6 and two urns U_1 and U_2.
U_1 contains 4 blue balls, 3 red balls and 1 green ball.
U_2 contains 4 blue balls, 2 red balls and 2 green balls.
A game consists of rolling the die once.

- If the die shows the face numbered 1 or 2, then three balls are randomly and simultaneously selected from U_1,
- Otherwise, three balls are randomly and simultaneously selected from U_2.

Consider the follwing events:
A : « the die shows the face numbered 1 or 2»
B : « the three selected balls have the same color »
C : « no red ball is obtained among the three selected balls »

1) a- Calculate the probability $P\left(B/A\right)$ and show that $P\left(A \cap B\right) = \frac{5}{168}$.
   b- Calculate P(B).
2) a- Verify that $P(C) = \frac{25}{84}$.
   b- Knowing that no red ball is obtained among the three selected balls, calculate the probability that the die shows a face with number greater than or equal to 3.
3) Let X be the random variable equal to the number of green balls obtained among the three selected balls.
   a- Determine the probability distribution of X.
   b- If this game is repeated 160 times, estimate then the number of green balls obtained.

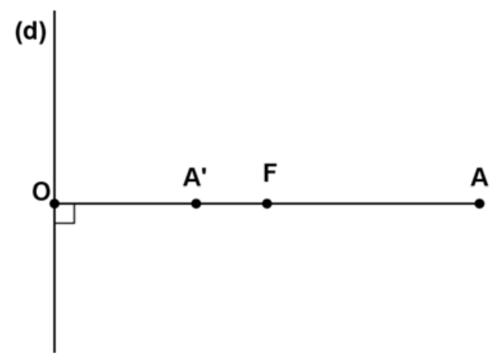
III- (2 points)

Consider the sequence (U_n) defined as : $U_n = \int_0^1 \frac{x^{2n}}{1+x^2}dx$ where $n \in \mathbb{N}$.

1) a- Calculate U_0.
   b- Calculate $U_0 + U_1$ and deduce U_1.
2) a- For all $n \in \mathbb{N}$, show that $U_n \geq 0$.
   b- For all $0 \leq x \leq 1$, prove that (U_n) is decreasing.
   c- Deduce that (U_n) is convergent.
3) a- For all $n \in \mathbb{N}$, show that $U_{n+1} + U_n = \frac{1}{1+2n}$.
   b- Deduce the limit of U_n as n tends to $+\infty$.

IV- (3 points)

Consider in a plane (P) a line (d) and a point F.
Let O be the orthogonal projection of F on (d) with OF = 3.
Let A be the symmetric of O with respect to F and A′ the point on segment [OF] such that OA′ = 2.
In the plane (P), consider the ellipse (E) with focus F, associated directrix (d) and eccentricity $\frac{1}{2}$.



Part A

1) a- Verify that A and A′ are two vertices of (E).
   b- Determine the center I of (E) and its second focus G.
2) Denote by B and B′ the vertices of (E) on the non-focal axis.
   a- Calculate AA′ and verify that BB′ = $2\sqrt{3}$.
   b- Draw (E).

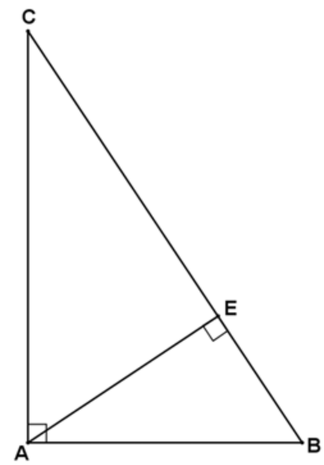
Part B

The plane (P) is referred to an orthonormal system $(O; \vec{i}, \vec{j})$ such that $\vec{i} = \frac{1}{3}\overrightarrow{OF}$.

1) Verify that an equation of (E) is: $3x^2 + 4y^2 - 24x + 36 = 0$.
2) Let L be the point of (E) with abscissa 3 $(y_L > 0)$.
   a- Write an equation of (T), the tangent at L to (E).
   b- Denote by K the point of intersection of (T) with the non-focal axis of (E).
      Calculate the area of the region inside triangle OIK and outside ellipse (E).

V- (4 points)

In an oriented plane, consider a triangle ABC right angled at A such that AB = 4 , AC = 6 and $(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{\pi}{2}\ [2\pi]$.



Denote by E the orthogonal projection of point A on line (BC).

Let S be the direct plane similitude that maps B onto A and A onto C.

1) Calculate the ratio (scale factor) k of S and find a measure of angle α of S.
2) a- Determine the image of line (AE) under S and the image of line (BC) under S.

   b- Deduce that E is the center of S.
3) Let F = S(C).

   a- Prove that A, E and F are collinear.

   b- Show that (CF) is parallel to (AB).

   c- Construct F and calculate CF.
4) Denote by h the dilation that maps A onto B and with ratio $\frac{-1}{3}$.

   a- Determine $S \circ h(A)$.

   b- $S \circ h$ is a direct plane similitude.
   Determine its center, its ratio and a measure of its angle.
5) The complex plane is referred to a direct orthonormal system $(A; \vec{u}, \vec{v})$ with $\vec{u} = \frac{1}{4}\overrightarrow{AB}$ and $\vec{v} = \frac{1}{6}\overrightarrow{AC}$.

   a- Write the complex form of $S \circ h$.

   b- Calculate the affix of point $B' = S \circ h(B)$.

   c- Let (P) be the parabola with vertex A and focus B and (P′) be the image of (P) under $S \circ h$.
   Write an equation of (P′).

VI- (6 points)

Part A

1) Verify that $\int \ln x dx = x\ln x - x + k$ where k is a real constant and $x > 0$.
2) Consider the differential equation (E) satisfied by y : $xy' + y = -1 - 2x - 2\ln x$ where y is a function of x ($x > 0$).

   Let $z = x\,y$.

   a- Form a differential equation (E′) satisfied by z and solve (E′).

   b- Deduce the particular solution of (E) such that $y(1) = 0$.

Part B

Consider the two functions g and f defined over $]0\,;+\infty[$ as $g(x) = 1 - x - 2\ln x$

and $f(x) = \dfrac{x + \ln x}{x^2}$ and denote by (C) the representative curve of f in an orthonormal

system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{\substack{x \to 0 \\ x > 0}} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$.

   b- Calculate $g'(x)$ and set the table of variations of g.

   c- Calculate $g(1)$, then discuss according to the values of x the sign of $g(x)$.
2) Determine $\lim\limits_{\substack{x \to 0 \\ x > 0}} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$. Deduce the asymptotes to (C).
3) Show that $f'(x) = \dfrac{g(x)}{x^3}$ and set up the table of variations of f.
4) Calculate the exact value of f(e) and draw the curve (C).
5) Use integration by parts to calculate $\int \dfrac{\ln x}{x^2} dx$.
6) a- For $x \in [1;+\infty[$, prove that the function f has an inverse function f^{-1} whose domain of definition is to be determined.

   b- Draw (Γ), the representative curve of f^{-1} in the same system as that of (C).

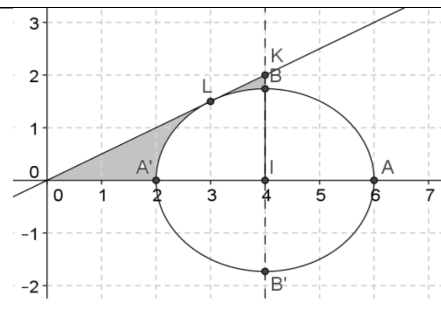
   c- Calculate the area of the region bounded by (Γ) and the three lines with equations $y = 1$, $x = \dfrac{e+1}{e^2}$ and $x = 1$.

عدد المسائل: ست — أسس تصحيح مادة الرياضيات

QI	Answers	Mark
1a	$m-1=-2$; $m=-1$ $y=2(m)=2(-1)=-2\neq y_E=0$ then $E\notin(d)$	0.25
1b	$E\in(P)$ and $(d)\subset(P)$.	0.5
2a	• $F\in(d)\subset(P)$ for $m=-1$ • $IF=\sqrt{3}$ = Radius • $\overrightarrow{IF}\cdot\overrightarrow{V_d}=0$	1
2b	• $IE=\sqrt{3}$ = Radius and $E\in(P)$ then $E\in(C)$ • $A(m-1\ ;\ 2m\ ;\ m+2)$ and $\overrightarrow{AE}\cdot\overrightarrow{IE}=0$ then $m=-\frac{1}{2}$ thus $A\left(-\frac{3}{2};-1;\frac{3}{2}\right)$	1
3a	$(\Delta)\perp(P)$ then $\overrightarrow{V}_{(\Delta)}=\overrightarrow{n}_{(P)}$ and $I\in(\Delta)$ thus $(\Delta):\begin{cases}x=t-3\\y=-1\\z=-t\end{cases}$; $t\in\mathbb{R}$	0.5
3b	$M(t-3\ ;-1\ ;-t)$ $\det\left(\overrightarrow{IM},\overrightarrow{IE},\overrightarrow{IF}\right)=\lvert 4t\rvert$ $V=\frac{1}{6}\left\lvert\det\left(\overrightarrow{IM},\overrightarrow{IE},\overrightarrow{IF}\right)\right\rvert=\frac{1}{6}\lvert 4t\rvert=2$ then $t=-3$ ou $t=3$ For $t=-3$, $M(-6\ ;-1\ ,3)$	0.75

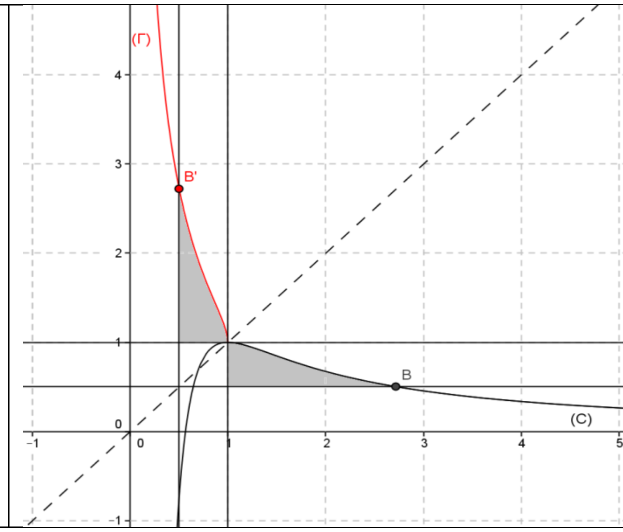
QII	Answers	Mark
1a	$P\left(B/A\right)=\frac{C_3^3+C_4^3}{C_8^3}=\frac{5}{56}$; $P(A\cap B)=P(A)\times P\left(B/A\right)=\frac{1}{3}\times\frac{5}{56}=\frac{5}{168}$	1
1b	$P(B)=P(A\cap B)+P(\overline{A}\cap B)=\frac{5}{168}+P(\overline{A})\times P\left(B/\overline{A}\right)=\frac{5}{168}+\frac{2}{3}\times\frac{C_4^3}{C_8^3}=\frac{13}{168}$	1
2a	$P(C)=P(A\cap C)+P(\overline{A}\cap C)=\frac{1}{3}\times\frac{C_5^3}{C_8^3}+\frac{2}{3}\times\frac{C_6^3}{C_8^3}=\frac{25}{84}$	1
2b	$P\left(\overline{A}/C\right)=\frac{P(\overline{A}\cap C)}{P(C)}=\frac{\frac{40}{168}}{\frac{25}{84}}=\frac{4}{5}$	1
3a	The values of X are 0, 1 and 2. $P(X=0)=\frac{1}{3}\times\frac{C_7^3}{C_8^3}+\frac{2}{3}\times\frac{C_6^3}{C_8^3}=\frac{75}{168}$; $P(X=1)=\frac{1}{3}\times\frac{C_7^2\times C_1^1}{C_8^3}+\frac{2}{3}\times\frac{C_6^2\times C_2^1}{C_8^3}=\frac{81}{168}$ $P(X=2)=\frac{2}{3}\times\frac{C_6^1\times C_2^2}{C_8^3}=\frac{12}{168}$	1
1a	$E(X)=\frac{5}{8}$ then the estimated number of green balls is $\frac{5}{8}\times160=100$.	1

QIII	Answers	Mark
1a	$U_0 = \int_0^1 \frac{1}{1+x^2}dx = \arctan x\big]_0^1 = \frac{\pi}{4}$	**0.5**
1b	$U_0 + U_1 = \int_0^1 \frac{1}{1+x^2}dx + \int_0^1 \frac{x^2}{1+x^2}dx = \int_0^1 \frac{1+x^2}{1+x^2}dx = x\big]_0^1 = 1$; $U_1 = 1 - U_0 = 1 - \frac{\pi}{4}$	**0.75**
2a	$\frac{x^{2n}}{1+x^2} \geq 0$ for $0 \leq x \leq 1$; then $U_n \geq 0$	**0.5**
2b	$U_{n+1} - U_n = \int_0^1 \frac{x^{2n+2} - x^{2n}}{1+x^2}dx = \int_0^1 \frac{x^{2n}\left(x^2 - 1\right)}{1+x^2}dx$ If $0 \leq x \leq 1$, then $0 \leq x^2 \leq 1$, thus $x^2 - 1 \leq 0$ Therefore $U_{n+1} - U_n \leq 0$ then (U_n) is decreasing	**0.75**
2c	(U_n) is decreasing and bounded below by 0, then (U_n) is convergent.	**0.5**
3a	$U_{n+1} + U_n = \int_0^1 \frac{x^{2n+2} + x^{2n}}{1+x^2}dx = \int_0^1 \frac{x^{2n}\left(x^2 + 1\right)}{1+x^2}dx = \int_0^1 x^{2n}dx = \left[\frac{x^{2n+1}}{2n+1}\right]_0^1 = \frac{1}{2n+1}$	**0.5**
3b	Let $L = \lim\limits_{n\to+\infty} U_n = \lim\limits_{n\to+\infty} U_{n+1}$, then $L + L = \lim\limits_{n\to+\infty} \frac{1}{2n+1} = 0$. Thus $L = 0$	**0.5**

QIV	Answers	Mark
A1a	$\frac{AF}{AO} = \frac{1}{2} = e$ and $A \in (OF)$ = focal axis, then A is a vertex of (E). $\frac{A'F}{A'O} = \frac{OF - OA'}{OA'} = \frac{1}{2} = e$ and $A' \in (OF)$ = focal axis, then A′ is a vertex of (E).	**0.5**
A1b	I is the midpoint of [AA′] ; G is the symmetric of F with respect to I	**0.5**
A2a	AA′ = A′F + FA = 1 + OF = 4 = 2a, then a = 2 FG = 2FI = 2(A′I – A′F) = 2 = 2c, then c = 1 $BB' = 2b = 2\sqrt{a^2 - c^2} = 2\sqrt{3}$	**1**
A2b		**1**
B1	a = 2; b = $\sqrt{3}$; focal axis is the abscissa axis; center I(4 ; 0) (E) : $\frac{(x-4)^2}{4} + \frac{y^2}{3} = 1$, thus $3x^2 + 4y^2 - 24x + 36 = 0$	**1**
B2a	$L\left(3\,;\frac{3}{2}\right)$; $y'_L = \frac{1}{2}$; $(T) : y = \frac{x}{2}$	**1**
B2b	K(4 ; 2) ; Area = Area(Triangle OIK) – $\frac{1}{4}$ Area(E) = $\frac{1}{2} \times OI \times IK - \frac{1}{4} \times \pi ab$ = $4 - \frac{\pi\sqrt{3}}{2}$ units of area.	**1**

QV	Answers	Mark
1	$k = \frac{AC}{BA} = \frac{3}{2}$ and $\alpha = (\overrightarrow{BA};\overrightarrow{AC}) = -\frac{\pi}{2}\ (2\pi)$	**0.5**
2a	S(A) = C, then the image of (AE) is a line passing through C and perpendicular to (AE), which is (BC). S(B) = A, then the image of (BC) is a line passing through A and perpendicular to (BC), which is (AE).	**1**
2b	$\{E\} = (AE) \cap (BC)$, then $\{S(E)\} = S((AE)) \cap S((BC)) = (BC) \cap (AE) = \{E\}$	**0.5**
3a	S(B) = A; S(C) = F; S(E) = E B, C and E are collinear, then A, F and E are collinear	**0.5**
3b	S(A) = C and S(C) = F, then $(CF) \perp (AC)$ and since $(AB) \perp (AC)$, Thus (CF) / / (AB)	**1**
3c	F is the common point between the parallel drawn from C to (AB) and (AE). S(A) = C and S(C) = F, then CF = k AC = 9	**1**
4a	$S \circ h(A) = S(h(A)) = S(B) = A$	**0.5**
4b	$S \circ h\left(A; \frac{1}{2}; \frac{\pi}{2}\right)$	**0.5**
5a	$z' = \frac{1}{2} iz$	**0.75**
5b	$z_B = 4$, then $z_{B'} = 2i$	**0.75**
5c	(P′) is a parabola with vertex A(0 ; 0) and focus B′(0 ; 2) $(P') : x^2 = 8y$	**1**

QVI	Answers	Mark
A1	$(x\ln x - x + k)' = \ln x$	**0.5**
A2a	$z = xy$, $z' = y + xy'$ $(E') : z' = -1 - 2x - 2\ln x$; $z = -x^2 + x - 2x\ln x + C$	**1**
A2b	$y = \frac{z}{x} = -x + 1 - 2\ln x + \frac{C}{x}$ y(1) = 0, then C = 0 ; thus $y = 1 - x - 2\ln x$	**0.75**
B1a	$\lim_{\substack{x \to 0 \\ x > 0}} g(x) = +\infty$ and $\lim_{x \to +\infty} g(x) = -\infty$	**0.5**
B1b	$g'(x) = -1 - \frac{2}{x} < 0$ x: 0, 1, $+\infty$ g′(x): $-$ g(x): $+\infty$ ↘ 0 ↘ $-\infty$	**0.75**
B1c	g(1) = 0 g(x) > 0 for 0 < x < 1 g(x) = 0 for x = 1 g(x) < 0 for x > 1	**1**
B2	$\lim_{\substack{x \to 0 \\ x > 0}} f(x) = -\infty$, then x = 0 is an asymptote.	**1**

	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(\frac{1}{x}+\frac{1}{x}\times\frac{\ln x}{x}\right) = 0$ (or using H.R.), then $y = 0$ is an asymptote.	
B3	$f'(x) = \frac{g(x)}{x^3}$ x: 0 ; 1 ; $+\infty$ $f'(x)$: ‖ $+$ 0 $-$ $f(x)$: ‖ $-\infty$ ↗ 1 ↘ 0	**1.25**
B4	$f(e) = \frac{e+1}{e^2}$ 	**1.25**
B5	$\int \frac{\ln x}{x^2}dx = -\frac{1}{x}\ln x - \int -\frac{1}{x^2}dx = -\frac{1}{x}\ln x - \frac{1}{x} + C$	**1**
B6a	f is continuous and strictly decreasing over $[1;+\infty[$, then f has an inverse function f^{-1} whose domain of definition is $]0\ ;\ 1]$	**1**
B6b	(Γ) is the symmetric of (C) with respect to line y = x.	**0.5**
B6c	$A = \int_{\frac{e+1}{e^2}}^{1}\left(f^{-1}(x)-1\right)dx = \int_1^e\left(\frac{1}{x}+\frac{\ln x}{x^2}-\frac{e+1}{e^2}\right)dx = \left[\ln x + \frac{-1-\ln x}{x}\right]_1^e - \frac{e+1}{e^2}\times(e-1) =$ $1-\frac{2}{e}+\frac{1}{e^2} = \left(\frac{e-1}{e}\right)^2 \approx 0.4$ units of area.	**1.5**