وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع: الاجتماع والاقتصاد	دورة العام ٢٠١٧ الاستثنائيّة الاثنين في ٧ آب ٢٠١٧
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The following table represents the results of a survey in a certain store about x_i, the price evolution of a laptop and y_i, the number of laptops sold:

Price of a laptop: x_i in **millions** LL	1.5	1.4	1.3	1.2	1.1
Number of laptops sold: y_i	5	7	10	14	20

A-

1) Determine the center of gravity $G(\overline{X}, \overline{Y})$ and write an equation of the regression line $(D_{y/x})$.
2) Represent the scatter plot $(x_i ; y_i)$ in a rectangular system of axes. Plot G, and draw $(D_{y/x})$ in the same rectangular system of axes.
3) Suppose that the above pattern remains valid till the price 800 000 LL.
   Estimate the number of laptops sold at the price of 900 000 LL.

B-

1) Use the table above to show that the revenue achieved upon selling 56 laptops is 69 100 000 LL.
2) Knowing that the store buys each laptop for 850 000 LL, calculate the profit achieved by the store upon selling these 56 laptops.

II- (4 points)

In a sports club,

- 60% of the members are men
- 25% of the men practice swimming
- 24% of members of this club practice swimming.

One member is randomly selected from the club. Consider the following events:

M: "The selected member is a man"
W: "The selected member is a woman"
S: "The selected member practices swimming".

1) a- Calculate the probability $P(M \cap S)$.
   b- Verify that $P(M \cap \overline{S}) = 0.45$ and deduce $P(W \cap \overline{S})$.
2) Knowing that the selected member does not practice swimming, calculate the probability that this member is a woman.
3) Each member who practices swimming should pay 3 000 000 LL as an annual membership.
   Each man who doesn't practice swimming should pay 2 500 000 LL as an annual membership.
   Each woman who doesn't practice swimming should pay 2 300 000 LL as an annual membership.
   Denote by X the random variable that is equal to the annual membership paid by each member.
   a- Determine the probability distribution of X.
   b- 500 members are subscribed in this club, estimate the annual revenue achieved by the club.

III- (4 points)

A factory F produces milk. In January 2014, the milk production of the factory F was 500 000 liters and this production increases monthly at the rate of 1%.

For every non-zero natural number n, denote by U_n the production in liters of this factory in the n^{th} month.

Thus, $U_1 = 500\,000$.

A-

1) Show that $U_n = 500\,000 \times (1.01)^{n-1}$.
2) Let $S_n = U_1 + U_2 + ... + U_{n-1} + U_n$.
   Knowing that $S_n = 50\,000\,000 \times (1.01)^n - 50\,000\,000$, after how many months will the total production of this factory exceed 30 000 000 liters for the first time? Justify.

B-

Another factory G produces also milk. In January 2014, the milk production of the factory G is 350 000 liters and this production increases by 10 000 liters monthly.

For every non-zero natural number n, denote by V_n the production in liters of this factory in the n^{th} month.

Thus, $V_1 = 350\,000$.

1) Show that (V_n) is an arithmetic sequence and show that $V_n = 10\,000n + 340\,000$.
2) In August 2017 only, which factory produces more milk? Justify.

IV- (8 points)

A- Let f and g be the functions defined over $[0,+\infty[$ as: $f(x) = \dfrac{x+2}{1+e^x}$ and $g(x) = \dfrac{e^x}{10}$. Denote by (C) the representative curve of f and by (G) the representative curve of g in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to+\infty} f(x)$. Deduce an asymptote to (C).

   b- Show that $f'(x) = \dfrac{1 - xe^x - e^x}{(1+e^x)^2}$, then copy and complete the adjacent table of variations of the function f.

x	0	$+\infty$
f′(x)	——	
f(x)		

   c- Draw (C).

2) a- Determine $\lim\limits_{x\to+\infty} g(x)$. Calculate g(3) and g(4).

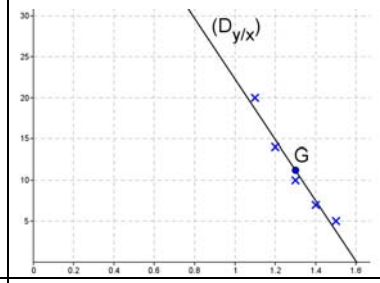
   b- Calculate $g'(x)$, then set up the table of variations of the function g.

3) The two curves (C) and (G) intersect at only one point E with abscissa α. Verify that $1.72 < \alpha < 1.73$.
4) Draw the curve (G) in the same system as that of (C).

B- A company produces vases. The demand function and the supply function are respectively modeled as: $f(p) = \dfrac{p+2}{1+e^p}$ and $g(p) = \dfrac{e^p}{10}$; where p is the unit price expressed in ten thousands LL, f(p) and g(p) expressed in thousands of vases with $p \in [0.5\,;\,4]$.

1) The selling price of each vase is 25 000 LL. Estimate the number of demanded vases.
2) Assume that $\alpha = 1.725$. Give an economical interpretation of α.
3) E(p) represents the elasticity of the demand with respect to the price p .
   a- Calculate E(2). Is the demand elastic for p = 2? Justify.
   b- Give an economical interpretation of E(2).

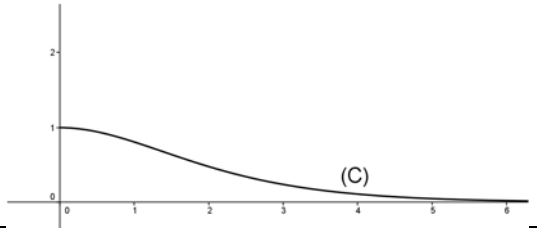
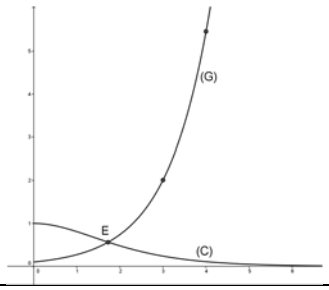
عدد المسائل: اربع — **أسس التصحيح**

I		Short answers	Note
A	1	$\overline{x}=1,3$; $\overline{y}=11,2$ and $\left(D_{y/x}\right): y=-37x+59,3$	1.5
	2		2
	3	For $x=0,9$ then $y=-37(0,9)+59,3=26$ so 26 Laptops	1
B	1	$5\times1,5+7\times1,4+10\times1,3+14\times1,2+20\times1,1=69,1$ so revenue= 69 100 000 LL	1.5
	2	$69\ 100\ 000-56\times850\ 000=21\ 500\ 000$ LL	1

II		Short answers	Note
1	a	$P(H\cap N)=P(H)\times P\left(N/H\right)=0,6\times0,25=0,15.$	1
	b	$P(H\cap\overline{N})=P(H)-P(H\cap N)=0,6-0,15=0,45$ or $P(H\cap\overline{N})=P(H)\times P\left(\overline{N}/H\right)=0,6\times(1-0,25)=0,45$ $P(F\cap\overline{N})=P(\overline{N})-P(H\cap\overline{N})=(1-0,24)-0,45=0,31$	1.5
2		$P\left(F/\overline{N}\right)=\dfrac{P(F\cap\overline{N})}{P(\overline{N})}=\dfrac{0,31}{0,76}=\dfrac{31}{76}$	1.5
3	a	$X=\{2\ 300\ 000\ ;\ 2\ 500\ 000\ ;\ 3\ 000\ 000\}$.$P(X=\ 2\ 300\ 000)=P(F\cap\overline{N})=0,31$ $P(X=\ 2\ 500\ 000)=P(H\cap\overline{N})=0,45$ and $P(X=\ 3\ 000\ 000)=P(N)=0,24$	2
	b	$E(X)=\sum x_ip_i=2\ 300\ 000\times0,31+2\ 500\ 000\times0,45+3\ 000\ 000\times0,240=2\ 558\ 000$ then $R=500\times E(X)=500\times2\ 558\ 000=1\ 279\ 000\ 000$ LL.	1

III		Short answers	Note
A	1	$U_{n+1}=U_n+0,01U_n=1,01U_n$ then (U_n) is a geometric sequence of common ratio $q=1,01$ so $U_n=U_1\times q^{n-1}=500000\times(1,01)^{n-1}$.	2
	2	$S_n>30\ 000\ 000$ so $50\ 000\ 000\times(1,01)^n-50\ 000\ 000>30\ 000\ 000$ then $(1,01)^n>1,6$ donc $n>\dfrac{\ln(1,6)}{\ln(1,01)}$ then $n>47,23$ so $\boxed{48\text{ months}}$.	2

B	1	$V_{n+1}=V_n+10\,000$ then $V_{n+1}-V_n=10\,000$ so (V_n) is an arithmetic sequence whose common difference is $d=10\,000$ $V_n=V_1+(n-1)d=350\,000+10\,000(n-1)=10\,000n+340\,000.$	1.5
	2	August 2017 corresponds to $n=43$ $U_{43}=500\,000\times(1{,}01)^{42}=759\,395$ et $V_{43}=10\,000\,(43)+340\,000=770\,000$ so the factory G produces more milk than factory F in August 2017.	1.5

IV			Short answers	Note
A	1	a	$\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{x+2}{e^x+1}=\lim\limits_{x\to+\infty}\dfrac{x}{e^x}=0$; then the axis of abscissa is an asymptote at $+\infty$.	1.5
		b	$f'(x)=\dfrac{1+e^x-e^x(x+2)}{(1+e^x)^2}=$ $\dfrac{1+e^x-xe^x-2e^x}{(1+e^x)^2}=\dfrac{1-xe^x-e^x}{(1+e^x)^2}.$ x: 0 … $+\infty$; $f'(x)$: 0 …; $f(x)$: 1 … 0	2
		c		1
	2	a	$\lim\limits_{x\to+\infty} g(x)=+\infty$ $g(3)\approx 2{,}008$ and $g(4)\approx 5{,}46$	1
		b	$g'(x)=\dfrac{e^x}{10}>0$ then g is strictly increasing: x: 0 … $+\infty$; g': $+$; g: $\frac{1}{10}$ … $+\infty$	1
	3		let $h(x)=f(x)-g(x)$. $h(1{,}72)=6{,}5\times10^{-3}>0$ and $h(1.73)=-2.3\times10^{-3}<0$ and h is continuous over $[1.72\,;\,1.73]$ then $1.72<\alpha<1.73$.	1
	4			0.5
B	1		$f(2{,}5)\approx 0{,}341$ so 341 vases	1.5
	2		17 250 LL represents the equilibrium price.	1.5
	3	a	$E(2)=-2\times\dfrac{f'(2)}{f(2)}\approx 1{,}26>1$ then the demand is elastic at the price 20 000LL	2
		B	At the price 20 0000L.L, an increase 1% in the price will cause a decrease 1,26% in the demand	1