

الاسم: مسابقة في مادة الفيزياء
الرقم: المدة: ثلاث ساعات

This exam is formed of four obligatory exercises in 4 pages.
The use of non-programmable calculator is recommended

Exercise 1 (7 ½ points)

Torsion pendulum

Consider a torsion pendulum formed of a homogeneous thin disk (D), suspended from its center O by means of a vertical massless torsion wire while the other end of the wire is fixed to point O' (Document 1).

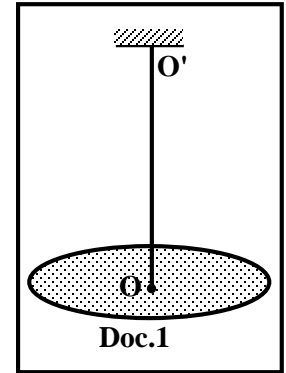
The aim of this exercise is to determine the moment of inertia I of (D) with respect to the axis (OO') and the torsion constant C of the wire.

The disk is in the equilibrium position. The disk is rotated from its equilibrium position in the horizontal plane about the vertical axis (OO'), by an angle θ_m and then it is released from rest at $t_0 = 0$. At an instant t, the angular abscissa of the disk relative to its

equilibrium position is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

Take the horizontal plane passing through the center of mass of the disk as a reference level of gravitational potential energy.

Neglect all frictional forces.



Doc.1

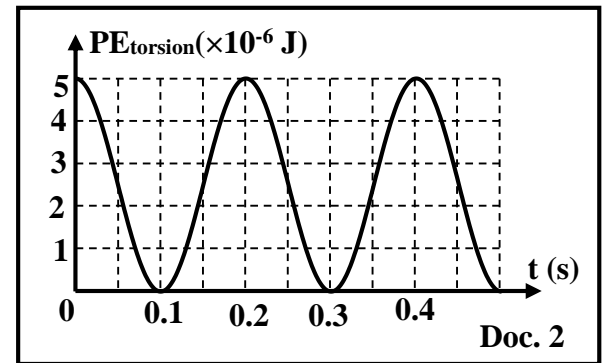
1- Theoretical study

- 1-1) Write, at the instant t, the expression of the mechanical energy ME of the system (torsion pendulum, Earth) in terms of I, C, θ and θ' .
- 1-2) Derive the differential equation in θ that describes the motion of the disk.
- 1-3) Deduce, in terms of C and I, the expression of the proper frequency f_0 .

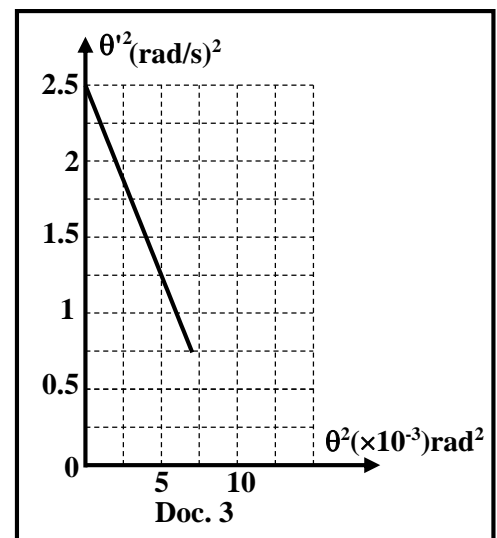
2- Experimental study

An appropriate apparatus allows us to trace the variation of the torsion potential energy of the torsion wire as a function of time as shown in document 2.

- 2-1) Using the graph of document 2:
 - 2-1-1) justify that the pendulum performs un-damped oscillations;
 - 2-1-2) determine the value of f_0 , knowing that $f_E = 2 f_0$, where f_E is the frequency of the torsion potential energy;
 - 2-1-3) determine the value of the mechanical energy ME of the system (torsion pendulum, Earth).
- 2-2) Write, using the expression of the mechanical energy, the expression of θ'^2 in terms of θ , C, I and ME.
- 2-3) The curve of document 3 represents the variation of θ'^2 as a function of θ^2 .
 - 2-3-1) Show that the curve of document 3 is in agreement with the expression of θ'^2 established in part (2-2).
 - 2-3-2) Determine, using the curve of document 3, the value of I.
- 2-4) Determine, by two different methods, the value of C.



Doc. 2



Doc. 3

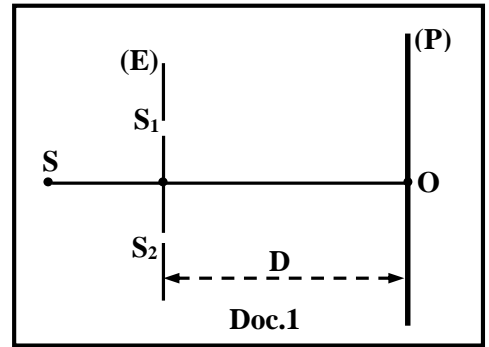
Exercise 2 (7 1/2 points)

Interference of light

Consider Young's double slit apparatus that is represented in document 1. (S_1) and (S_2) are two parallel thin slits separated by a distance $a = S_1S_2$. (P) is the screen of observation that placed parallel to the plane of the slits (E) at a distance D .

(S) is a point source of monochromatic radiation of wavelength λ in air placed on the perpendicular bisector of [S_1S_2].

The aim of this exercise is to determine the expression of the interfringe distance « i ».



1- Interference pattern

1-1) (S_1) and (S_2) satisfy two properties in order to obtain the phenomenon of interference. What are these two properties?

1-2) Describe the interference pattern observed on the screen (P).

2- Expression of the interfringe distance « i »

2-1) Using different monochromatic point sources of different wavelengths, we measure the distance between the centers of the first and the eleventh fringe of the same nature, for each wavelength. The results obtained are listed in the table of document 2.

Doc.2						
λ (nm)	400	500	600	650	700	750
10 i (mm)	36	45	54	58.5	63	68.5
i (mm)						

2-1-1) Copy and complete the table of document 2.

2-1-2) Draw the graph representing the variation of the interfringe distance « i » in terms of the wavelength λ using the scale:

On abscissa axis: 1 cm \leftrightarrow 100 nm; On ordinate axis: 1 cm \leftrightarrow 1 mm.

2-1-3) Determine, using the preceding graph, the expression of « i » in terms of λ .

2-2) We propose the following six expressions, for « i » (C is a unit less positive constant).

(a)	(b)	(c)	(d)	(e)	(f)
$i = C \lambda D a$	$i = C \frac{D}{\lambda a}$	$i = C \frac{\lambda D}{a}$	$i = C \lambda \frac{D^2}{a^2}$	$i = C \lambda \frac{a^2}{D^2}$	$i = C \lambda^2 \frac{D}{a}$

2-2-1) Based on the preceding experimental study, the expressions (b) and (f) should be eliminated. Justify.

2-2-2) The analysis of units permits us to eliminate expression (a). Justify.

2-2-3) By increasing the distance D , the interfringe distance « i » also increases. Specify the expression among (c) , (d) and (e), which does not satisfy this result.

2-2-4) To choose the correct expression of « i » between the two remaining expressions; we double the distance D , we notice that « i » is also doubled. Specify the correct expression of « i ».

2-2-5) Deduce the value of C knowing that $D = 1.8$ m and $a = 0.2$ mm.

Exercise 3 (7 1/2 points)

Solar spectrum

In 1814, Fraunhofer discovered the absorption lines present in the solar spectrum. He studied 570 lines and designated the main of these lines by the letters A, B, C, etc...(Doc.1). He aimed to identify the elements in the solar atmosphere.

Doc.1										
Rays	A	B	C	a	D-Doublet		E	F	G	h
Wavelength (nm)	759.370	686.719	657.289	627.661	589.592	588.410	527.039	486.881	434.715	410.805

Given: speed of light in vacuum $c = 2.998 \times 10^8$ m/s; Planck's constant $h = 6.626 \times 10^{-34}$ J.s; $1 \text{ eV} = 1.60 \times 10^{-19}$ J.

The energy levels of the hydrogen atom are given by the relation: $E_n = -\frac{E_0}{n^2}$; with $E_0 = 13.6 \text{ eV}$ and n is a non-zero whole positive number.

1- Solar spectrum

Justify the presence of absorption lines (dark lines) in the solar spectrum.

2- Balmer series of the hydrogen atom

The Balmer series is a series of spectral lines of hydrogen atom. The line « C » of the spectrum of document 1 corresponds to the alpha line (α) of this series. Three other lines beta, gamma and delta (β , γ and δ) of the same series are in this document.

2-1) To what domain, visible, infrared or ultraviolet, do the lines of Balmer series belong?

2-2) Each line of this series corresponds to an absorption from the first excited state E_2 to a higher energy level E_n .

2-2-1) Show that the wavelengths of the lines of this series are given by: $\lambda = \frac{4n^2 hc}{E_0(n^2 - 4)}$.

2-2-2) λ_α , λ_β , λ_γ and λ_δ are the wavelengths of α , β , γ and δ respectively. The line α corresponds to $n = 3$. Indicate the values of n corresponding to the other three lines beta, gamma and delta and calculate the values of the wavelengths λ_β , λ_γ , λ_δ , knowing that $\lambda_\beta > \lambda_\gamma > \lambda_\delta$.

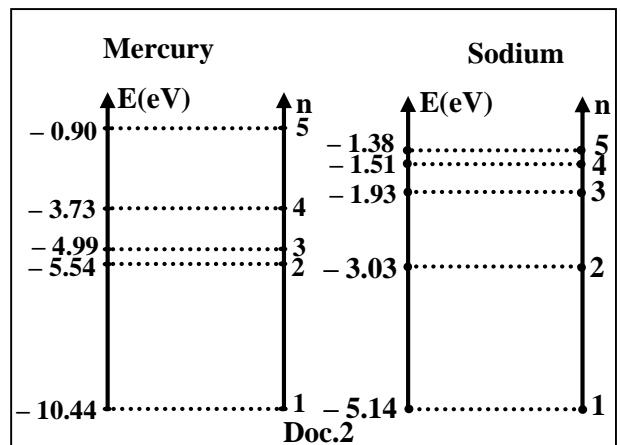
2-2-3) Deduce, using document 1, which lines in the solar spectrum are those of Balmer series.

3- D-doublet of an atom

The D-doublet in document 1 corresponds to the transition of a certain atom from its fundamental state to the first excited state.

3-1) Calculate the energy of each photon corresponding to each line of the D-doublet in document 1.

3-2) Document 2 shows two simplified diagrams of the energy levels of sodium and mercury atoms. Show that one of the lines of the D-doublet corresponds to one of the two atoms.



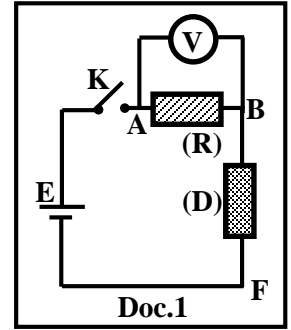
Exercise 4 (7 1/2 points)

Pacemaker

The aim of this exercise is to identify an electric component (D), and to study its usage in medicine. (D) may be a resistor, a coil with negligible resistance or a capacitor.

1- Identification of the component (D)

The component (D) is connected in series with a resistor of resistance $R = 8 \times 10^5 \Omega$ across an ideal DC generator of e.m.f. E . A voltmeter (V) is connected across R to measure the voltage $u_R = u_{AB}$ as shown in document 1. The switch K is closed at an instant $t = 0$ and the readings of the voltmeter are tabulated in the table below:

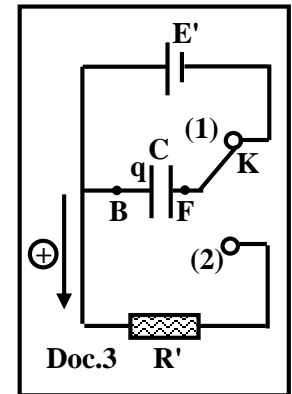


Doc. 2									
t(s)	0	0.4	0.8	1.2	1.6	2	2.4	2.8	3.2
u_R (V)	12	7.28	4.44	2.68	1.62	1	0.6	0.36	0.2

- 1-1) Show that, using document 2, (D) is a capacitor.
- 1-2) Deduce the value of E .
- 1-3) Let $u_C = u_{BF}$ be the voltage across the capacitor at an instant t . Calculate the ratio $\frac{u_C}{E}$ at $t = 0.8$ s.
- 1-4) Deduce, referring to document 2, the value of the time constant τ of the circuit.
- 1-5) Show that the capacitance of the capacitor is $C = 1 \mu F$.

2- Usage of the capacitor in medicine: Pacemaker

When the human heart does not function correctly, the surgery permits to implant in the human body an artificial stimulator called pacemaker sending artificial electric pulses to the heart. This pacemaker can be modeled by an electric circuit as shown in document 3. This circuit consists of:



An ideal DC generator of e.m.f. E' , a resistor of resistance R' , the capacitor of capacitance $C = 1 \mu F$ and an electronic double switch (K).

At $t = 0$ the switch is at position 1, the capacitor is totally charged instantly, then the switch is turned automatically to position 2 and the capacitor discharges slowly through R' . At an instant t_1 the voltage across the capacitor is $u_C = u_{BF} = 2.08$ V, the circuit sends an electric pulse to the heart to get one beat, at this instant the switch turns automatically to the position 1 and so on (Doc. 4).

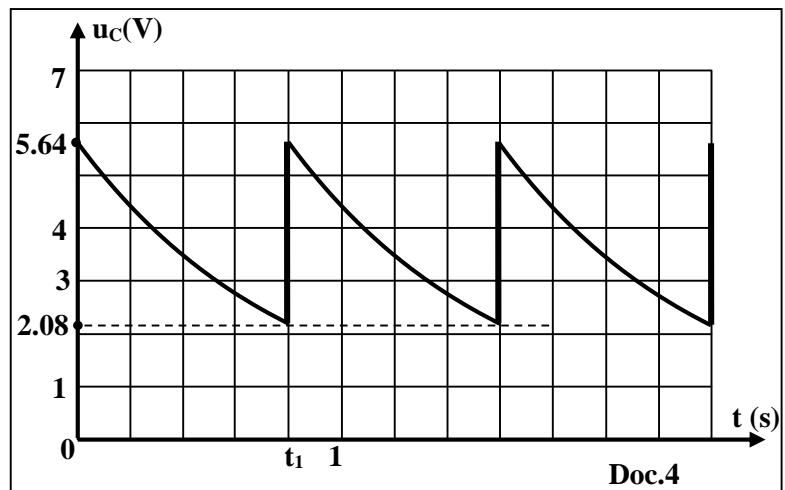
- 2-1) Establish the differential equation of $u_C = u_{BF}$ during the discharging.
- 2-2) The solution of the obtained differential

equation has the form of $u_C = a + b e^{-\frac{t}{\tau}}$. Determine the expressions of the constants a , b and τ , in terms of R' , E' and C .

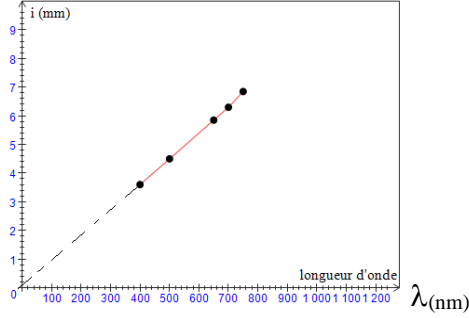
- 2-3) Determine graphically the value of τ .
- 2-4) Deduce the value of R' .

3- Heart beat

- 3-1) Indicate, referring to document 4, the value of t_1 .
- 3-2) Deduce the duration Δt separating two successive pulses.
- 3-3) Deduce the number of beats of the heart per one minute.



Exercise 1 : torsion pendulum		7½		
1	1-1	$ME = \frac{1}{2}I\theta'^2 + \frac{1}{2}C\theta^2$	1	
	1-2	Since no friction (or work done by the non conservative forces is zero) then $ME = \text{const, so } \frac{dME}{dt} = 0 ; I\theta'\theta'' + C\theta\theta' = 0 \Rightarrow \theta'' + \frac{C}{I}\theta = 0$	1	
	1-3	The differential equation is of the form : $\theta'' + \omega_0^2\theta = 0 ; \omega_0 = \sqrt{\frac{C}{I}}$ $\omega_0 = 2\pi f_0 , \text{ then } f_0 = \frac{1}{2\pi} \sqrt{\frac{C}{I}}$	1	
2	2-1	2-1-1	$(E_{\text{ptorsion}})_{\text{max}} = \text{constant}$ so the oscillations are undamped	¼
		2-1-2	Since $f_E = 2f_0$, then $= 2T_E = 2 \times 0.2 = 0.4 \text{ s} ; f_0 = \frac{1}{T_0} = \mathbf{2.5 \text{ Hz}}$	1
		2-1-3	$ME = (E_{\text{ptorsion}})_{\text{max}} = 5 \times 10^{-6} \text{ J.}$	½
	2-2	$ME = \frac{1}{2}I\theta'^2 + \frac{1}{2}C\theta^2 ; \theta'^2 = -\frac{C}{I}\theta^2 + \frac{2ME}{I}$	½	
	2-3	2-3-1	Decreasing straight line does not passing through the origin that is compatible with the expression of θ'^2 that is of the form ; $y = -bx + c$	½
		2-3-2	for $\theta = 0 ; \theta'^2 = \frac{2ME}{I} = 2.5$ so $I = \frac{2ME}{\theta'^2} = \frac{2 \times 5 \times 10^{-6}}{2.5} = \mathbf{4 \times 10^{-6} \text{ kg. m}^2}$	½
2-4	First method : $f_0 = \frac{1}{2\pi} \sqrt{\frac{C}{I}}$, so $C = \mathbf{10^{-3} \text{ N.m/rad}}$ Second method : slope of the straight line = $-\frac{C}{I} = -\frac{2.5}{0.01} = -250 ;$ Therefore, $C = 250 \times 4 \times 10^{-6} = \mathbf{10^{-3} \text{ N.m/rad}}$	1 ¼		

Exercise 2 : Interference of light (7.5 pts)				
1		1.1	(S ₁) and (S ₂) are two synchronous and coherent secondary sources	0.5
		1.2	Rectilinear band, bright and dark, their centers are equidistant and parallel to the slits.	1
2	2.1	2.1.1	3.6 – 4.5 – 5.4 – 5.85 – 6.3 – 6.85	0.5
		2.1.2		1.25
		2.1.3	The form of $i(\lambda)$ is a straight line and passing through the origin slope $\alpha = 9000$ so $i = 9000 \lambda$.	0.25 0.75
	2.2	2.2.1	i is directly proportional to λ . In expression (b): i and λ are inversely proportional;	0.5
			In expression (f): i is proportional to the square of λ .	0.5
2.2.2		In expression (a): The unit of i is m^3	0.5	
2.2.3		Expression (e) since: as D increases, i decreases.	0.5	
2.2.4		The interfringe- distance and D are proportional that is satisfied in the equation (c).	0.5	
	2.2.5	For any couple of values in the table : (500 nm, 4.5 mm) $C = \frac{i \times a}{\lambda D} = \frac{4.5 \times 10^{-3} \times 0.2 \times 10^{-3}}{500 \times 10^{-9} \times 1.8} = 1$	0.75	

Exercise 3 : Solar spectrum (7.5 pts)			7 ½
1		the presence of the lines in the absorption spectrum is due to the absorption of photons of the gas in the atmosphere (each missing line corresponds to a certain transition in an atom of the gas in the atmosphere from a lower level to a higher level).	0.5
2	2.1	visible	0.5
2.2	2.2.1	$E_{\text{photon}} = E_n - E_2$, so $\frac{hc}{\lambda} = \frac{-E_0}{n^2} + \frac{-E_0}{2^2}$, then $\frac{1}{\lambda} = \frac{E_0}{hc} \left(\frac{1}{4} - \frac{1}{n^2} \right)$ Therefore, $\lambda = \frac{4n^2 hc}{E_0(n^2 - 4)}$	1.5
	2.2.2	line $\alpha \rightarrow n = 3$ (given) ; line $\beta \rightarrow n = 4$; line $\gamma \rightarrow n = 5$; line $\delta \rightarrow n = 6$ $\lambda_\alpha = 657.289 \text{ nm}$ (given); $\lambda_\beta = 486.881 \text{ nm}$; $\lambda_\gamma = 434.715 \text{ nm}$; $\lambda_\delta = 410.805 \text{ nm}$	1.5
	2.2.3	line $\alpha \rightarrow C$; line $\beta \rightarrow F$; line $\gamma \rightarrow G$; line $\delta \rightarrow h$	0.75
3	3.1	for $\lambda = 589.592 \text{ nm}$; $E_{\text{photon}} = \frac{hc}{\lambda} = \mathbf{2.10 \text{ eV}}$. for $\lambda = 588.410 \text{ nm}$; $E_{\text{photon}} = \mathbf{2.11 \text{ eV}}$.	1.5
	3.2	Mercury : $E_2 - E_1 = 4.9 \text{ eV}$ Sodium : $E_2 - E_1 = 2.11 \text{ eV} = E_{\text{photon}}$ So the gas is the sodium .	1.25

Exercise 4 Pacemaker (7.5 pts)			
1	1-1	u_D increase with time, $u_D = E - u_R$	0.5
	1-2	$E = 12 \text{ V}$ at $t = 0$ $u_R = 12 \text{ V}$ $u_C = 0$	0.5
	1-3	$\frac{u_C}{E} = \frac{12 - 4.44}{12} = 0.63$	0.5
	1-4	$\tau = 0.8 \text{ s}$ since at $t = \tau: u_C = 0.63E$	0.5
	1.5	$\tau = RC$ so $C = \frac{\tau}{R} = \frac{0.8}{8 \times 10^5} = 10^{-6} \text{ F}$	0.75
2	2.1	$U_{BF}(C) = U_{BF}(R)$, then $u_C = Ri$ $i = -\frac{dq}{dt} = -C \frac{du_C}{dt}$, then $u_C + RC \frac{du_C}{dt} = 0$	1
	2.2	$u_C = a + be^{-\frac{t}{\tau'}}$; $\frac{du_C}{dt} = -\frac{b}{\tau'} e^{-\frac{t}{\tau'}}$, so $-\frac{b}{\tau'} e^{-\frac{t}{\tau'}} + \frac{1}{RC} \left(a + be^{-\frac{t}{\tau'}} \right) = 0$ $be^{-\frac{t}{\tau'}} \left(+\frac{1}{RC} - \frac{1}{\tau'} \right) + \frac{a}{RC} = 0$, then $\tau' = R'C$ and $a = 0$ At $t = 0: u_C = E'$, therefore, $b = E'$	1.5
	2-3	Graphically: at $t = \tau'$: $u_C = 0.37 \times E' = 2.086 \text{ V}$, so $\tau' = t_1 = 0.8 \text{ s}$	0.75
	2-4	$R' = \frac{\tau'}{C} = \frac{0.8}{10^{-6}} = 800000 \text{ } \Omega$	0.5
3	3-1	$t_1 = 0.8 \text{ s}$	0.25
	3-2	$\Delta t = \text{time of charging} + \text{time of discharging} = t_1 + 0 = 0.8 \text{ s}$	0.25
	3-3	$N_b = \frac{60}{0.8} = 75$	0.5