


المادة: الرياضيات الشهادة: المتوسطة نموذج رقم - ٤ - المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز العلمي للبحوث والأبحاث
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نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
 - يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

### I-(2 points)

Answer "True" (T) or "False" (F) for each of the following statements. Justify your answer.

- 1) The number  $\sqrt{3} - 1$  is a solution of the equation  $x^2 + 2x - 2 = 0$ .
- 2) The price of an item is reduced by 20%, and then by 20%. After the two reductions, the price of the item is reduced by 40%.
- 3) For any positive real number  $x$ , the number  $[(x + 1)^2 - (x - 1)^2]$  is positive.
- 4) If a triangle ABC is right-angled at A, then  $\cos^2 \hat{B} + \cos^2 \hat{C} = 1$ .

### II- (5.5 points)

In an orthonormal system of axes  $(x'Ox)$  and  $(y'Oy)$ , consider the three points  $A(-1, -3), B(-5, 1)$ , and  $E(2, 0)$ .

- 1)
  - a) Plot the three points A, B, and E.
  - b) Let (D) be a line with equation  $y = x + 2$ . Show that (D) is the perpendicular bisector of the segment [AB].
- 2) Let (C) be the circle with diameter [BE].
  - a) Calculate the coordinates of K, the center of (C), and show that A is a point on (C).
  - b) Let M be the point with coordinates  $(-\frac{3}{2}, m)$ , where  $m$  is a real number. Calculate  $m$  so that M is on (C).
- 3) Let F be translate of E under the translation with vector  $\overrightarrow{AB}$ .
  - a) Determine the coordinates of point F, and determine the nature of the quadrilateral ABFE.
  - b) Write an equation of the line (D'), the translate of line (D) by the translation of vector  $\overrightarrow{KA}$ .

### III-(2.5 points)

Before the World Cup, a company has 1500 TVs in its stores. For one week, the director of the company registered, each day, the percentage of televisions sold in terms of the stock purchased. The table below shows the sales within this week.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Payment percentage	8%	10%	15%	12%	20%	30%

- 1) Calculate the number of televisions sold every day.
- 2) What is the percentage of televisions that are sold?
- 3) Draw the bar graph that represents the distribution.

**IV- (3 points)**

- 1) Let  $Q(x) = (x - 2)^2 - 5(x - 3)(x - 2) + x^2 - 4$ .
  - a) Develop, reduce, and order  $Q(x)$ .
  - b) Solve the equation  $Q(x) = -30$ .
  - c) Show that  $Q(x) = (x - 2)(-3x + 15)$ .
- 2) Let  $D(x) = x^2 - 4x + 4 + (2x - 4)(x + 3)$  and  $F(x) = \frac{Q(x)}{D(x)}$ .
  - a) Factorize  $D(x)$ .
  - b) Determine the values of  $x$  for which  $F(x)$  is defined.
  - c) Simplify  $F(x)$ , then solve the equation  $F(x) = 2$ .

**V- (2 points)**


My cousin told her husband: "I bought 4 identical shirts and 3 identical pants for 100 000 LL. But if I had bought six of the same shirts and 5 of the same pants, I would have paid 120 000 LL."  
He replied: "your story is impossible."

- 1) Write a system of equations to model the speech of my cousin.
- 2) Who was right: My cousin or her husband? Justify your answer.

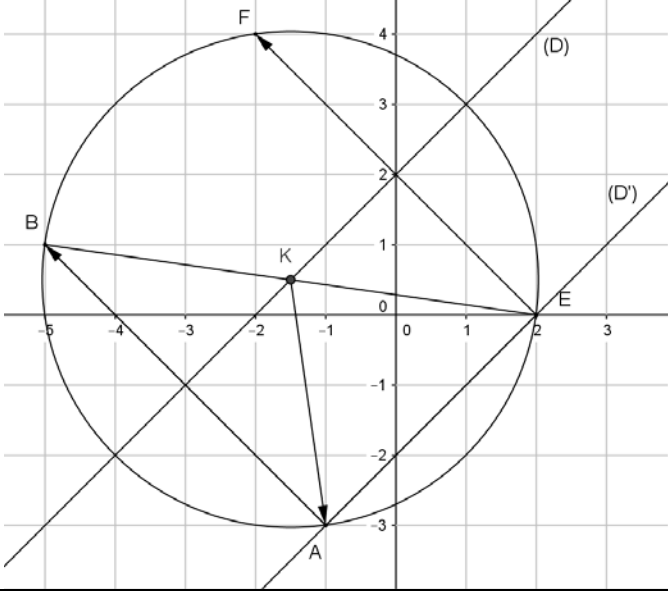
**VI -(5 points)**

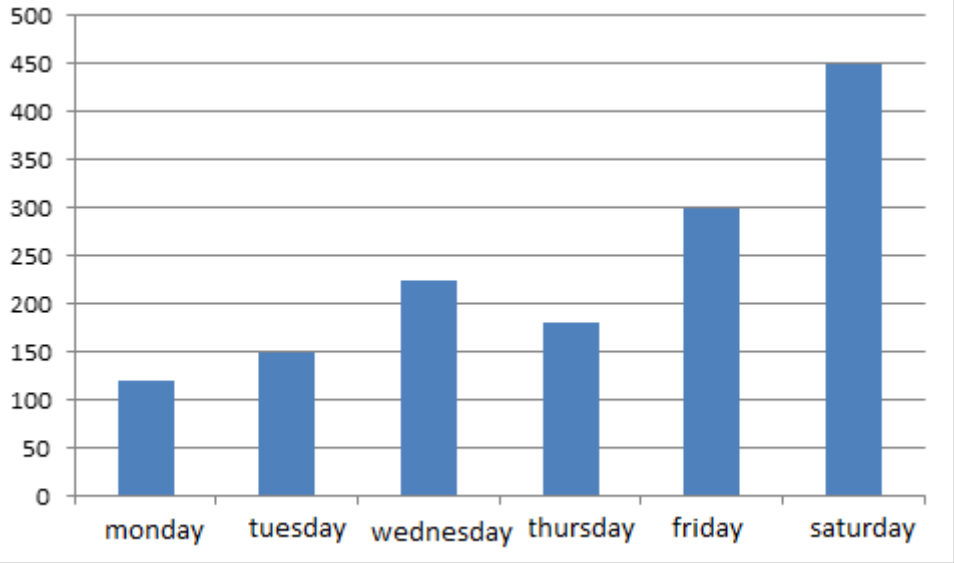
Let  $(C)$  be a semicircle with center  $O$  and diameter  $[AB]$  such that  $AB = 6$  cm. The perpendicular to  $[AB]$  at  $O$  intersects  $(C)$  at  $F$ . Let  $M$  be a point on the arc  $\widehat{BF}$ . The segment  $[AM]$  intersects the segment  $[OF]$  at  $D$ . The bisector of angle  $\widehat{FOM}$  intersects the segment  $[AM]$  at  $I$  and  $(C)$  at  $E$ .

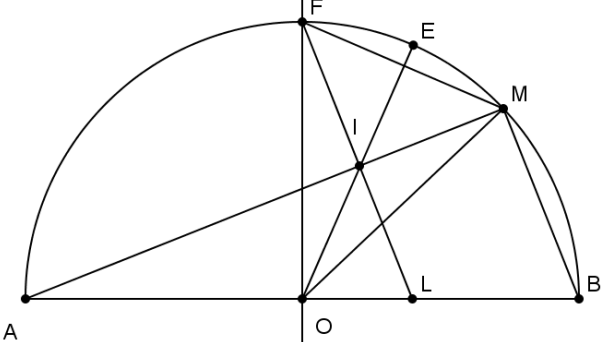
- 1) Draw a figure.
- 2)
  - a) Prove that the two triangles  $AMB$  and  $AOD$  are similar.
  - b) Then calculate  $AD \times AM$ .
- 3) Prove that the triangle  $FIM$  is right isosceles.
- 4) The line  $(FI)$  intersects  $(AB)$  at  $L$ .  
Prove that  $\frac{LA}{LB} = \frac{IA}{IF}$ .
- 5)  $H$  is the orthogonal projection of  $M$  on segment  $[AB]$ . Prove that  $MH = \frac{3MB}{AD}$ .

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أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

Question	اسس التصحيح	Note
I	1) True Justification: substitute x by $(\sqrt{3} - 1)$ the equation is verified.	0.5
	2) False Justification : if x is the initial price, after first the price reduction is $0.8x$ ; after the second the price is $0.8 \cdot (0.8x) = 0,64x$ . the price reduction is 36%.	0.5
	3) True Justification : $(x+1)^2 - (x-1)^2 = 4x$ positive ,product of 2 positives numbers.	0.5
	4) True. Justification : $\cos^2 \hat{C} + \cos^2 \hat{B} = \frac{AC^2}{BC^2} + \frac{AB^2}{BC^2} = \frac{AC^2+AB^2}{BC^2} = 1$	0.5
II	1) 	0.5
	b) Slope of (AB) = -1, $a_{(AB)} \times a_{(D)} = -1$ then (D) $\perp$ to (AB). The midpoint of [AB] have coordinates $(-3, -1)$ et $-1 = -3 + 2$ which is on (D) then (D) is the perpendicular bisector of [AB].	1
	a) K midpoint of [EB] then $K(-3/2, 1/2)$ Radius of the circle = $\frac{EB}{2} = \frac{\sqrt{50}}{2}$ or $AK = \sqrt{\frac{50}{4}} = \frac{EB}{2} =$ radius, K is on (C).	1
	2) b) $KM = \sqrt{(m - \frac{1}{2})^2}$ , M is on (C) then $KM =$ radius, $\sqrt{(m - \frac{1}{2})^2} = \sqrt{\frac{50}{4}}$ then $(m - \frac{1}{2})^2 = \frac{50}{4}$ $(m - \frac{1}{2}) = \frac{\sqrt{50}}{2}$ or $(m - \frac{1}{2}) = \frac{-\sqrt{50}}{2}$ then $m = \frac{\sqrt{50}}{2} + \frac{1}{2}$ or $m = \frac{-\sqrt{50}}{2} + \frac{1}{2}$ .	1
	3) a) $\vec{EF} = \vec{AB}$ , $x_F - 2 = -4$ and $y_F = 4$ then F $(-2, 4)$ . ABFE is a rectangle.(parm + right angle)	1

		<p>b) K point on (D) and A translate of K, (AE) // to (D) then (D')= (AE) .</p> <p>slope of (AE)=<math>\frac{y_E - y_A}{x_E - x_A} = 1</math> or <math>a_{(D')} = a_{(D)} = 1</math>, <math>y = x + b</math> or A is on (D') then <math>y_A = x_A + b</math>, <math>b = -2</math></p>	1														
III	1)	<table border="1"> <thead> <tr> <th>Day</th> <th>Monday</th> <th>Tuesday</th> <th>Wednesday</th> <th>Thursday</th> <th>Friday</th> <th>Saturday</th> </tr> </thead> <tbody> <tr> <td>Frequency</td> <td>120</td> <td>150</td> <td>225</td> <td>180</td> <td>300</td> <td>450</td> </tr> </tbody> </table>	Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Frequency	120	150	225	180	300	450	1,25
	Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday										
	Frequency	120	150	225	180	300	450										
2)	The percentage of televisions and not sales is $\frac{75}{1500} = 5\%$	0.5															
3)		0,75															
IV	1)	<p>a- <math>Q(x) = (x - 2)^2 + 5(x - 3)(2 - x) + x^2 - 4</math>.</p> $= x^2 - 4x + 4 + 5(-x^2 + 5x - 6) + x^2 - 4$ $= -3x^2 + 21x - 30$	0.5														
		<p>b- <math>Q(x) = -30</math> then <math>Q(x) + 30 = 0</math> then <math>x = 0</math> or <math>x = 7</math></p>	0.5														
		<p>c- <math>Q(x) = (x - 2)^2 + 5(x - 3)(2 - x) + (x - 2)(x + 2)</math></p> $= (x - 2)[(x - 2) - 5(x - 3) + (x + 2)]$ $= (x - 2)(-3x + 15).$	0.5														
	2)	<p>a- <math>D(x) \neq 0</math> then <math>x^2 - 4x + 4 + (2x - 4)(x + 3) \neq 0</math></p> <p><math>(x - 2)(3x + 4) \neq 0</math>. Then <math>x \neq 2</math> and <math>x \neq \frac{-4}{3}</math></p>	0.75														
		<p>b- <math>F(x) = \frac{(x - 2)(-3x + 15)}{(x - 2)(3x + 4)} = \frac{-3x + 15}{3x + 4}</math></p> $F(x) = 2, \frac{-3x + 15}{3x + 4} = 2$ $-3x + 15 = 2(3x + 4)$ $x = \frac{7}{9}$	0.25														
			0.5														
V		<p>Let <math>x</math> be the price of tishirt and <math>y</math> the price of pants.</p> $\begin{cases} 4x + 3y = 100\,000 \\ 6x + 5y = 120\,000 \end{cases}$	1,25														
		$y = -80\,000$ LL impossible my cousin is wrong .	0,75														

	1)		0,5
VI	2)	<p>a) <math>\widehat{AMB} = 90^\circ</math> (AMB inscribed in semicircle (C)) <math>\widehat{AOD} = 90^\circ</math></p> <p><math>\widehat{ADO} = \widehat{ABM} = 90 - \widehat{ADO}</math></p>	1
		b) $\frac{AM}{AO} = \frac{AB}{AD}$ then $AM \times AD = AB \times AO = 6 \times 3 = 18$ .	0,5
	3)	<p>(OE) axis of symmetry in the isosceles triangle OFM. I point of the axis, then <math>IF = IM</math> then the triangle IFM is isosceles.</p> <p><math>\widehat{AMF} = \widehat{IMF} = 90 / 2 = 45^\circ</math>, then FIM right isosceles triangle.</p>	1
	4)	<p>(FI) // (MB) two perpendiculars to (MB). Thales: <math>\frac{LA}{LB} = \frac{IA}{IM}</math> or <math>IM = IF</math> then <math>\frac{LA}{LB} = \frac{IA}{IF}</math>.</p>	1
	5)	<p>In the triangle AMB: <math>MH \times AB = MA \times MB</math></p> <p><math>MH \times 6 = \frac{18}{AD} \times MB</math> then <math>MH = \frac{18 \times MB}{6 \times AD} = \frac{3 \times MB}{AD}</math></p>	1