ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points

A(3; 1; 0), B(2; 0; 1) and S(3; −1; −2). Denote by (d) the line defined as: $\begin{cases} x = t \\ y = t+1 \\ z = -t \end{cases}$; $t \in IR$.

1) a- Verify that the point A is not on (d) and that the two lines (AB) and (d) are parallel.
   b- Show that y + z – 1 = 0 is an equation of the plane (P) determined by (AB) and (d).
2) a- Prove that A is the orthogonal projection of the point S on the plane (P).
   b- Denote by S′ the symmetric of S with respect to (P). Calculate the area of the triangle BSS′.
3) Consider in the plane (P) the circle (C) with center A and radius 3.
   The line (d) intersects the circle (C) in two points E and F.
   a- Find the coordinates of E and F.
   b- Write a system of parametric equations of a bisector of the angle $\widehat{EAF}$.

II- (4 points)

A bag U contains nine balls:

- three red balls numbered **0**
- two green balls numbered **1**
- four blue balls numbered **2**.

Part A

Three balls are randomly and simultaneously selected from this bag.
Consider the following events:
M: « the three selected balls have the same color »;
N: « the product of numbers on the three selected balls is equal to zero ».

1) Calculate P(M), the probability of the event M.
2) a- Verify that $P(N) = \frac{16}{21}$.
   b- Calculate $P(M \cap N)$ and verify that $P(\overline{M} \cap N) = \frac{3}{4}$.
3) Knowing that the three selected balls don't have the same color, calculate the probability that the product of numbers on these three balls is equal to zero.

Part B

In this part, one ball is randomly selected from the bag U.
This ball is not replaced back in U.

- If the selected ball is numbered **0**, then two balls are randomly and simultaneously selected from U. *(We get then 3 balls)*
- If the selected ball is not numbered **0**, then one ball is randomly selected from U. *(We get then 2 balls.)*

Calculate the probability that the sum of numbers on the selected balls is 3.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M' with respective affixes i, –2i, z and z', such that $z'=\dfrac{-2iz}{z-i}$ with $z\neq i$.

1) a- Prove that $(z'+2i)(z-i)$ is a real number.

   b- Deduce that $AM\times BM'=2$.

   c- If M moves on the circle with center A and radius 3, show that M' moves on a circle with center and radius to be determined.

2) In the case where $z'=2i$, write z in exponential form.

3) Let z = x + iy and z' = x' + iy' where x, y, x' and y' are real numbers.

   a- Show that $x'=\dfrac{2x}{x^2+(y-1)^2}$ and $y'=\dfrac{-2\left(x^2+y^2-y\right)}{x^2+(y-1)^2}$.

   b- If $AM=\sqrt{2}$, prove that $x = x'$.

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ as $f(x)=(1-x)e^x+2$.

Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$. Deduce an asymptote (d) to (C).

   b- Determine $\lim\limits_{x\to+\infty} f(x)$, then calculate $f(1)$ and $f(2)$.

2) a- Verify that $f'(x)=-xe^x$ and set up the table of variations of the function f .

   b- Prove that the curve (C) has an inflection point I whose coordinates should be determined.

3) Draw (d) and (C).

4) Denote by (Δ) the line with equation $y=2x$.

   a- Verify that $f(x)-2x=\left(e^x+2\right)(1-x)$. Study, according to the values of x, the relative positions of (C) and (Δ).

   b- Find an antiderivative F of the function f .

   c- Draw (Δ), then calculate the area of the region bounded by the curve (C), the y-axis and the line (Δ).

5) Let g be the function given as $g(x)=\ln\left[f(x)-2\right]$.

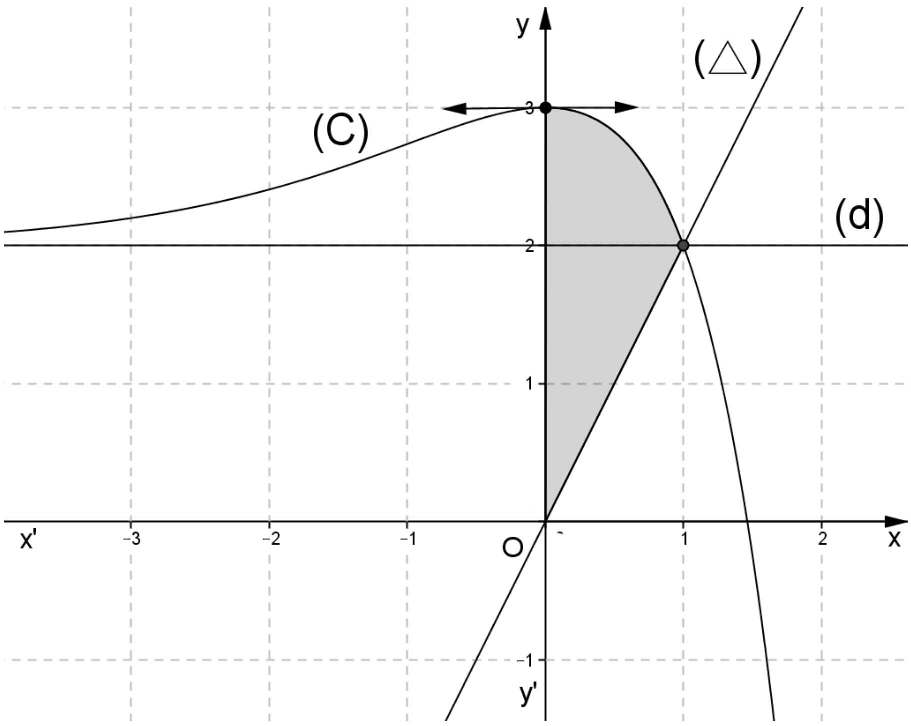
   Denote by (G) the representative curve of g in the system $(O;\vec{i},\vec{j})$.

   a- Verify that the domain of definition of g is $]-\infty\,;1[$.

   b- Is there a point on (G) where the tangent to (G) is parallel to the line (Δ)? Justify.

أسس تصحيح مسابقة الرياضيات

Q.I	Answers	4 pts
1.a	3 = t and 1 = t + 1 then t = 3 and t = 0 which is impossible, thus A ∉ (d). $\overrightarrow{AB}(-1\ ;\ -1\ ;\ 1) = -\overrightarrow{V}(1\ ;\ 1\ ;\ -1)$ thus (AB) // (d).	¾
1.b	A ∈ (P) : 1 + 0 – 1 = 0, 0 = 0. B ∈ (P) : 0 + 1 – 1 = 0, 0 = 0. (d) ⊂ (P) : t + 1 + (−t) – 1 = 0, 0 = 0.	½
2.a	A ∈ (P) and $\overrightarrow{AS}(0\ ;\ -2\ ;\ -2) = -2\overrightarrow{n_{(P)}}(0\ ;\ 1\ ;\ 1)$ thus (AS) ⊥ (P) at A	¾
2.b	Area(BSS') = 2.Area(BSA) = $2.\frac{1}{2}$.AB.AS = $\sqrt{3}\times 2\sqrt{2} = 2\sqrt{6}$ square units.	½
3.a	E ∈ (d) thus E(t ; t + 1 ; −t). AE = 3, $(t-3)^2 + t^2 + (-t)^2 = 9$ then t = 0 and t = 2. Thus, E(0 ; 1 ; 0) and F(2 ; 3 ; −2)	1
3.b	AE = AF = radius, then AEF isosceles at A. Let I be the midpoint of [EF] then I(1 ; 2 ; −1). $\overrightarrow{AI}(-2\ ;\ 1\ ;\ -1)$ is a directing vector of the bisector. Hence, x = −2k + 3, y = k + 1, z = −k (k ∈ ℝ). **Another method :** AE = AF = 3 then $\overrightarrow{W} = \overrightarrow{AE} + \overrightarrow{AF}$ is a directing vector of a bisector. $\overrightarrow{AE}(-3\ ;\ 0\ ;\ 0)$ and $\overrightarrow{AF}(-1\ ;\ 2\ ;\ -2)$ then $\overrightarrow{W}(-4\ ;\ 2\ ;\ -2)$. Hence, x = −4k' + 3, y = 2k' + 1, z = −2k' (k' ∈ ℝ).	½
Q.II	**Answers**	**4 pts**
A.1	$P(M) = \frac{C_3^3}{C_9^3} + \frac{C_4^3}{C_9^3} = \frac{5}{84}$	½
A.2.a	$P(N) = 1 - P(\overline{N}) = 1 - \frac{C_6^3}{C_9^3} = \frac{16}{21}$	½
A.2.b	$P(M\cap N) = \frac{C_3^3}{C_9^3} = \frac{1}{84}$. $P(\overline{M}\cap N) = P(N) - P(M\cap N) = \frac{16}{21} - \frac{1}{84} = \frac{3}{4}$	1
A.3	$P(N/\overline{M}) = \frac{P(\overline{M}\cap N)}{P(\overline{M})} = \frac{\frac{63}{84}}{1-\frac{5}{84}} = \frac{63}{79}$	1
B	P(S = 3) = P(R∩(G and B)) + P(B∩(G)) + P(G∩(B)) $= \frac{C_3^1}{C_9^1}\times\frac{C_2^1\times C_4^1}{C_8^2} + \frac{C_4^1}{C_9^1}\times\frac{C_2^1}{C_8^1} + \frac{C_2^1}{C_9^1}\times\frac{C_4^1}{C_8^1} = \frac{20}{63}$	1
Q.III	**Answers**	**4 pts**
1.a	$(z'+2i)(z-i) = \left(\frac{-2iz}{z-i}+2i\right)(z-i) = \frac{2}{z-i}(z-i) = 2$	¾
1.b	$AM.BM' = \|z-i\|\|z'+2i\| = 2$	½
1.c	$AM = 3, \|z-i\| = 3$, then $BM' = \frac{2}{3}$ Thus, M' varies on the circle with center B and radius $\frac{2}{3}$	½
2	$2i = \frac{-2iz}{z-i}$ then $z = \frac{1}{2}i = \frac{1}{2}e^{i\frac{\pi}{2}}$.	¾
3.a	$x'+iy' = \frac{2i(x+iy)}{i-x-iy} = \frac{-2y+2ix}{-x+i(1-y)} = \frac{(-2y+2ix)(-x-i(1-y))}{x^2+(y-1)^2}$ $x' = \frac{2x}{x^2+(y-1)^2}$, $y' = \frac{-2(x^2+y^2-y)}{x^2+(y-1)^2}$	1
3.b	If AM = $\sqrt{2}$ then $x^2+(y-1)^2 = 2$ so, $x' = x$.	½

<table>
<tr><th>Q.IV</th><th>Answers</th><th>8 pts</th></tr>
<tr><td>1.a</td><td>$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}(e^x - xe^x + 2) = 2$. Thus, $y = 2$ is an asymptote to (C) at $-\infty$.</td><td>½</td></tr>
<tr><td>1.b</td><td>$\lim\limits_{x\to+\infty} f(x) = +\infty$. $f(1) = 2$, $f(2) = 2 - e^2 = -5.33$.</td><td>¾</td></tr>
<tr><td>2.a</td><td>$f'(x) = -xe^x$
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>0</td><td></td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td></td><td>+</td><td>0</td><td>$-$</td><td></td></tr>
<tr><td>f(x)</td><td>2</td><td>↗</td><td>3</td><td>↘</td><td>$-\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td>2.b</td><td>$f''(x) = -(x+1)e^x$ vanishes and changes sign at $x = -1$.
Thus, $I(-1\,;\,2e^{-1} + 2)$ is an inflection point.</td><td>¾</td></tr>
<tr><td>3</td><td>
</td><td>1</td></tr>
<tr><td>4.a</td><td>$f(x) - 2x = (1-x)e^x + 2 - 2x = (1-x)(e^x + 2)$.
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>-1</td><td></td><td>$+\infty$</td></tr>
<tr><td>f(x) – 2x</td><td></td><td>+</td><td>0</td><td>$-$</td><td></td></tr>
<tr><td>Position</td><td></td><td>(C) is above (d)</td><td>(C) and (d) intersect at $(-1\,;\,2)$</td><td>(C) is below (d)</td><td></td></tr>
</table></td><td>1</td></tr>
<tr><td>4.b</td><td>$\int f(x)dx = (2-x)e^x + 2x + c$</td><td>1</td></tr>
<tr><td>4.c</td><td>L’aire = $\int_0^1 [f(x) - 2x]dx = (e-1)u^2$</td><td>¾</td></tr>
<tr><td>5.a</td><td>$f(x) - 2 > 0$ then $x \in]-\infty; 1[$</td><td>½</td></tr>
<tr><td>5.b</td><td>$g'(x) = 2$, $\frac{f'(x)}{f(x)-2} = 2$, $x = 2$ rejected since $2 \notin]-\infty; 1[$.</td><td>¾</td></tr>
<tr><td></td><td></td><td></td></tr>
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