ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I - (4 points)

A store sells LCDs and laptops only.
A survey conducted on customers who visit this store revealed that:

- 20% of these customers buy an LCD
- 60 % of the customers who buy an LCD, buy also a laptop
- 20% of the customers who don't buy an LCD buy a laptop.

A customer may buy one laptop, one LCD, both or none.

Part A

A customer is randomly selected from the surveyed customers and interviewed.
Consider the following events:

D : " the customer bought an LCD" L : " the customer bought a laptop "

1) a- Calculate the probability $P(D \cap L)$.
   b- Show that the probability that the customer bought a laptop is 0.28.
2) The customer did not buy a laptop, calculate the probability that this customer bought an LCD.

Part B

The profit of the store from selling an LCD is 150 000 LL and from selling a laptop is 250 000 LL.

Let X be the random variable equal to the profit of the store from each customer.

1) Determine the 4 possible values of X.
2) Calculate the probability $P(X = 0)$.
3) Determine the probability distribution of X.
4) Estimate the average profit obtained by this store for a number of customers equal to 200.

II- (4 points)

A factory produces a certain liquid detergent. The daily production is 200 liters which are poured every morning in a storage container of capacity 520 liters.
During the day, 40% of the quantity stored in the container is sold.

Denote by U_n the quantity of detergent stored in the container in the morning of the nth day just after the daily production is added to the container. Thus $U_1 = 200$.
We admit that $U_{n+1} = 0.6U_n + 200$.

1) Calculate U_3.
2) For all $n \geq 1$, consider the sequence (V_n) defined as $V_n = 500 - U_n$.
   a – Show that (V_n) is a geometric sequence. Calculate its first term V_1 and its common ratio r.
   b - Express V_n then U_n in terms of n.
   c- Will this factory need a second container to store its production? Justify.
3) Each liter of this detergent is sold for 4 000 LL. Calculate the revenue at the end of the first five days.

III- (4 points)

The following table represents Bashir's monthly salary in each year from 2004 to 2010:

Year	2004	2005	2006	2007	2008	2009	2010
Rank of the year: x_i	1	2	3	4	5	6	7
Monthly Salary: y_i (in thousands LL)	1650	1720	1740	1750	1820	1850	1950

Part A

1) Justify that there is a strong positive linear correlation between the two variables x and y.

2) Determine an equation of the regression line $(D_{y/x})$.

3) Assume that the above pattern remains valid till the year 2021.
   a- Estimate Bashir's salary in the year 2012.
   b- Would Bashir's monthly salary reach 2 200 000 LL before 2015? Justify.

Part B

Bashir intends to borrow a loan of 60 000 000LL from a bank.
This bank proposes to him to pay back this loan in terms of monthly payments for 10 years at an annual interest rate of 8% compounded monthly. The bank accepts to offer Bashir this loan only if the amount of the monthly payment does not exceed one third of his monthly salary.

1) Calculate the amount of each monthly payment.
2) Based on the model above and the conditions of the bank, in which year can Bashir get this loan? Justify.

IV- (8 points)

Part A

Consider the function defined over $[0; +\infty[$ as $f(x) = 4xe^{-x+1}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x \to +\infty} f(x)$. Deduce an asymptote to (C).
2) Verify that $f'(x) = 4(1-x)e^{-x+1}$ and set up the table of variations of f.
3) Calculate $f(2)$, $f(3)$ and draw (C).
4) The line (D) with equation $y = 2.5$ intersects (C) in two points with abscissas α and β.
   We admit that $0.31 < \alpha < 0.33$, verify that $2.30 < \beta < 2.32$.

In what follows, let $\alpha = 0.32$ and $\beta = 2.31$.

Part B

A factory produces toys. The demand, expressed in thousands of toys, is modeled as $D(x) = 4e^{-x+1}$ where x is the unit price of one toy in thousands of LL.

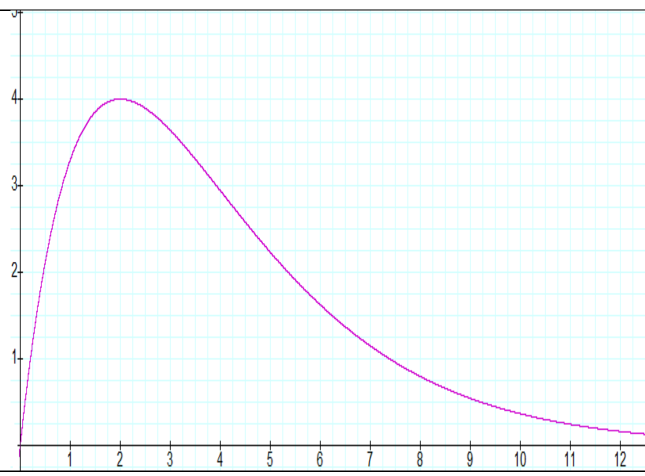
1) Verify that f(x) represents the revenue obtained from selling the demanded quantity and that it is expressed in millions LL. $(0.2 \le x \le 6)$
2) a- Find the unit price of a toy so that the revenue is maximal. Determine, in millions LL, this maximal revenue.
   b- Calculate E(x), the elasticity of the demand with respect to the price.
   c- Determine the value of this elasticity when the revenue is maximal. Interpret.
3) a- If the revenue exceeds 2 500 000 LL, in which interval will the unit price of a toy vary?
   b- Deduce the interval in which the demanded quantity of toys will vary.

مشروع معيار التصحيح **مسابقة في مادة الرياضيات**

QI	Short Answers	M
A1a	$p(D\cap L)=P(L/D)\times P(D)=\frac{12}{100}=0.12$	1
A1b	$p(L) = p(L\cap D) + p(L\cap \overline{D}) = (0.2)(0.6) + (0.8)(0.2) = 0.28$	1
A 2	$P(\overline{D}/L)=\frac{P(\overline{D}\cap L)}{P(L)}=\frac{0.8\times 0.2}{0.28}=0.571$	1
B1	$X\in \{0;\ 150000;\ 250000\ ;\ 400000\}$	0.5
B2	$p(X=0) = p(\overline{D}\cap\overline{L}) = 0.64$	1
B3	$p(X=250000) = p(\overline{D}\cap L) = 0.2\times 0.8 = 0.16$ $p(X=150000) = p(D\cap\overline{L}) = 0.2\times 0.4 = 0.08$ $p(X=400000) = p(D\cap L) = 0.12$ $X=x_i$: 0 ; 150000 ; 250000 ; 400000 $P_i = p(X=x_i)$: 0.64 ; 0.08 ; 0.16 ; 0.12	1.5
B4	E (X) = 0 +12000 + 40000 + 48000 = 100000 L.L. ; 200 × E(X) = 20 000 000 LL. So the average profit achieved in the store is 20 000 000 LL.	1

QII	Short Answers	M
1	$U_2 = 0.6U_1 + 200 = 120 + 200 = 320$ $U_3 = 0.6\,U_2 + 200 = 192 + 200 = 392$	1
2a	$V_{n+1}=500-U_{n+1}=500-0.6\,U_n-200=-0{,}6U_n+300=-0.6(500-U_n)+300=0{,}6V_n$. Therefore (V_n) is a geometric sequence with common ratio r = 0.6 and first term V_1= 300.	2
2b	$V_n=V_1.q^{n-1}=300\times(0.6)^{n-1}$. $U_n=500-300\times(0.6)^{n-1}$.	1
2c	No since $U_n-500=-300\times 0.6^{n-1}<0$	1.5
3	$U_1=200; U_2=320; U_3=392; U_4=435.2; U_5=461.12$ $U_1+U_2+U_3+U_4+U_5=1808.32$ $R = 4000\times 0.4\times 1808.32=2{,}893{,}312$ LL.	1.5

QIII	Short Answers	M
A1	$r = 0.97 \approx 1$	1
A2	$D_{y/x}$: y = 44,285x +1605.71.	1
A3a	For x = 9 ; y =2,0042,65 LL	1
A3b	x =12 ; y =2137128 LL < 2200000 LL Then it won't reach 2,200,000 LL in 2015	1
B1	$V = R\dfrac{1-(1+i)^{-n}}{i}$; $60\ 000\ 000 = R\dfrac{1-\left(1+\dfrac{8}{100\times 12}\right)^{-120}}{\dfrac{8}{100\times 12}}$; $60\ \ 000\ 000 = R\times\dfrac{1-0.45052346}{0.0066666}$; $60\ \ 000\ 000 = R\times 82.504$; $R = 727237.46$ LL.	1.5
B2	727237,46×3 = 2181712,38 LL 2181,71238= 44.285x +1605.71 $\Rightarrow$ x=13.006 >13 Thus x=14 that is in 2017.	1.5

QIV	Short Answers	N
A1	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} 4xe^{-x+1} = \lim\limits_{x\to+\infty}\dfrac{4x}{e^{x-1}} = \lim\limits_{x\to+\infty}\dfrac{4}{e^{x-1}} = 0$ (L'HR) The axis of abscissas is an asymptote to (C).	1
A2	$f(x) = 4xe^{-x+1}$; $f'(x) = 4e^{-x+1} - 4xe^{-x+1}$ $f'(x) = 4(1-x)e^{-x+1}$ x: 0, 1, $+\infty$ f '(x): + 0 − f (x): 0 ↗ 4 ↘ 0	2
A3	 $f(2) = 8e^{-1} \approx 2,94$ $f(3) = 12e^{-2} \approx 1,6$	2.5
A4	$(f(2,3)-2,5)\times(f(2,32)-2,5) = 0,07\times(-0,03) < 0$	1
B1	The revenue is $(x)(1000)\times\big(d(x)\big)(1000) = f(x)$ in millions.	1
B2a	The revenue is maximum for a price of 1000 LL and it is equal to 4 000 000 LL	1.5
B2b	$d(x) = 4e^{-x+1}$ so $d'(x) = -4e^{-x+1}$ then $E(x) = -x\dfrac{d'(x)}{d(x)} = x$	1
B2c	For x=1 the revenue is maximum ; Unit Elasticity	1.5
B3a	Between 321 LL and 2309 LL	1.5
B3b	$d'(x) < 0$; 0.32 <x<2.31 ; and d is decreasing therefore d(2.31)< d(x)< d(0.32). Thus the demand is between 1080 and 14973 toys.	1