وزارة التربية والتعليم العالي	امتحانات الشهادة المتوسطة	دورة العـام 2017 الاستثنائيّة
المديرية العامة للتربية		الخميس فـي 27 تموز 2017
دائرة الامتحانات الرسمية		

الاسم:	مسابقة في مادة الرياضيات	
الرقم:	المدة: ساعتان	

I – (2 points)

In the following table, only one of the proposed answers to each question is correct.

Write the number of the question and its corresponding answer. Justify your choice.

	Questions	Answers		
		a	b	c
1	$3\sqrt{2} - \sqrt{50} + \sqrt{8} =$	$10\sqrt{2}$	0	$-30\sqrt{2}$
2	$\dfrac{1}{\sqrt{5}-2} =$	$\sqrt{5}+2$	$\dfrac{\sqrt{5}+2}{3}$	-1
3	ABCD is a parallelogram, then $\overrightarrow{AB} + \overrightarrow{DA} =$	$\overrightarrow{BC}$	$\overrightarrow{CA}$	$\overrightarrow{DB}$
4	After an increase of 15%, the price of an article becomes 23 000 L.L. The original price of this article is:	17 000 L.L.	20 000 L.L.	19 550 L.L.

II – (3.5 points)

Given $A(x) = (x-3)^2 - (x-3)(2x-7)$.

1) Prove that $A(x) = (x-3)(4-x)$.

2) Let $B(x) = (16 - x^2) + A(x)$.

Factorize $B(x)$.

3) Let $F(x) = \dfrac{A(x)}{(4-x)(2x+1)}$.

 a. For what values of x, is $F(x)$ defined?

 b. Simplify $F(x)$, then solve the equation $F(x) = \dfrac{2}{3}$.

 c. Does the equation $F(x) = x$ have a solution? Justify.

III – (3 points)

1) Solve the following system: $\begin{cases} 5x + 2y = 12\,000 \\ x + 2y = 8\,000. \end{cases}$

2) A restaurant sells 10 green salads and 4 vegetarian pizzas for 24 000 L.L.

 The same restaurant sells 6 green salads and 12 vegetarian pizzas for 48 000 L.L.

 Show that this text is modeled by the system given in question **1)**.

3) Nadine orders 8 green salads and 6 vegetarian pizzas, how much will she pay?

IV – (5.5 points)

In an orthonormal system of axes (x'ox, y'oy), consider the points $A(4;2)$, $B(-1;2)$ and $E(1;3)$.

Let (d) be the line with equation $y = 3x$.

1) a. Plot A, B and E.

b. Verify that E is a point on the line (d). Draw (d).

2) a. Calculate OB and show that OA = 2OB.

b. Show that OAB is a right angled triangle.

3) a. Determine the coordinates of the point L, the symmetric of O with respect to B.

b. Verify that E is the midpoint of [AL].

4) Let (d') be the line passing through A and perpendicular to (OA).

Show that the equation of (d') is $y = -2x + 10$.

5) Let F be the point with coordinates $(2;6)$.

a. Verify that F is the intersection point of (d) and (d').

b. Prove that the quadrilateral OAFL is a square.

V – (6 points)

In the adjacent figure:
- (C) is a circle with center O, radius 5 and diameter [EB]
- A is a point on (C) so that AE = 6
- (d) is the tangent at B to (C)
- [AL] is an altitude in the triangle ABE.

1) Copy the figure that will be completed in the remaining parts of the problem.

2) a. Calculate AB.

b. Verify that $sin\ \widehat{AEB} = \dfrac{4}{5}$.

3) The parallel through L to (AB) intersects [EA] at M and the line (d) at F.

a. Prove that the two triangles EML and FBL are similar.
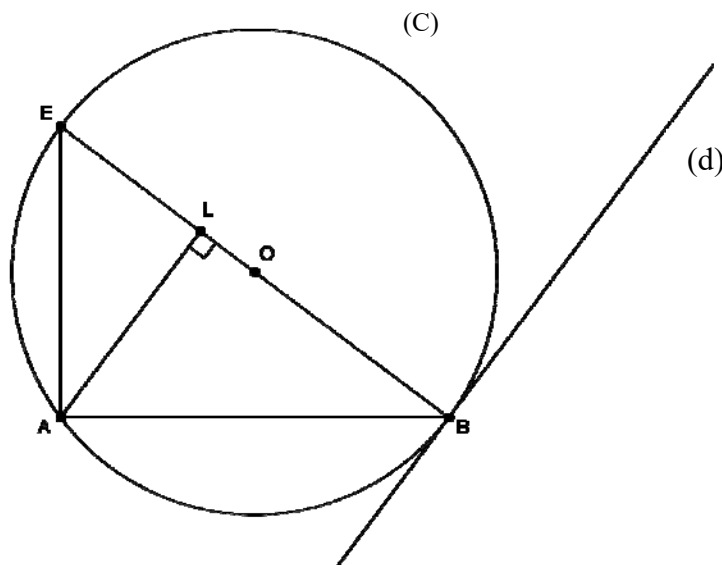
b. Calculate, rounded to the nearest degree, the measure of angle $\widehat{BFL}$.

4) Prove that the points E, M, B and F are on the same circle whose center I should be determined.

5) Prove that the quadrilateral ALFB is a parallelogram.

6) The diagonals [AF] and [BL] of the parallelogram ALFB intersect at J.

Prove that (IJ) is perpendicular to (AB).

		Correction	
Part		**Question I**	**Note**
1	$3\sqrt{2}-\sqrt{50}+\sqrt{8}=0$	**b)**	0.5
2	$\dfrac{1}{\sqrt{5}-2}=\dfrac{1}{\sqrt{5}-2}\times\dfrac{\left(\sqrt{5}+2\right)}{\left(\sqrt{5}+2\right)}=\sqrt{5}+2$	**a)**	0.5
3	$\overrightarrow{DA}+\overrightarrow{AB}=\overrightarrow{DB}$	**c)**	0.5
4	$23000\div1.15=20000$	**b)**	0.5
Part		**Question II**	**Note**
1	$A(x)=(x-3)^2-(x-3)(2x-7)=(x-3)(x-3-2x+7)=(x-3)(4-x)$ 0.25+0.25		0.5
2	$B(x)=\left(16-x^2\right)+A(x)=\left(16-x^2\right)+(x-3)(4-x)$ 0.25+0.25 $B(x)=(4-x)(4+x)+(x-3)(4-x)=(4-x)(2x+1)$ 0.25+0.25		1
3	**a.** F(x) is defined for $x\neq4$ et $x\neq-\dfrac{1}{2}$ 0.25 + 0.25		0.5
	b. $\dfrac{(x-3)(4-x)}{(4-x)(2x+1)}=\dfrac{(x-3)}{(2x+1)}$; $3x-9=4x+2$ alors $x=-11$ 0.25+0.5		0.75
	c. $\dfrac{x-3}{2x+1}=x$ then $(2x+1)x=(x-3)$ and $x^2=-\dfrac{3}{2}$ No solution 0.25+0.25+0.25		0.75

Part	**Question III**		**Note**
1	x = 1 000 and y = 3 500	0.5 + 0.5	1
2	$\begin{cases}10x+4y=24\,000\,(\div2)\\6x+12y=48\,000\,(\div6)\end{cases}$	(0.5+0.25) (0.25+0.25)	1.25
3	The price of a green salad is 1 000 L.L. and the price of a vegetarian pizza is 3 500L.L. 0.25 so Nadine will pay 8(1000) +6(3500) = 29 000 L.L 0.5		0.75

Part	**Question IV**		**Note**
1		One point (0.25) Two points (0.5)	0.5
	b. $y_E=3x_E$ then E is a point on the line (d)	0.25+0.25	0.5

2	**a.** $OB = \sqrt{5}$ and $OA = 2\sqrt{5}$ then OA=2OB $\qquad$ 0.25+0.25+0.25		0.75
	b. AB=5 and by Pythagorean theorem $AB^2 = OA^2 + OB^2$ $\qquad$ 0.25+0.25		0.5
3	**a.** $x_B = \dfrac{x_L + x_O}{2} = -1$ and $y_B = \dfrac{y_L + y_O}{2} = 2$ then L(-2;4)		0.5
	b. $x_E = \dfrac{x_A + x_L}{2} = 1$ and $y_E = \dfrac{y_A + y_L}{2} = 3$ verified		0.5
4	$a_{(OA)} = \dfrac{1}{2}$ and $a_{(OA)} . a_{(d')} = -1$ then $a_{(d')} = -2$; (d'):y = - 2 x +10 $\qquad$ 0.25+0.5+0.25		1
5	**a.** F belongs to (d) since $y_F = 3x_F$ and F belongs to (d') since $y_F = -2x_F + 10$ $\qquad$ 0.25+0.25		0.5
	b. E midpoint of [LA] and E miodpoint of [OF] then OAFL is a parallelogram. OL=2OB=OA and $\hat{O} = 90^0$ then OAFL is a square. $\qquad$ 0.25+0.25+0.25		0.75

Part	Question V	Note
1		0. 5
2	**a.** Pythagorean Theorem : $AB^2 = EB^2 - AE^2 = 100 - 36$ so $AB = 8$	0.5
	b. $\sin \hat{A}EB = \dfrac{AB}{EB} = \dfrac{8}{10} = \dfrac{4}{5}$	0.5
3	**a.** $\hat{E}LM = \hat{B}LF$ (vertically opposite angles) $\qquad$ 0.5 $\hat{E}ML = \hat{L}BF = 90^0$ $\qquad$ 0.5 So the two triangles are similar	1
	b. $\sin \hat{B}FL = \sin \hat{A}EB = 0.8$ so $\hat{B}FL \square 53^0$ $\qquad$ 0.5+0.5	1
4	The two triangles EMF and EBF are two right triangles facing the same hypotenuse [EF] so the four points E,M,B and F are on the same circle of center I the midpoint of [EF]. $\qquad$ 0.25+0.25+0.25	0.75
5	$(AL) \perp (EB)$ and $(BF) \perp (EB)$ so $(AL)//(BF)$ $\qquad$ 0.5+0.25 And we have (LF)//(AB) then ALFB is a parallellogram.	0.75
6	I midpoint of [EF] and J midpoint of [AF] so (IJ)//(EA) and $(EA) \perp (AB)$ so $(IJ) \perp (AB)$ $\qquad$ 0.5+0.5	1