وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: العلوم العامة	دورة العام ٢٠١٧ العاديّة الإثنين ١٢ حزيران ٢٠١٧
عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (2 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

	Questions	**Answers**			
		a	**b**	**c**	**d**
1	f is the function defined over $\left]-\frac{5}{2};\frac{5}{2}\right[$ as f(x) = $\frac{1}{\sqrt{25-4x^2}}$. An antiderivative of f is:	$\arcsin\frac{2x}{5}$	$\arcsin 2x$	$\frac{1}{2}\arcsin\frac{2x}{5}$	$\frac{2}{5}\arcsin\frac{2x}{5}$
2	If $T(x)=\int_1^{2x}\sqrt{1+3\ln^2 t}\,dt$ with x > 0, then $T'\left(\frac{e}{2}\right)=$	1	2	3	4
3	z and z' are two complex numbers. If $z'=\frac{z-2i}{iz+2}$ with $z\neq 2i$, then $\lvert z'\rvert=$	1	$\sqrt{\frac{5}{3}}$	$\sqrt{5}$	2
4	In the complex plane referred to an orthonormal system, M and M' are two points with respective non-zero affixes z and z'. If $z'\sqrt{2}=(1-i)z$, then triangle OMM' is:	right	isosceles	equilateral	right isosceles

II- (2.5 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the following points:

A (1 ; –1 ; 2), B (–1 ; 1 ; 3) and E (–1 ; 4 ; $\frac{3}{2}$).

Let (P) be the plane with equation $2x + y + 2z - 5 = 0$.

Let (Δ) be the perpendicular bisector of [AB] in (P).

1) Verify that the points A and B are in the plane (P).

2) a- Verify that $\vec{V}(1\,;\,2\,;-2)$ is a direction vector to (Δ).

   b- Write a system of parametric equations of the line (Δ).

3) Let I be a point on (Δ) such that $x_I > 0$.

   Consider, in the plane (P), the circle (C) with center I and radius 3 that is tangent to (AB).

   a- Determine the coordinates of I.

   b- Verify that E is on the circle (C).

4) Denote by (D) the line defined as $\begin{cases} x = 2t - 1 \\ y = 4t + 4 \\ z = -4t + \frac{3}{2} \end{cases}$ where $t \in \mathbb{R}$.

   Show that the line (D) is tangent to the circle (C) at E.

III- (2.5 points)

Consider two urns U_1 and U_2.

U_1 contains two red balls and one green ball.

U_2 contains four red balls and three green balls.

Each red ball holds the number 1 and each green ball holds the number -1.

One ball is randomly selected from U_1.

- If this ball is red, then one ball is randomly selected from U_2. *(Hence, we get two balls)*
- If it is green, then two balls are randomly and simultaneously selected from U_2. *(Hence, we get three balls)*

Consider the following events :

R_1 : « One red ball is selected from U_1 » ,

R_2 : « One red ball is selected from U_2 » ,

D: « The selected balls have the same color».

1) Calculate the probability $P(R_1 \cap R_2)$.

2) Verify that $P\left(\overline{D}\right) = \frac{4}{7}$.

3) Let S be the sum of numbers on the selected balls.

   a- Verify that the possible values of S are : –3 ; –1 ; 0 ; 1 ; 2.

   b- Calculate P(S < 0).

   c- Knowing that S < 0, calculate the probability that the selected balls don't have the same color.

IV- (3 points)

In the plane referred to an orthonormal system $\left(O;\vec{i},\vec{j}\right)$, consider the point $E(2;0)$ and the two variable points $M(m\,;0)$ and $N(0\,;n)$ such that $OM = EN$ with m and n being two real numbers ($m \le -2$ or $m \ge 2$). Let P be the point defined as $\overrightarrow{NP} = \frac{1}{2}\overrightarrow{OM}$.

Part A

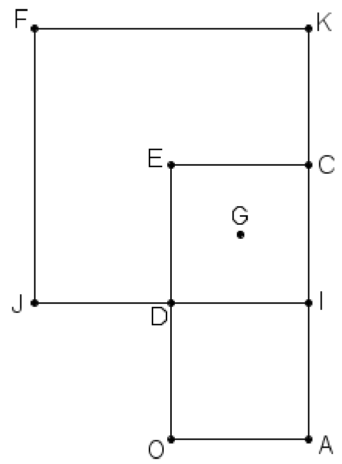
1) Verify that $m^2 = n^2 + 4$.
2) a- Find the coordinates of P in terms of m and n.
   b- Show that P moves on the hyperbola (H) with equation $4x^2 - y^2 = 4$.
3) Denote by A and A' the vertices of (H), and by F and F' its foci.
   a- Find the coordinates of A, A', F and F' ($x_A > 0$ and $x_F > 0$).
   b- Write the equations of the asymptotes of (H) and draw (H).

Part B

Let (E) be the ellipse so that A, A' and $B(0;4)$ are three of its vertices.

1) Draw (E) in the system $\left(O;\vec{i},\vec{j}\right)$.
2) The tangent at B to (E) intersects (H) at L with $x_L > 0$.
   a- Show that OFLB is a rectangle.
   b- Calculate the area of the region interior to quadrilateral OALB and exterior to (E).
3) Let G be the point defined as $\overrightarrow{OG} = \frac{1}{5}\overrightarrow{OF}$. Show that the line (LG) is tangent to (H).

V- (3 points)



In the figure to the right:

- DICE and JIKF are two direct squares with centers G and E respectively.
- A is the symmetric of C with respect to I.
- O is the symmetric of E with respect to D.

Let S be the direct plane similitude that maps A onto I and I onto E.

Part A

1) a- Show that the ratio of S is equal to $\sqrt{2}$ and that $\frac{\pi}{4}$ is an angle of S.
   b- Determine S(C).
2) a- $S \circ S$ is a similitude. Find an angle of $S \circ S$ and calculate its ratio.
   b- Find $S \circ S(A)$ and deduce that O is the center of S.
3) The two straight lines (OC) and (AD) intersect at L.
   Let L′ = S(L).
   Prove that the three points I, D and L′ are collinear.

Part B

The plane is referred to a direct orthonormal system $\left(O;\overrightarrow{OA},\overrightarrow{OD}\right)$.

1) Write the complex form of S and determine the affix of G' such that G'= S(G).
2) Let (T) be the ellipse with center I. The points O and G are two of its vertices.
   Denote by (T′) be the image of (T) under S. Write an equation of (T′).

VI- (7 points)

Part A

Consider the differential equation $(E): y' + y = 2 - e^{-x}$.

Let $y = z + 2 - xe^{-x}$.

1) Form the differential equation (E′) satisfied by z.
2) Find the particular solution of (E) whose representative curve in an orthonormal system passes through the point A(– 2 ; 2).

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = 2 - (x+2)e^{-x}$. Denote by (C) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$.

   b- Determine $\lim\limits_{x \to +\infty} f(x)$. Deduce an asymptote (d) to (C).

2) a- Calculate f ′(x), and set up the table of variations of f.

   b- Show that the equation f(x) = 0 has two roots α and 0.

   Verify that $-1.6 < \alpha < -1.5$.

3) a- Show that (C) has an inflection point whose coordinates are to be determined.

   b- Write an equation of (Δ), the tangent to (C) at its inflection point.

4) Let (d') be the line with equation y = –x.

   a- Verify that $f(x) + x = (x+2)\left(1 - e^{-x}\right)$.

   b- Study, according to the values of x, the relative positions of (d') and (C).

5) Draw (d), (Δ), (d′) and (C).

6) a- Use the differential equation (E) to find an antiderivative of f.

   b- Deduce the area of the region bounded by (C), (d′) and the two lines with equations x = α and x = 0.

7) Let g be the function defined as g(x) = ln(–x – f(x)).

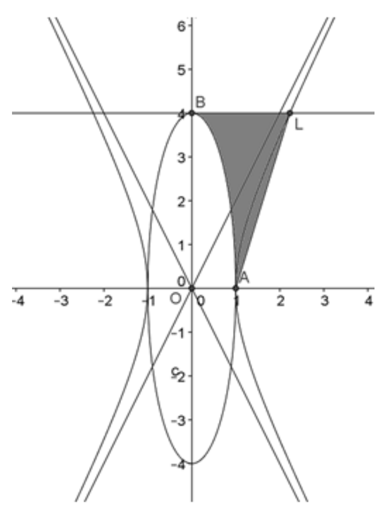
   Determine the domain of definition of g.

عدد المسائل: ست — أسس التصحيح مادة الرياضيات

QI	Answers	M
1	$\int \frac{dx}{\sqrt{25-4x^2}} = \frac{1}{5}\int \frac{dx}{\sqrt{1-(2x/5)^2}} = \frac{1}{2}\arcsin\frac{2x}{5}$ **c**	**1**
2	$T'(x) = 2\sqrt{1+3(\ln 2x)^2}$; $T'(\frac{e}{2}) = 2\sqrt{1+3(\ln e)^2} = 2\sqrt{4} = 4$ **d**	**1**
3	$\lvert z'\rvert = \left\lvert\frac{z-2i}{iz+2}\right\rvert = \frac{\lvert z-2i\rvert}{\lvert iz+2\rvert} = 1$ **a**	**1**
4	$\frac{z'}{z} = e^{-i\frac{\pi}{4}}$; then triangle OMM' is isosceles at O **b**	**1**

QII	Answers	M
1	$2x_A + y_A + 2z_A - 5 = 0$ and $2x_B + y_B + 2z_B - 5 = 0$	**0.5**
2a	$\vec{V}\cdot\overrightarrow{AB} = 0$ and $\vec{V}\cdot\vec{N}_P = 0$ So $\overrightarrow{AB}\wedge\vec{N}_P \,//\, \vec{V}$	**1**
2b	$J(0\,;0\,;\frac{5}{2})$ midpoint of [AB] ; $(\Delta)\begin{cases} x = k \\ y = 2k \\ z = -2k+\frac{5}{2}\end{cases}$ where $k \in \mathbb{R}$	**0.5**
3a	IJ = 3 ; $IJ^2 = 9$; $k^2 + 4k^2 + 4k^2 = 9$; $k = \pm 1$; k = 1 since $x_I > 0$; then $I(1\,;2\,;\frac{1}{2})$	**1**
3b	$2x_E + y_E + 2z_E - 5 = 0$, So $E \in (P)$; $IE^2 = 4 + 4 + 1 = 9$, IE = 3 = R	**1**
4	• $2t - 1 = -1$; $4t + 4 = 4$; $-4t + \frac{3}{2} = \frac{3}{2}$ gives t = 0, then $E \in (D)$. • $\vec{V}_D\cdot\overrightarrow{IE} = 0$ then $(D) \perp (IE)$ • $2(2t-1) + (4t + 4) + 2(-4t + \frac{3}{2}) - 5 = 0$, so $(D) \subset (P)$ Thus (D) is tangent to (C) at E.	**1**

QIII	Answers	M
1	$P(R_1 \cap R_2) = \frac{2}{3}\times\frac{4}{7}=\frac{8}{21}$	**1**
2	$P(D) = P(R_1 \cap R_2) + P(V \cap 2V) = \frac{8}{21}+\frac{1}{3}\times\frac{C_3^2}{C_7^2}=\frac{3}{7}$, then $P(\overline{D})=\frac{4}{7}$	**1**
3a	–3 (V, 2V) ; –1 (V and (R , V)) ; 0 (R_1 and V) ; 1 (V , 2R) ; 2 (R_1 and R_2)	**1**
3b	P(S < 0) = P(S = –3) + P(S = –1) = P(V , 2V) + P(V , (R V)) = $\frac{1}{3}\times\frac{C_3^2}{C_7^2}+\frac{1}{3}\times\frac{4\times3}{C_7^2}=\frac{5}{21}$	**1**
3c	$P\left(\overline{D}/_{S<0}\right)=\frac{P\left(\overline{D}\cap(S<0)\right)}{P(S<0)}=\frac{P(S=-1)}{P(S<0)}=\frac{\frac{4}{21}}{\frac{5}{21}}=\frac{4}{5}$	**1**

<table>
<tr><th>QIV</th><th colspan="3">Answers</th><th>M</th></tr>
<tr><td>A1</td><td colspan="3">$EN^2=OE^2+ON^2$ then $m^2=4+n^2$</td><td>0.5</td></tr>
<tr><td>A2a</td><td colspan="3">$P\left(\frac{m}{2};n\right)$</td><td>0.5</td></tr>
<tr><td>A2b</td><td colspan="3">$m=2x$ and $n=y$ thus $4x^2-y^2=4$ or $x^2-\frac{y^2}{4}=1$</td><td>0.5</td></tr>
<tr><td>A3a</td><td colspan="3">$A(1;0)$ and $A'(-1;0)$; $F(\sqrt{5};0)$ and $F'(-\sqrt{5};0)$</td><td>0.5</td></tr>
<tr><td>A3b</td><td colspan="3">Asymptotes: $y=2x$ and $y=-2x$. Drawing of (H)</td><td>1</td></tr>
<tr><td>B2a</td><td>Tangent at B: $y=4$.
$4x^2=16+4=20$; $x=\sqrt{5}$; $L(\sqrt{5};4)$;
$x_F=x_L$;
$\overrightarrow{BL}=\overrightarrow{OF}$ and $B\hat{O}F=90^\circ$ So OFLB is a rectangle.</td><td>0.5</td><td rowspan="3">B1

</td><td rowspan="3">0.5</td></tr>
<tr><td>B2b</td><td>$\text{Area of OALB}=\frac{(1+\sqrt{5})\times4}{2}=2(1+\sqrt{5})$
Area =
$2(1+\sqrt{5})-\frac{1}{4}\text{Area of (E)}=2(1+\sqrt{5})-\pi$
units of area</td><td>1</td></tr>
<tr><td>B3</td><td>$G\left(\frac{\sqrt{5}}{5};0\right)$; $L(\sqrt{5};4)$;
slope of $(GL)=\sqrt{5}$; $4x^2=y^2+4$
So $8x=2yy'$ and $y'=\frac{4x}{y}$.
$y'_L=\frac{4\sqrt{5}}{4}=\sqrt{5}=$ slope of (GL).</td><td>1</td></tr>
</table>

QV	Answers	M
A1a	S: A $\mapsto$ I I $\mapsto$ E $\frac{IE}{AI}=\frac{IE}{IC}=\sqrt{2}$. Angle of S = $(\overrightarrow{AI};\overrightarrow{IE})=(\overrightarrow{IC};\overrightarrow{IE})=\frac{\pi}{4}$.	**0.5**
A1b	S(C) = F since C is the symmetric of A with respect to I then S(C) is the symmetric of I of with respect to E.	**0.5**
A2a	$S\circ S$ is a similitude with ratio 2 and angle $\frac{\pi}{2}$.	**0.5**
A2b	$S\circ S$(A) = S (I) = E, and we have OE = 2 OA ; $(\overrightarrow{OA};\overrightarrow{OE})=\frac{\pi}{2}$; therefore O is the center of $S\circ S$, hence O is the center of S.	**1.5**
A3	S(A) = I, then $S((AD))$ is a line passing through I making an angle $\frac{\pi}{4}$ with (AD), then it is line (ID). $L\in(AD)$, thus $S(L)=L'\in(ID)$.	**1**
B1	z´= az + b , S has O as a center, then b = 0, thus z´ = a z. a = $\sqrt{2}e^{i\frac{\pi}{4}}=1+i$; thus z´ = (1 + i) z. $z_G=\frac{1}{2}+\frac{3}{2}i$ then $z_{G'}=(1+i)\left(\frac{1}{2}+\frac{3}{2}i\right)=-1+2i$.	**1**
B2	(T) has I as a center and O and G as vertices; then (T´) has S(I) = E as a center and S(O) = O and S(G) = G´ as vertices. Therefore, the focal axis of (T´) is (OE) / / to the axis of ordinates, E (0 ; 2), a = OE = 2 and b = EG´ = 1. Thus, an equation of (T´) is $x^2+\frac{(y-2)^2}{4}=1$.	**1**

QVI	Answers	Note
A1	$y'+y=2-e^{-x}$; $y=z+2-xe^{-x}$; $y'=z'-(e^{-x}-xe^{-x})$; $(E'):z'+z=0$	**0.5**
A2	The general solution of (E') is: $z=ke^{-x}$; the general solution of (E) is: $y=ke^{-x}+2-xe^{-x}$. y(–2)= (k + 2)e^2 + 2 = 2, then k = –2. f(x) = 2 – (x + 2)e^{-x} The particular solution of (E) is: $y(-2)=2$, then $k=-2$; thus, $y=2-(x+2)e^{-x}$	**1**
B1a	$\lim_{x\to-\infty} f(x)=+\infty$	**0.5**
B1b	$\lim_{x\to+\infty} f(x)=2$; (d): $y=2$ horizontal asymptote.	**1**

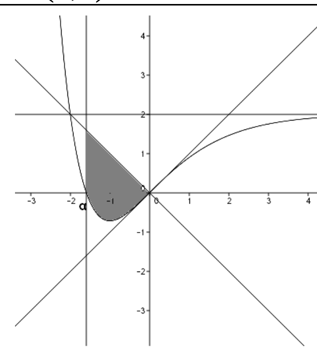
<table>
<tr><td>B2a</td><td>

$f'(x)=(x+1)e^{-x}$

<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>-1</td><td></td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td></td><td>−</td><td>0</td><td>+</td><td></td></tr>
<tr><td>f(x)</td><td>$+\infty$</td><td>↘</td><td>$2-e \approx -0.7$</td><td>↗</td><td>2</td></tr>
</table>

</td><td>1</td></tr>
<tr><td>B2b</td><td>On] $-\infty$, -1[: f is continuous and strictly decreasing from $+\infty$ to –0.7, then the equation f(x) = 0 has one unique solution α ∈] $-\infty$, -1[and $f(-1.6)\times f(-1.5)=0.0187\times(-0.24)<0$, then -1.6 < α < -1.5. Moreover $f(0)=0$.</td><td>1.5</td></tr>
<tr><td>B3a</td><td>f "(x) = $-e^{-x}(x+1)+e^{-x}=-xe^{-x}$
f "(x) = 0 for x = 0 and changes its sign from positive to negative, then O(0,0) is an inflection point of (C).</td><td>1</td></tr>
<tr><td>B3b</td><td>$f'(0)=1$; $y-0=1(x-0)$; $(\Delta): y=x$ is tangent to (C).</td><td>0.5</td></tr>
<tr><td>B4a</td><td>$f(x)+x=2-(x+2)e^{-x}+x=(x+2)(1-e^{-x})$.</td><td>0.5</td></tr>
<tr><td>B4b</td><td>

<table>
<tr><td>x</td><td>$-\infty$</td><td>-2</td><td></td><td>0</td><td>$+\infty$</td></tr>
<tr><td>x + 2</td><td>−</td><td>0</td><td>+</td><td></td><td>+</td></tr>
<tr><td>$1-e^{-x}$</td><td>−</td><td></td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>f(x) + x</td><td>+</td><td>0</td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>position</td><td>(C) is above (d')</td><td>(C) cuts (d') in (-2;2)</td><td>(C) is below (d')</td><td>(C) cuts (d') in (0;0)</td><td>(C) is above (d')</td></tr>
</table>

</td><td>1.5</td></tr>
<tr><td>B5</td><td>$\lim\limits_{x\to+\infty}\frac{f(x)}{x}=-\infty$ asymptotic direction parallel to y'y.
</td><td>1.5</td></tr>
<tr><td>B6a</td><td>f '(x) + f(x) = 2 – e^{-x} ; f(x) = 2 – e^{-x} – f '(x) ; $\int f(x)dx=2x+e^{-x}-f(x)+c$, then an antiderivative of f is $2x+e^{-x}-2+(x+2)e^{-x}=2x-2+(x+3)e^{-x}$.</td><td>1</td></tr>
<tr><td>B6b</td><td>$A=\int_{\alpha}^{0}[-x-f(x)]dx=-\frac{x^2}{2}-2x+2-(x+3)e^{-x}]_{\alpha}^{0}=-3+\frac{\alpha^2}{2}+2\alpha+(\alpha+3)e^{-\alpha}$ units of area.</td><td>1.5</td></tr>
<tr><td>B7</td><td>$-x-f(x)>0;\ x+f(x)<0;$ Using part B-4-b, $-2<x<0$ OR graphically.</td><td>1</td></tr>
</table>