المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: الثانوية العامة الفرع: العلوم العامة نموذج رقم -٣- المدّة : أربع ساعات

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

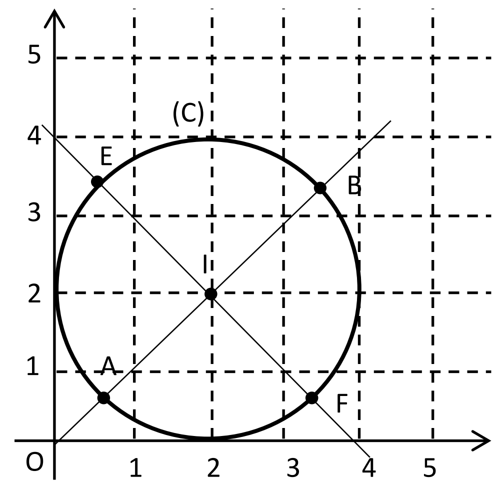
I- (2 points)

In the complex plane referred to the orthonormal system $(O ;\vec{u};\vec{v})$
(C) is the circle with the center I(2 ;2) and radius 2 ;

In the next figure :

- (D) and (D') are two lines with respective equations :
  y = x and y = - x + 4 ;
- (D) intersects (C) at A and B;
- (D') intersects (C) at E and F.

Answer to each question by true or false and jutify :



1) The affix of B is : $Z_B = (\sqrt{2}+2)e^{i(\frac{\pi}{4})}$
2) The affixes of the points I, F and B verify the relation: $Z_B = i\,(Z_F - 2 - 2i)$.
3) Let S be the direct plane similitude that maps A onto B and I onto F, then the measure of the angle of SoS is $\frac{\pi}{4}$.
4) The set of the points M with affix z verifying the two conditions: $|z-1| = |z-i|$ and $|z-2-2i| = 2$ is the segment [AB].

II- (3 points)

In the space referred to the system $(O\ ;\vec{i}\ ,\ \vec{j}\ ;\vec{k})$. Consider the points A(2;1;0) ; B(0;1;3),

the line (d) : $\begin{cases} x = 4t \\ y = 2 \\ z = -3t+3 \end{cases}$ $(t \in IR)$ and the plane (P) with equation : $3x - 4z = 0$

1) **a-** Show that (AB) and (d) are skew.
   b- Show that an equation of the plane (Q) containing (d) and parallel to (AB) is $y - 2 = 0$.
   c- Calculate the distance from A to (Q).
2) **a-** Show that (P) and (Q) are perpendicular and give the parametric equations of (Δ).The intersection line of (P) and (Q).
   b. Let S $(1;2;\frac{-3}{2})$ be a point in the space. Show that S is equidistant from (P) and (d).

III- (2 points)
An urn contains 4 black balls , 3 white balls and n red balls . (n> 1)

Part A:

In this part , suppose that n = 2. We select randomly and simultaneously 3 balls from the urn.

1) Calculate the probability to select three balls having same color.

2) Let E be the event: "Amoung the three balls selected, we obtain exactly two balls with same color.
Show that P(E) is equal to $\frac{55}{84}$.

Partie B:

We select randomly and simultaneously two balls from the (n + 7) balls.
Denote by X the random variable that is equal to the number of red balls obtained.

Show that $P(X= 2) = \frac{n(n-1)}{(n+6)(n+7)}$.

1) Determine the probability distribution of X.

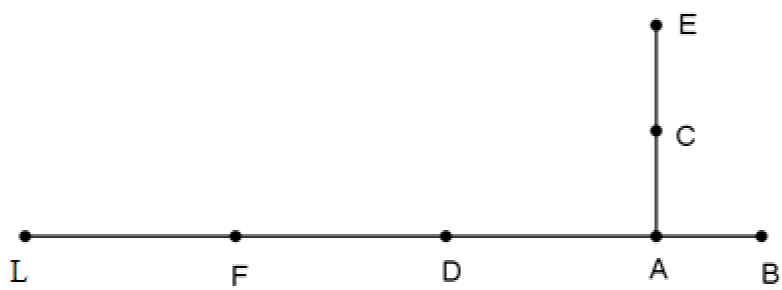
2) Knowing that X = 0, prove that the probability to select two balls of different color is independent of n .

3) Calculate n so that the mathematic expected value E(x) is equal to 1.

IV-(3 points)

In the next figure, (AE) and (BL) are two perpendicular lines so that:

AB = AC = 1, AE = AD = DF = FL = 2.



Let S be the direct similitude of the plane that maps A onto D and C onto F.

1) Determine the ratio and the angle of S.

2)

a- G is a point so that $\overrightarrow{DG} = \overrightarrow{AE}$, prove that S(B)=G.

b- Find S(E) .

3) Let H and K the respective midpoints of [BE] and [GL].The lines (AH) and (DK) intersect at I. The lines (AH) and (DG) intersect at O.

a- Prove that S(H)=K and S(D) = O.

b- Deduce that I is the center of S.

4) R is a rotation with center B and angle $\frac{\pi}{2}$. J is the point of intersection of (BG) and (AE).

a-What is the nature of SoR ?

b- Prove that J is the center of SoR.

5) The complex plane is referred to the orthonormal sytem $(A;\overrightarrow{AB},\overrightarrow{AC})$.

a- Give the complex form of S and deduce the affix of I.

b- Determine the affix of O then compare $\overrightarrow{IO}$ and $\overrightarrow{IA}$.

c - M is a variable point so that $Z_M = x+2(1-x)\,i$ and M'= S(M).

Determine $Z_{M'}$, and deduce that M' moves on a line to determine its equation .

V- (3 points)

Consider a right angled triangle OBF at O with OF=3 and OB=4.

M is a variable point so that $\left\|\overrightarrow{MO}\wedge\overrightarrow{MB}\right\| = 2MF$.

<u>Part A</u>

1) Prove that M moves on a Hyperbol (H) with focus F , directrice (OB) and eccentricity e=2

Determine the focal axis of (H).

2) A is a point so that $\overrightarrow{OA}=\frac{1}{3}\overrightarrow{OF}$ and A' is the symmetric of F with respect to O.

a- Prove that A and A' are the vertices of (H) .

b- Deduce the center I of (H) and the second focus F'.

3) The circle with center I and radius 2 intersects (OB) at G and G'

Prove that (IG) and (IG') are asymptotes to (H).

4) C is a point defined as $\overrightarrow{FC}=\frac{3}{2}\overrightarrow{OB}$.

a- Prove that C is a point on (H).

b- Calculate CF'-CF and deduce CF'.

c-Prove that (OC) is a bissector of $F\hat{C}F'$.

d-Plot (H).

Part B

Consider the orthonormal system $(I,\vec{i},\vec{j})$ with $\vec{i}=\frac{1}{3}\overrightarrow{OF}$ and $\vec{j}=\frac{1}{4}\overrightarrow{OB}$.

1) Write an equation of (H).

2) Write the equations of the asymptotes to (H).

3) (P) is a parbola with vertex V(0,2) and focus R(0,3).

a- Write an equation of (P).

b- Show that (P) is tangent to (H) at L(4 :6) and another point to be determined.

VI- (7 points):

Let f be the function defined over IR as $f(x)=x-\ln(x^2+1)$ and (C) its representative curve

in the orthonormal sytem $(O;\vec{i};\vec{j})$.

Part A

1) Calculate $\lim\limits_{x\to -\infty} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$.

2) Show that the curve (C) has a parabolic branch in $+\infty$ and $-\infty$ parallel to the line (d) with equation y=x.

a- Show that $f'(x)=\dfrac{(x-1)^2}{x^2+1}$ and set up the table of variations of f.

b- Verify that (d) is tangent to(C) at O and determine a tangent in x=1.

c -Draw (C) and the line (d).

3) The function f has over IR an inverse function f^{-1} and the point of intersection of (C_f) and $(C_{f^{-1}})$ is on the first bisector.

a- Solve the inequality $f(x)<x$. Deduce the values of x so that $f^{-1}(x)>x$.

b-Determine the parabolic branch of f^{-1}.

c- Plot the curve of f^{-1} in the same system of that of f.

4) Calculate $\int f(x)dx$. Deduce the area of the domain bounded by the curves of f and f^{-1} and the lines with equations x = 4 and y = 4.

5) Let g be the function defined over IR as $g(x)=\dfrac{4}{1+x^2}$,and let h = fog.

a- Show that h is defined over IR.

b-Calculate $\lim\limits_{x\to -\infty} h(x)$ and $\lim\limits_{x\to +\infty} h(x)$. Deduce an equation of an asymptote to the curve of h.

c- Calculate h ‘(1).

part B

Let (u_n) the sequence defined as $\begin{cases} u_0 = 1 \\ u_{n+1} = u_n - \ln(u^2_n + 1) \end{cases}$ with $n \in N$.

1) a-Show that if $x \in [0;1]$ then $f(x) \in [0;1]$.

b-Deduce that for all $n \in N$, $u_n \in [0;1]$.

2) Discuss the variations of the sequence (u_n) .

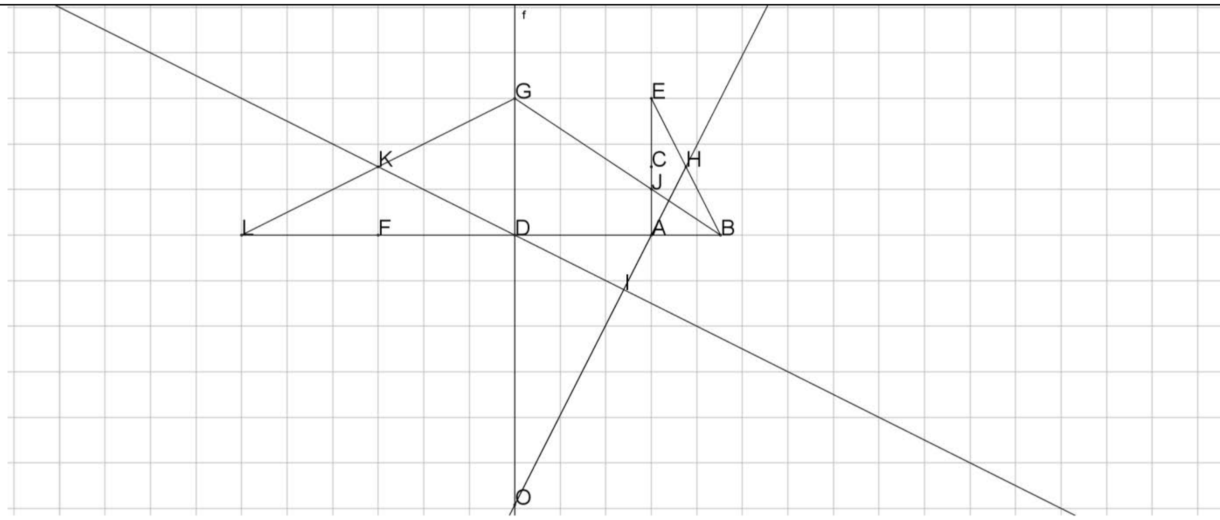
3) a- Show that the sequence (u_n) is convergent.

b- Determine the limit of the sequence (U_n).

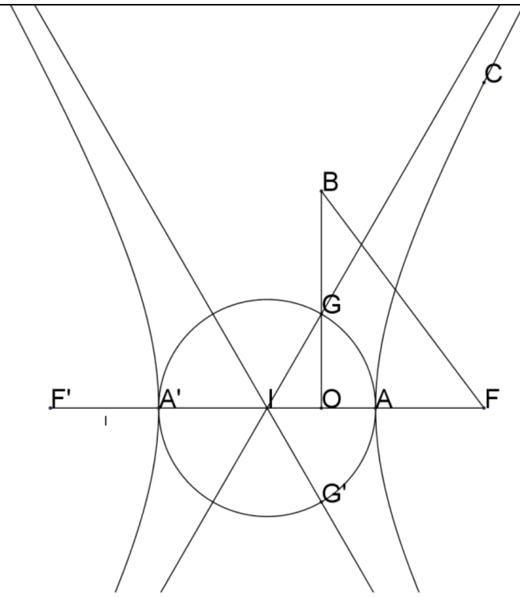
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

	Solution	
I-	**Question I**	**4 pts**
1)	False. The affix of B is : $Z_B = (2\sqrt{2}+2)e^{i(\frac{\pi}{4})}$	**1**
2)	False. The affixes of the points I, F and B satisfy the relation: $Z_{IB} = i\, Z_{IF}$. $Z_B = i\,(Z_F - 2 - 2i) + 2 + 2i$	**1**
3)	False . A measure of the angle of S o S is $\frac{\pi}{2}$	**1**
4)	False. The set of th points M with affix z verifing the two conditions: $\lvert z-1\rvert = \lvert z-i\rvert$ and $\lvert z-2-2i\rvert = 2$ is {A,B}.	**1**
	Question II	**6pts**
1-a	K(0;2;3) is a point on (d) and $\vec{KA}.(\vec{AB}\wedge\vec{V}_d) \neq 0$ Then (d) and (AB) are skew.	**1**
1-b	$(d)\subset(Q)$ since $y = 2$; $\vec{AB} \perp \vec{n}_Q$ since $\vec{AB}.\vec{n}_Q = 0$. K(0 ; 2 ; 3)$\in$(d) ; $\vec{KM}.(\vec{V_d}\wedge\vec{AB}) = \begin{vmatrix} x & y-2 & z-3 \\ 4 & 0 & -3 \\ -2 & 0 & 3 \end{vmatrix} = 0$; y-2 = 0.	**1.5**
1-c	$d(A;(Q)) = \frac{\lvert 1-2\rvert}{\sqrt{1}} = 1$	**0.5**
2-a	$\vec{N}_P \perp \vec{n}_Q$ because $\vec{N}_P.\vec{n}_Q = 0$. (P) and (Q) are perpendicular .. $(P)\cap(Q) = (\Delta)$ and (P):3x $-$4z =0 , (Q): Y $-$2 =0 and z = m therefore	**1.5**

	x= $\frac{4}{3}m$ and y=2 then (Δ) : $\begin{cases} x = \frac{4}{3}m \\ y = 2 \\ z = m \end{cases}$	
2-b	S(1;2;$\frac{3}{2}$) : K(0 ; 2 ; 3) then $\vec{SK} \wedge \vec{V}_d = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & \frac{3}{2} \\ 4 & 0 & -3 \end{vmatrix} = 3\vec{j}$ d(S ;(P)) = $\frac{\lvert 3-6 \rvert}{\sqrt{25}} = \frac{3}{5}$ d(S ;(d)) = $\frac{\lVert \vec{SK} \wedge \vec{V}_d \rVert}{\lVert \vec{V}_d \rVert} = \frac{\lvert 3 \rvert}{\sqrt{25}} = \frac{3}{5}$	**1**
	Question III	**4pts**
A-1	$\frac{C_3^3 + C_4^3}{C_9^3} = \frac{5}{84}$	**0.5**
A-2	P(E)= $\frac{C_3^2.C_6^1}{C_9^3} + \frac{C_4^2.C_5^1}{C_9^3} + \frac{C_2^2.C_7^1}{C_9^3} = \frac{55}{84}$	**1**
B-1	P(X=2) = $\frac{C_n^2}{C_{n+7}^2} = \frac{n(n-1)}{(n+6)(n+7)}$	**0,5**
B-2 **B-3**	P(X=0) = $\frac{C_7^2}{C_{n+7}^2} = \frac{42}{(n+6)(n+7)}$ et P(X=1) = $\frac{C_n^1 C_7^1}{C_{n+7}^2} = \frac{14n}{(n+6)(n+7)}$ P(two different colors /X = 0) = $\frac{C_4^1 \times C_3^1}{C_7^2} = \frac{4}{7}$.	**1**
B-4	E(X)=$\frac{2n^2+12n}{(n+6)(n+7)}$=1 then n=7	**1**

	Question IV	
		
1	S is the similitude that maps A onto D and C onto F. Ratio $\frac{DF}{AC}=2$ and $\left(\overrightarrow{AC},\overrightarrow{DF}\right)=\frac{\pi}{2}+2k\pi$	**1**
2-a	$\left(\overrightarrow{AB},\overrightarrow{DG}\right)=\frac{\pi}{2};\frac{DG}{AB}=2$ then S (B) = G.	**0.5**
2-b	S(A)=D and S(C)=F but C midpoint of [AE] then S(C) is the midpoint of S([AE]) then S(E)=L	**0.5**
3-a	H midpoint of [BE] then S(H)=K midpoint of [GL]. Since S(A)=D then (AH)$\perp$(DK) and $\left(\overrightarrow{AD},\overrightarrow{DO}\right)=\frac{\pi}{2}+2k\pi$; $\frac{DO}{DA}=tanD\hat{A}O=tanA\hat{B}E=2$ then S(D)=O	**1**
3-b	$S(I)=S\left((AH)\cap(DK)\right)=(DK)\cap(OH)=I$	**0.5**
4-a	$SoR=S'(?,2,\pi)=h(?,-2)$.	**0.5**
4-b	$B\overset{R}{\rightarrow}B\overset{S}{\rightarrow}G$ but $\overrightarrow{JG}=-2\overrightarrow{JB}$ then J center of SoR	**0.5**
5-a	z'=2iz-2 et $Z_I=\frac{-2}{5}-\frac{4i}{5}$	**0.5**
5-b	$Z_O=-2-4i\Rightarrow\overrightarrow{IO}=-4\overrightarrow{IA}$	**0.5**

5-c	$Z_{M'} = -6+4x+2ix$ and M moves on a line with equation x-2y+6=0 .	**0.5**
	Question V	**6pts**
	<u>**Part A**</u>	
1	$\left\lVert \overrightarrow{MO} \wedge \overrightarrow{MB} \right\rVert = 2A_{MOB} = MH \times OB$ with (MH) $\perp$ (OB) MH$\times$4=2MF $\Rightarrow \dfrac{MF}{d(M,OB)} = 2$; then M' moves on a hyperbola with focus F, directrix (OB) , e=2 and focal axis (OF).	**0.5**
2-a	AF=2AO , AF'=6=2A'O but Aand A'are on the focal axis . A and A ' are the vertices of (H).	**0.5**
2-b	I midpoint of [AA '] and F' symmetric of F with respect to I.	**0.5**
3	$tan O\hat{I}G = \sqrt{3} = Slope\ of\ the\ asymptote$ then (IG) and (IG) are asymptotes to (H).	**0.5**
4-a	FC=6=2d(C,OB) then C is a point on (H).	**0,5**
4-b	CF'-CF=AA'=4 then CF'=10	**0.5**
4-c	$tan O\hat{C}F = \dfrac{1}{2}\ and\ tan F\hat{C}F' = \dfrac{4}{3}$ $tan(2O\hat{C}F) = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$ then $F\hat{C}F' = 2O\hat{C}F$, therefore (CO) bisector of $F\hat{C}F'$.	**0,5**

4-d		**0.5**
	<u>Part B</u>	
1	$\frac{x^2}{4}-\frac{y^2}{12}=1$	**0.5**
2	$y=x\sqrt{3}$ and $y=-x\sqrt{3}$	**0.5**
3-a	$x^2=4(y-2)$.	**0.5**
3-b	L(4 ;6) is a common point of (P) and (H). Slope of the tangent at L to (H) =Slope of the tangent at L to (P) = 2 then (P) and (H) are tangent at L but (IV) is an axis of symmetry of (H) and (P) hence (H) and (P) are tangent at L'(-4 ;6).	**0.5**
	Question VI	**Note**
A-1	$\lim\limits_{x\to +\infty} f(x)=+\infty$ $\lim\limits_{x\to -\infty} f(x)=-\infty$	**0.5**
A-2	$\lim\limits_{x\to -\infty}\frac{f(x)}{x}=1$ $\lim\limits_{x\to +\infty}\frac{f(x)}{x}=1$, $\lim\limits_{x\to -\infty}[f(x)-x]=-\infty$ and $\lim\limits_{x\to +\infty}[f(x)-x]=-\infty$ then (C) has parabolic branch parallel to the line with equation y = x.	**0.5**

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<td>A-3.a</td>
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$$f'(x) = \frac{(x-1)^2}{x^2+1}.$$

Table of variations.

<table>
<tr><td>X</td><td>$-\infty$</td><td>1</td><td>$+\infty$</td></tr>
<tr><td>f'(x)</td><td>+</td><td>0</td><td>+</td></tr>
<tr><td>f(x)</td><td>$-\infty$</td><td>↗</td><td>$+\infty$</td></tr>
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<td>A3b</td>
<td>f(0) =0 and y=x is tangent at 0 and y=1 is a tangent at x=1.</td>
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<td>A-3.c</td>
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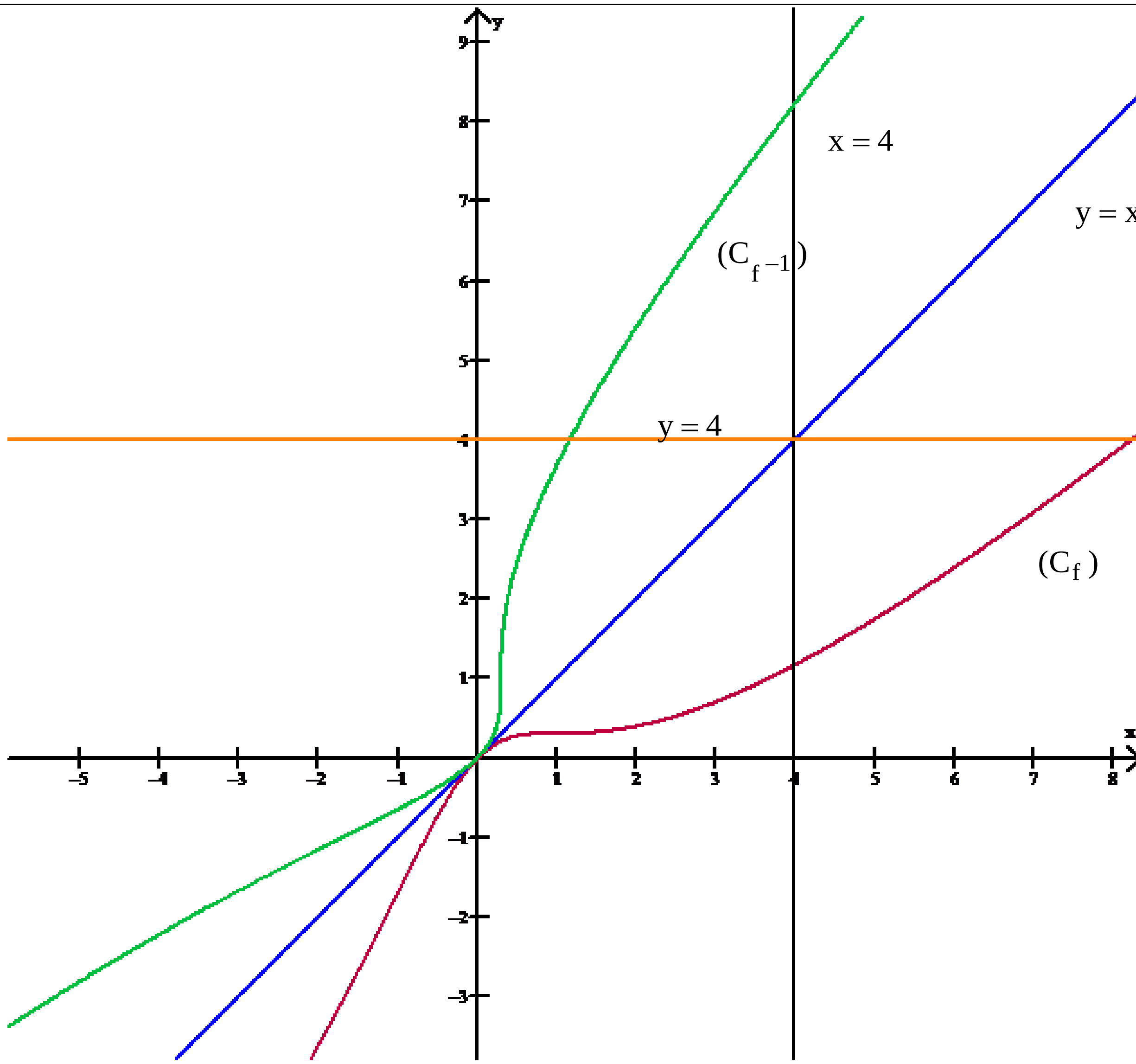

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A4a	f(x) < x then $x \in IR \setminus \{0\}$, $f^{-1}(x) > x$ then $x \in IR \setminus \{0\}$.	**0.5**
A4b	Asymptotic direction parallel to the line y = x .	**0.5**
A4c	The curves of f and f^{-1} are symmetric with respect to the first bissector.	**1**
A5	$\int f(x)dx = \frac{x^2}{2} - x\ln(x^2+1) + 2x - 2\arctan x + c$ $Area = 16 - 2\left[\frac{x^2}{2} - x\ln(x^2+1) + 2x - 2\arctan x\right]_0^4 = 8\ln 17 + 4\arctan 4 - 16$	**1.5**
A6a	$x \in D_g$, then $x \in R$ and $g(x) \in D_f$ therefore $D_h = R$. $x \in D_g$	**0.5**
A6b	$x \to \pm\infty, g(x) \to 0$ and $h(x) \to 0$;y = 0 HA	**0.5**
A6c	f is differentiable at g(x) and $h'(x) = f'(g(x) \times g'(x)$ hence h '(1) = $\frac{-2}{5}$.	**0.5**
B1a	If $x \in [0;1]$ then $f(x) \in [0;1]$	**0.5**
B1b	Mathematical induction.	**1**
B2	(U_n) strictly decreasing.	**1**
B3a	(U_n) strictly decreasing and having 0 as`lower bound then U_n is convergent.	**1**
B3b	Limit = 0.	**0.5**