**إرشادات عامة:** - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

## I – (3 points)

***The questions 1) and 2) are independent.***
***All the steps of calculation must be shown.***

**1)** Given $A=\dfrac{2-\dfrac{1}{3}}{2+\dfrac{1}{3}}$ and $B=\dfrac{24\times10^3\times5\times10^6}{8\times\left(10^3\right)^3}$.

**a.** Calculate A and give the result as a fraction in its simplest form.
**b.** Show that B is a natural number.

**2)** Given $C=\dfrac{\sqrt{45}-\sqrt{180}+9}{3+\sqrt{5}\times\sqrt{35}-5\sqrt{7}}$ and $D=\left(1-\sqrt{5}\right)^2$.

**a.** Write C in the form $n-\sqrt{5}$ where $n$ is a natural number.
**b.** Calculate D, then verify that $D=2\times C$.

## II – (3 points)

Given $A(x)=\left(2x-3\right)^2+\left(x-5\right)\left(3-2x\right)$.

**1)** Factorize $A(x)$.

**2)** Let $B(x)=2x^2-5x+3$.
Verify that $B(x)=\left(2x-3\right)\left(x-1\right)$.

**3)** Let $F(x)=\dfrac{\left(2x-3\right)\left(x+2\right)}{B(x)}$.

**a.** For what values of x, is $F(x)$ defined ?
**b.** Simplify $F(x)$.
**c.** Does the equation $F(x)=7$ have a solution? Justify.

## III – (3 points)

**1)** Solve the following system: $\begin{cases} x+y=35 \\ 9x+8y=300. \end{cases}$

**2)** The number of students (girls and boys) of a certain class is 35.
When 10% of the girls and 20% of the boys leave this class to participate in a sportive activity, the number of remaining students is then 30.

**a.** Denote by $x$ the number of girls and by $y$ that of boys of this class.
Write a system of two equations with two unknowns to model the text above.

**b.** Find the number of girls and that of boys in this class.

**IV – (5.5 points)**

In an orthonormal system of axes x'Ox and y'Oy , consider the points $A(-2\,;2)$, $B(0\,;-2)$, $C(5\,;3)$ and $I(-1\,;0)$. Let (d) be the line with equation $y=\frac{1}{2}x+\frac{1}{2}$.

**1)** **a.** Plot the points A, B, C and I.

**b.** Verify that C and I are two points on the line (d). Draw (d).

**2)** Prove that I is the midpoint of [AB].

**3)** **a.** Find the equation of the line (AB).

**b.** Prove that (AB) is perpendicular to the line (d).

**c.** Show that the triangle ABC is isosceles.

**4)** Consider the point $F(7\,;-1)$.

Show that F is the translate of C under the translation with vector $\overrightarrow{AB}$.

**5)** Denote by E the point on the line (AB) so that $x_E=1$.

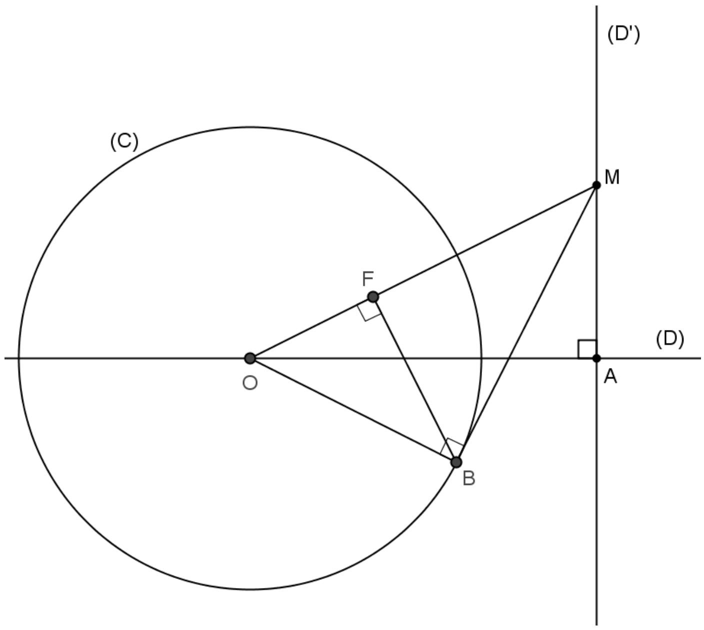
**a.** Show that $y_E=-4$.

**b.** Prove that the quadrilateral CIEF is a rectangle.

**V – (5.5 points)**

In the adjacent figure :

- (D) and (D') are two perpendicular lines at A
- O is a point on (D) so that OA = 6
- (C) is a circle with center O and radius 4
- M is a point on the line (D') so that AM = 3
- (MB) is a tangent through M to the circle (C)
- [BF] is an altitude in the triangle OBM.




**1)** Copy the figure that will be completed in the remaining parts of the problem.

**2)** Show that $OM=3\sqrt{5}$.

**3)** **a.** Show that the two triangles OFB and OBM are similar.

**b.** Deduce that $OF\times OM=16$.

**c.** Calculate OF.

**4)** The two segments [BF] and [OA] intersect at I.

**a.** Write in the two triangles FOI and MOA the ratios equal to $\cos MOA$.

**b.** Deduce that $OI\times OA=16$.

**c.** Calculate OI.

**5)** The line (FB) intersects (D') at E.

Show that (MI) is perpendicular to (OE).

| وزارة التربية والتعليم العالي<br>المديرية العامة للتربية<br>دائرة الامتحانات الرسمية | امتحانات الشهادة المتوسطة | دورة العام 2017 العادية<br>الثلاثاء 6 حزيران 2017 |
|---|---|---|
| عدد المسائل: خمس | مسابقة في مادة الرياضيات | مشروع اسس التصحيح - إنكليزي |

| Part of the ques. | Answer Key | Grade |
|---|---|---|
| | **Question I** | |
| **1a** | $A=\dfrac{2-\frac{1}{3}}{2+\frac{1}{3}}=\dfrac{\frac{6}{3}-\frac{1}{3}}{\frac{6}{3}+\frac{1}{3}}=\dfrac{\frac{5}{3}}{\frac{7}{3}}=\dfrac{5}{7}$ **0.25 + 0.25 + 0.25** | **0.75** |
| **1b** | $B=\dfrac{24\times10^3\times5\times10^6}{8\times\left(10^3\right)^3}=\dfrac{3\times5\times10^9}{10^9}=15$ is a natural number. **0.25 + 0.25 + 0.25** | **0.75** |
| **2a** | $C=\dfrac{\sqrt{45}-\sqrt{180}+9}{3+\sqrt{5}\times\sqrt{35}-5\sqrt{7}}=\dfrac{3\sqrt{5}-6\sqrt{5}+9}{3+5\sqrt{7}-5\sqrt{7}}=\dfrac{-3\sqrt{5}+9}{3}=-\sqrt{5}+3=3-\sqrt{5}$ with $n=3$ (natural number) **0.25 + 0.25 + 0.25** | **0.75** |
| **2b** | $D=\left(1-\sqrt{5}\right)^2=1-2\sqrt{5}+5=6-2\sqrt{5}$ **0.5**<br>$2\times C=2\left(3-\sqrt{5}\right)=6-2\sqrt{5}=D$ **0.25** | **0.75** |
| | **Question II** | |
| **1** | $A(x)=\left(2x-3\right)^2+\left(x-5\right)\left(3-2x\right)$ **0.25** (*change sign*)<br>$A(x)=\left(2x-3\right)\left[\left(2x-3\right)-\left(x-5\right)\right]$ **0.5** (common factor)<br>$A(x)=\left(2x-3\right)\left(2x-3-x+5\right)$<br>$A(x)=\left(2x-3\right)\left(x+2\right)$ **0.25** | **1** |
| **2** | $B(x)=\left(2x-3\right)\left(x-1\right)$<br>$B(x)=2x^2-2x-3x+3$ **0.25**<br>$B(x)=2x^2-5x+3$ **0.25** | **0.5** |
| **3a** | $F(x)=\dfrac{A(x)}{B(x)}=\dfrac{\left(2x-3\right)\left(x+2\right)}{\left(2x-3\right)\left(x-1\right)}$<br>$F(x)$ is defined if $x\neq\dfrac{3}{2}$ and $x\neq1$ **0.25 + 0.25** | **0.5** |
| **3b** | $F(x)=\dfrac{x+2}{x-1}$ **0.25** | **0.25** |
| **3c** | $F(x)=7$<br>$\dfrac{x+2}{x-1}=7$ gives $x=\dfrac{9}{6}=\dfrac{3}{2}$ (rejected since $F(x)$ is not defined) . $F(x)=7$ does not admit any solution. **0.25 + 0.25 + 0.25** | **0.75** |

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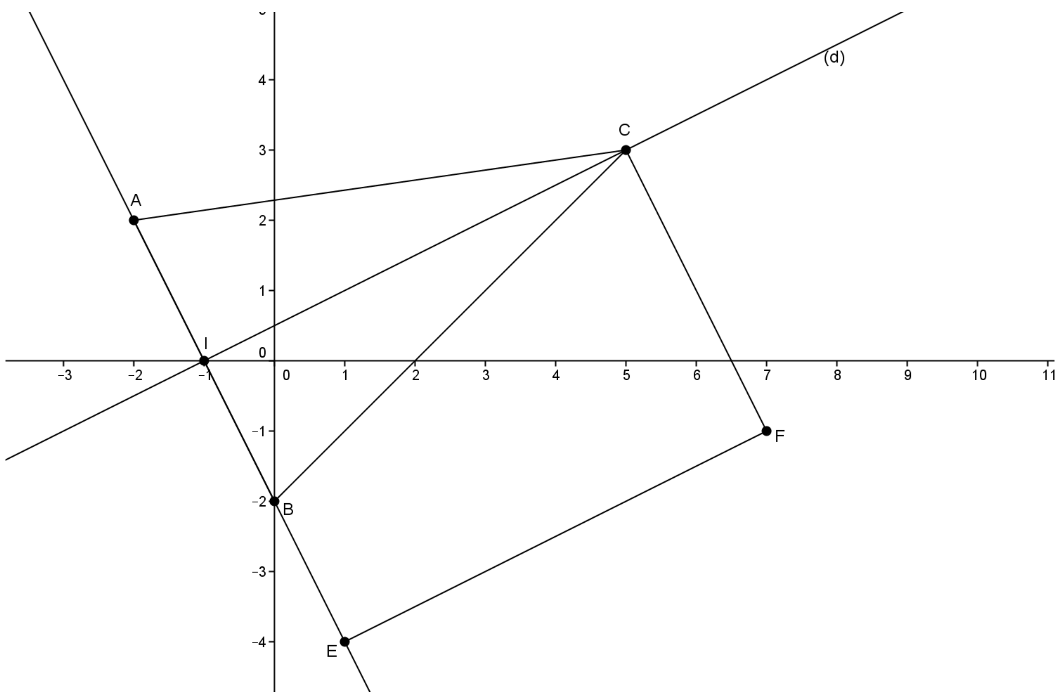
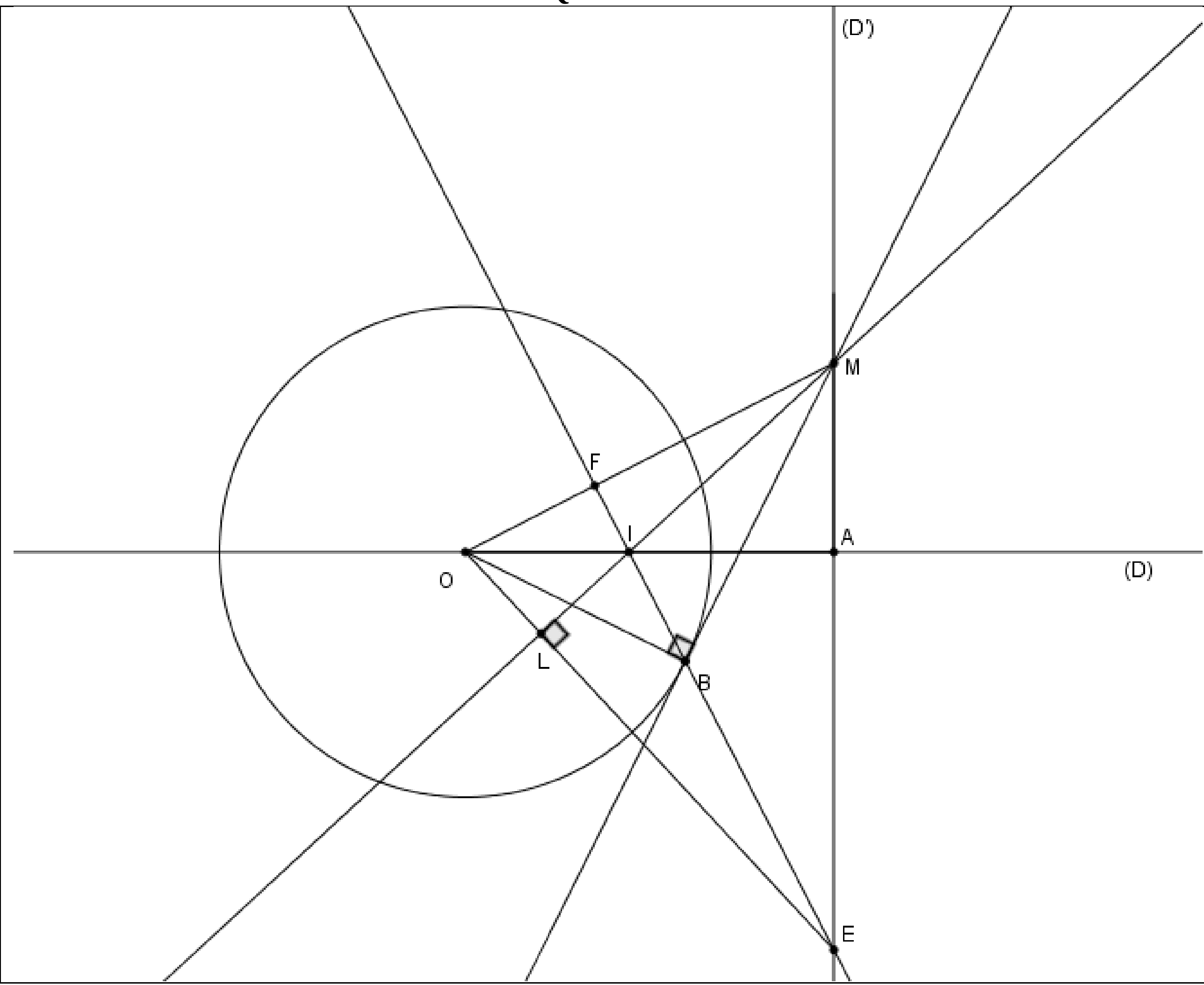
| | **Question III** | |
| **1** | $\begin{cases} x+y=35 \\ 9x+8y=300 \end{cases}$<br>$x=20\ ;\ y=15$ **0.5 + 0.5** | **1** |
| **2a** | $\begin{cases} x+y=35 \\ 0.9x+0.8y=30 \end{cases}$ **0.25**<br>**0.75** | **1** |
| **2b** | $\times 10 \begin{cases} x+y=35 \\ 0.9x+0.8y=30 \end{cases}$ **0.5**<br>$\begin{cases} x+y=35 \\ 9x+8y=300 \end{cases}$ **0.25**<br>$x=20\ ;\ y=15$ **0.25**<br>The number of girls is 20 and that of boys is 15. | **1** |
| | **Question IV** | |
| **1a** |  | **0.5** |
| **1b** | $y_C = \frac{1}{2}x_C + \frac{1}{2}$<br>$3 = \frac{1}{2}(5) + \frac{1}{2}$ **0.25**<br>$\mathbf{3 = 3}$<br><br>$y_I = \frac{1}{2}x_I + \frac{1}{2}$<br>$0 = \frac{1}{2}(-1) + \frac{1}{2}$ **0.25**<br>$0 = 0$ | **0.5** |

| | | |
|---|---|---|
| **2** | $x_I = \frac{x_A+x_B}{2}$<br>$-1 = \frac{-2+0}{2}$<br>$-1 = -1$<br><br>$Y_I = \frac{Y_A+Y_B}{2}$<br>$0 = \frac{2-2}{2}$<br>$0 = 0$ | **0.5** |
| **3a** | $a_{(AB)} = -2$ ; $(AB)$: $y = -2x - 2$ **0.5 + 0.25** | **0.75** |
| **3b** | $a_{(AB)} \times a_{(d)} = -1$ | **0.5** |
| **3c** | (CI) ⊥ (AB) at its midpoint I then ABC is an isosceles triangle of vertex C. | **0.75** |
| **4** | $\overrightarrow{CF}(2;-4) = \overrightarrow{AB}(2;-4)$ then F is the translate of C under the translation with vector $\overrightarrow{AB}$ **0.25 + 0.5** | **0.75** |
| **5a** | $y_E = -2x_E - 2 = -2(1) - 2 = -4$ | **0.5** |
| **5b** | $\overrightarrow{CF}(2;-4) = \overrightarrow{IE}(2;-4)$ then CIEF is a parallelogram **0.5**<br>and $\widehat{CIF} = 90^0$ then it is a rectangle. **0.25** | **0.75** |
| | **Question V** | |
| **1** |  | **0.5** |

| | | |
|---|---|---|
| **2** | OMA is a right triangle at A.<br>$OM^2 = OA^2 + AM^2 = 36 + 9 = 45$ (Pythagorean theorem) **0.25**<br>$OM = \sqrt{45} = 3\sqrt{5}$ **0.5** | **0.75** |
| **3a** | The two triangles OFB and OBM have :<br>$\widehat{OFB} = \widehat{MBO} = 90^0$ **0.5**<br>$\widehat{MOB} = \widehat{FOB}$ (common angle) **0.5**<br>Therefore they are similar | **1** |
| **3b** | $\frac{OF}{OB} = \frac{OB}{OM}$ ; $OF \times OM = OB^2 = 4^2 = 16$ **0.25 + 0.25** | **0.5** |
| **3c** | $OF \times OM = 16$<br>$OF \times 3\sqrt{5} = 16$<br>$OF = \frac{16}{3\sqrt{5}} = \frac{16\sqrt{5}}{15}$ | **0.5** |
| **4a** | $\cos \widehat{MOA} = \frac{OA}{OM}$ **0.5**<br>$\cos \widehat{FOI} = \frac{OF}{OI}$ **0.25** | **0.75** |
| **4b** | $\frac{OA}{OM} = \frac{OF}{OI}$ gives $OI \times OA = OF \times OM = 16$ | **0.5** |
| **4c** | $16 = OI \times OA$.<br>$16 = 6OI$<br>$OI = \frac{8}{3}$ | **0.25** |
| **5** | In the triangle OME we have :<br>[OA] is the first altitude. **0.5**<br>[EF] is the second altitude.<br>[OA] and [EF] intersect at I which is the orthocenter of this triangle. **0.25**<br>[ML] passes through I, then it is the third altitude.<br>Then (MI) $\perp$ (OE) | **0.75** |