وزارة التربية والتعليم العالي   امتحانات الشهادة الثانوية العامة   دورة العام ٢٠١٧ الاستثنائيّة
المديرية العامة للتربية   فرع: علوم الحياة   الثلاثاء في ٨ آب ٢٠١٧
دائرة الامتحانات الرسميّة

عدد المسائل: أربع   مسابقة في مادة الرياضيات   الاسم:
المدة: ساعتان   الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the three points A (9; –1; 4), B(5; 1; 2) , C(3; 2; 2) and the plane (P) with equation x + 2y – 7 = 0 determined by A, B and C.

1) Let (Q) be the plane passing through A and B and perpendicular to plane (P).
   Show that an equation of (Q) is 2x – y – 5z + 1 = 0.
2) Denote by (d) the line of intersection of (P) and (Q).
   Write a system of parametric equations of (d).
3) (L) is the line with parametric equations: $\begin{cases} x=-t+6 \\ y=-2t+3 \\ z=2 \end{cases}$; $(t \in \mathbb{R})$.
   a- Verify that B is on (L).
   b- Verify that (L) is in (Q) and that (L) is perpendicular to (d).
   c- Determine the coordinates of the point E on line (L) such that the area of triangle BCE is equal to 5 square units. $(y_E > 0)$

II- (4 points)

An urn U contains ten balls:

- **five white balls** numbered 1, 2, 3, 4, 5
- **three black balls** numbered 6, 7, 8
- **two green balls** numbered 9, 10.

Part A

A player selects randomly and simultaneously two balls from the urn U.
Consider the following events:

A: “The two selected balls hold odd numbers”
B: “The two selected balls have the same color”
C: “The two selected balls hold odd numbers and have the same color”
D: “The two selected balls hold odd numbers and have different colors”.

1) Calculate the probability P(A) and verify that P(B) = $\frac{14}{45}$.
2) a- Calculate P(C).
   b- Are the events A and B independent? Justify.
3) Verify that P(D) = $\frac{7}{45}$.
4) Knowing that the player has selected two balls with different colors, what is the probability that these two balls hold odd numbers?

Part B

In this part, the player selects randomly, successively and with replacement, two balls from the urn U. The player scores +1 point for each white ball selected, –1 point for each black ball selected and 0 points for each green ball selected.
Calculate the probability that the sum of scored points is equal to zero.

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A, M and M′ with respective affixes 2i, z and z′ such that: $z'=\dfrac{2\,i-z}{iz}$ with $z\neq 0$.

Let B be the midpoint of segment [OA].

1) Write z′ in algebraic form in the case where z = 1 + i.
2) a- Show that $OM'=\dfrac{AM}{OM}$.

   b- Show that, if M moves on the line (d) with equation y = 1, then M′ moves on a circle with center O and radius to be determined.
3) Verify that $z'-i=\dfrac{2}{z}$.
4) Let z = $e^{-i\frac{\pi}{4}}$.

   a- Write z′ – i in exponential form and algebraic form.

   b- Prove that the two lines (OM) and (BM′) are perpendicular.

IV- (8 points)

Let f be the function defined on ℝ as: $f(x)=x+2-2e^{x}$. Denote by (C) its representative curve in an orthonormal system (O; $\vec{i}$, $\vec{j}$).

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$.

   b- Show that the line (D) with equation y = x + 2 is an asymptote to (C).

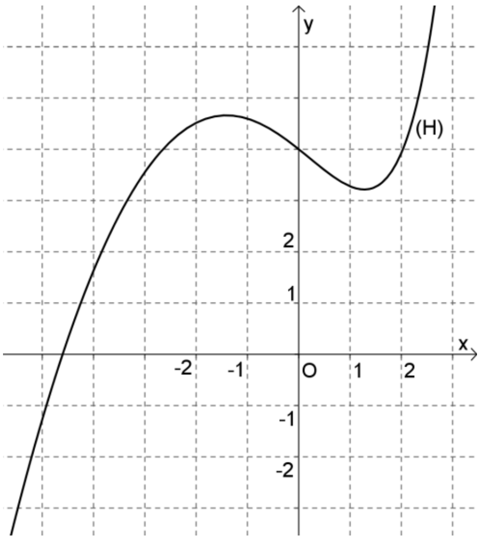
   c- For all x in ℝ, show that the curve (C) is below the line (D).
2) Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate f(1.5).
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Show that the equation f(x) = 0 has, in ℝ, exactly two roots 0 and α.

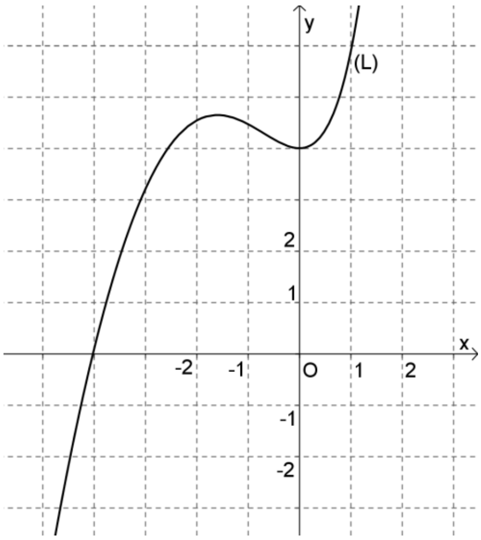
   Verify that –1.6 < α < –1.5.
5) Draw (D) and (C).
6) Denote by $A(\alpha)$ the area of the region bounded by (C) and the x-axis.

   Show that $A(\alpha)=\left(-\dfrac{\alpha^{2}}{2}-\alpha\right)$ square units.
7) Let g be a function defined on ℝ with: $g'(x)=-2f(x)$.

   One of the two curves (H) and (L) given below represents the function g.

   Choose it with justification.



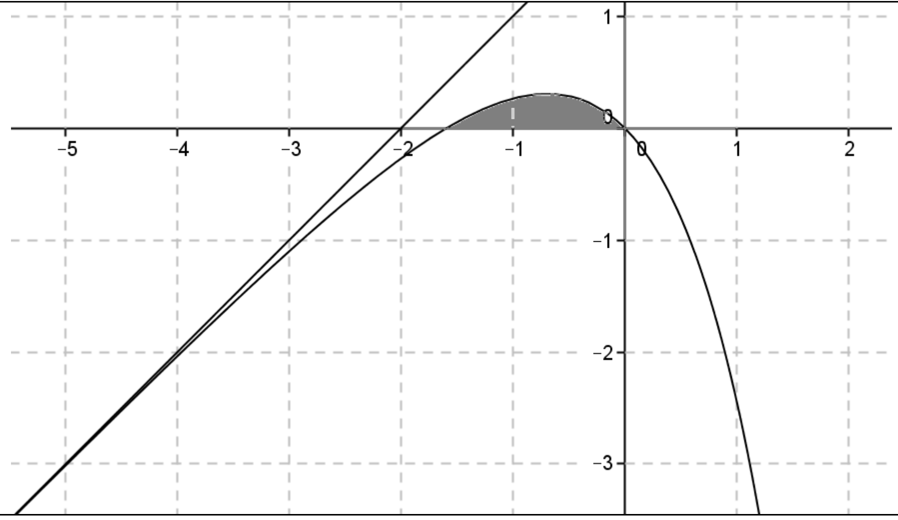


وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠١٧ الاستثنائيّة الثلاثاء في ٨ آب ٢٠١٧
عدد المسائل: أربع	أسس تصحيح مادة الرياضيات	

I	Answers	Grade
1	$A \in (Q): 2(x_A) - (y_A) - 5(z_A) + 1 = 0$, $2(9) - (-1) - 5(4) + 1 = 0$, $0 = 0$ $B \in (Q): 2(5) - (1) - 5(4) + 1 = 0$, $0 = 0$. $\overrightarrow{n_Q}.\overrightarrow{n_P} = (2)(1) + (-1)(2) + (-5)(0) = 0$.	**0.75**
2	A ∈ (Q)∩(P) and B ∈ (Q)∩(P) then (d) is the line (AB). Hence (d) : $\begin{cases} x = -4k + 9 \\ y = 2k - 1 \\ z = -2k + 4 \end{cases}$ where $k \in \mathbb{R}$	**0.75**
3a	B ∈ (L) for t = 1.	**0.5**
3b	$(L) \subset (Q): 2(-t + 6) - (-2t + 3) - 5(2) + 1 = 0$, $0 = 0$. $\overrightarrow{V_L}.\overrightarrow{V_d} = (-1)(-4) + (-2)(2) + (0)(-2) = 0$	**1**
3c	E ∈ (L) then $E(-t + 6 ; -2t + 3 ; 2)$, $\overrightarrow{BC}(-2; 1; 0)$, $\overrightarrow{EB}(t-1; 2t-2; 0)$. Area of (EBC) = $\frac{1}{2}\left\Vert\overrightarrow{EB} \wedge \overrightarrow{BC}\right\Vert = 5$ then $\frac{1}{2}\left\Vert 5(t-1)\vec{k}\right\Vert = 5$, $\frac{1}{2}5\vert t-1\vert = 5$, donc $\vert t-1\vert = 2$, so t = 3 then (3; –3; 2) rejected **or** t = –1 then (7; 5; 2) accepted, hence E(7; 5; 2). **Another method:** (L)⊂(Q), (Q)∩(P) = (d), (L)⊥(d) at B and (P)⊥(Q) thus (L)⊥(P) but (BC)⊂(P) so (L)⊥(BC) at B . Consequently, EBC is a right triangle with vertex B. Area of (EBC) = $\frac{1}{2}$EB.BC = 5. E ∈ (L) then $E(-t + 6 ; -2t + 3 ; 2)$. $\frac{1}{2}\sqrt{(t-1)^2 + 4(t-1)^2}.\sqrt{5} = 5$ then $\vert t-1\vert = 2$ so t = 3 then (3; –3; 2) rejected **or** t = –1 then (7; 5; 2) accepted, hence E(7; 5; 2).	**1**

II	Answers	Grade
A1	$P(A) = \frac{C_5^2}{C_{10}^2} = \frac{2}{9}$, $P(B) = P(ww) + P(bb) + P(gg) = \frac{C_5^2 + C_3^2 + C_2^2}{C_{10}^2} = \frac{14}{45}$	**1**
A2a	$P(C) = \frac{C_3^2}{C_{10}^2} = \frac{3}{45} = \frac{1}{15}$	**0.5**
A2b	$P(A \cap B) = P(C) = \frac{1}{15} \neq P(A).P(B) = \frac{28}{405}$ then A and B are not independent.	**0.5**
A3	$P(D) = P(A \cap \overline{B}) = P(A) - P(A \cap B) = \frac{2}{9} - \frac{1}{15} = \frac{7}{45}$	**0.5**
A4	$P(A/\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{P(D)}{1 - P(B)} = \frac{7}{31}$	**0.75**
B	$P(Sum = 0) = P(wb \text{ or } bw) + P(gg) = 2\left(\frac{5 \times 3}{10^2}\right) + \frac{2 \times 2}{10^2} = 0.34$	**0.75**

III	Answers	Grade
1	$z' = \frac{2i - (1+i)}{i(1+i)} = 1$	**0.5**
2a	$OM' = \vert z'\vert = \frac{\vert z - 2i\vert}{\vert i\vert\vert z\vert} = \frac{AM}{OM}$	**0.75**
2b	M ∈ (d) then M(x; 1). A(0; 2) thus $OM' = \frac{AM}{OM} = \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = 1$. Therefore M' moves on the circle with center O and radius 1. **Another method:** (d) is the perpendicular bisector of [OA] and M ∈ (d) then MA = MO so OM′ = 1.	**0.75**
3	$z' - i = \frac{2i - z}{iz} - i = \frac{2}{z}$	**0.5**
4a	$z' - i = 2e^{i\frac{\pi}{4}} = \sqrt{2} + i\sqrt{2}$	**0.75**
4b	$(\overrightarrow{OM}; \overrightarrow{BM'}) = (\overrightarrow{OM}; \vec{u}) + (\vec{u}; \overrightarrow{BM'})\ (2\pi) = -\arg(z) + \arg(z' - i)\ (2\pi) = \frac{\pi}{4} + \frac{\pi}{4}\ (2\pi) = \frac{\pi}{2}\ (2\pi)$ **Another method :** B(0; 1), $\overrightarrow{OM}(\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$ and $\overrightarrow{BM'}(\sqrt{2}; \sqrt{2})$. $\overrightarrow{OM}.\overrightarrow{BM'} = 0$	**0.75**

IV	Answers	Grade
1a	$\lim\limits_{x\to-\infty} f(x) = -\infty$	**0.25**
1b	$\lim\limits_{x\to-\infty}\left(f(x)-x-2\right) = \lim\limits_{x\to-\infty}\left(-2e^x\right) = 0$ then (D) is an asymptote to (C)	**0.5**
1c	$f(x) - x - 2 = -2e^x < 0$ then (C) is below (D)	**0.5**
2	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} e^x\left(\frac{x}{e^x}+\frac{2}{e^x}-2\right) = -\infty$; $f(1.5) = -5.463$	**0.75**
3	$f'(x) = 1 - 2e^x$ $\begin{array}{c\|ccccccccc} x & -\infty & & \alpha & & -\ln 2 & & 0 & & +\infty \\ \hline f'(x) & & & + & & 0 & & - & & \\ \hline & & & & & 0.306 & & & & \\ f(x) & & \nearrow & 0 & \nearrow & & \searrow & 0 & \searrow & \\ & -\infty & & & & & & & & -\infty \end{array}$	**1.25**
4	- On $]-\infty;-\ln 2[$: f is continuous and strictly increasing from $-\infty$ to 0.306 > 0 then the equation f(x) = 0 has a unique solution α. f(–1.6) × f(–1.5) = (–0.003) × (0.053) < 0, then –1.6 < α < –1.5. - On $]-\ln 2;+\infty[$: f is continuous and strictly decreasing from 0.306 > 0 to $-\infty$ then the equation f(x) = 0 has a unique solution β. But since f(0) = 0, then β = 0. Therefore, the equation f(x) = 0 has exactly two solutions 0 and α.	**1.25**
5		**1.25**
6	$A(\alpha) = \int_{\alpha}^{0} f(x)dx = \left[\frac{x^2}{2}+2x-2e^x\right]_{\alpha}^{0} = -2-\frac{\alpha^2}{2}-2\alpha+2e^{\alpha}$. But f(α) = 0, then $2e^{\alpha} = \alpha + 2$, therefore A(α) = $\left(-\frac{\alpha^2}{2}-\alpha\right)$ square units.	**1.25**
7	$\begin{array}{c\|ccccccc} x & -\infty & & \alpha & & 0 & & +\infty \\ \hline g'(x) = -2f(x) & & + & 0 & - & 0 & + & \\ \hline g(x) & & \nearrow & \| & \searrow & \| & \nearrow & \end{array}$ For x = 0, g ′(x) = 0 and g′(x) changes sign, then the curve of g has an extremum. Consequently, (H) does not represent g. So, (L) represents g.	**1**