


المادة: الرياضيات الشهادة: الثانوية العامة - فرع الاجتماع والاقتصاد نموذج رقم - ١ - المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز التربوي للبحوث والإنماء
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نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
 - يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

### I- (4 points)

The table below shows the VAT on cloths  $y_i$ , in the last 6 years in a certain country

Year	2010	2011	2012	2013	2014	2015
Rank of year $x_i$	3	4	5	6	7	8
VAT $y_i$ (in millions LL)	600	700	750	950	1100	1350

- 1) Calculate the averages  $\bar{x}$  and  $\bar{y}$  of the two statistical variables  $x_i$  and  $y_i$  respectively.
- 2) Represent graphically the scatter plot as well as the center of gravity  $G(\bar{x}; \bar{y})$  of the points  $(x_i; y_i)$  in a rectangular system.
- 3) Write an equation of the regression line  $D_{y/x}$  of  $y$  in terms of  $x$  and draw this line in the preceding system.
- 4) Suppose that the above pattern remains valid until the year 2020, Estimate the VAT on cloths in the year 2020.

### II- (4 points)

A shop sells products (perfumes, hair gel and shampoo) of two kinds A and B.

10% of kind A are “perfumes”, 30 % are “hair gel”, and the rest are “shampoo”

50% of kind B are “perfumes”, 20% are “hair gel”, and the rest “shampoo”

A client chooses one product at random.

Consider the events:

**A:** “The product is of kind A”

**B:** “The product is of kind B”

**H:** “The product is a hair gel”

**F:** “The product is a perfume”

**S:** “The product is a shampoo”

Suppose that  $P(A) = \frac{2}{3}$  and  $P(B) = \frac{1}{3}$ .

1)

- a- Calculate the following probabilities:  $P(A \cap F)$ ,  $P(A \cap H)$ ,  $P(A \cap S)$ , and  $P(F)$ .
- b- Calculate the probability of the event: “The chosen product is of kind A, given that it is a perfume”

2) The prices of the products are given in the table below.

	Shampoo	Perfume	Hair Gel
A	LBP15 000	LBP80 000	LBP10 000
B	LBP10 000	LBP50 000	LBP5 000

Designate by  $X$  the random variable that is equal to the amount paid by the client.

- Determine the probability distribution of  $X$ .
- Calculate the mathematical expectation of  $X$ . Interpret the result.

### III- (4 points)

In order to secure the future of their new-born, a bank proposes to parents the following offer:

For a deposit of 10 000 000 LL, an annual interest rate of 8 % is to be compounded annually, and to which a constant premium of 400 000 LL is to be added at the end of each year.

Designate by  $C_0$  the initial capital ( $C_0 = 10\,000\,000$ ), and by  $C_n$  the capital obtained at the end of the  $n$ th year.

1)

- Verify that  $C_1 = 11\,200\,000$  and calculate  $C_2$ . Deduce that the sequence  $(C_n)$  is neither arithmetic nor geometric.
- Express  $C_{n+1}$  in terms of  $C_n$ .

2) Consider the sequence  $(U_n)$  defined by:  $U_n = C_n + 5\,000\,000$ .

- Prove that  $(U_n)$  is a geometric sequence of common ratio 1.08 and whose first term is to be determined.
- Express  $U_n$  in terms of  $n$ . Deduce  $C_n$  in terms of  $n$ .
- How much shall be, after 18 years, the capital of a child whose parents accepted the offer of this bank?

### IV-(8points)

The adjacent curve  $(C)$  is the representative of a continuous and strictly decreasing function  $h$  that is defined on  $]0 ; +\infty[$  by:

$$h(x) = a + bx - \ln(x) \text{ where } a \text{ and } b \text{ are two real numbers.}$$

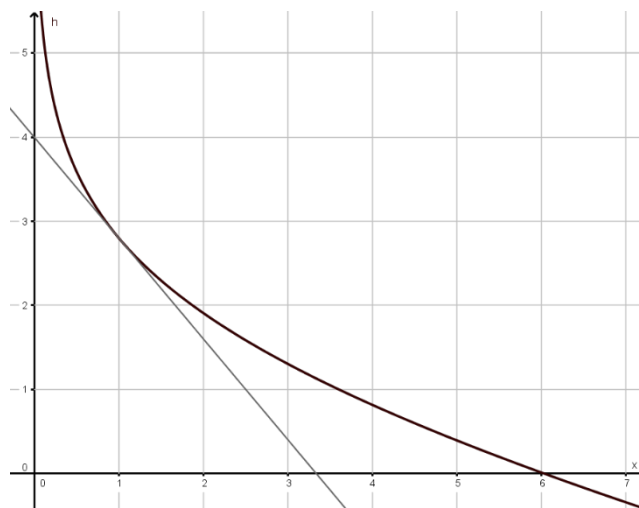
**Indication:** the line  $(d)$  of equation:  $y = -1.2x + 4$  is tangent to the curve  $(C)$  at the point  $(1; 2.8)$

1. Prove that  $a = 3$  and  $b = -0.2$
2. Set up the table of variations of  $h$ .

**B)** Let  $g$  be the function defined over  $[0 ; +\infty[$  by:

$$g(x) = 3(1 - e^{-0.2x}). \text{ Let } (C_1) \text{ be the representative curve of } g \text{ in an orthonormal system}$$


1. Calculate  $\lim_{x \rightarrow +\infty} g(x)$  and deduce an asymptote of  $(C)$ .
2. Study the variation of  $g$  and setup the table of variations.
3.  $(C_1)$  cuts  $(C)$  at a point of abscissa  $\alpha$ . verify that  $2.93 < \alpha < 2.95$
4. Draw  $(C_1)$  and  $(C)$  on the same curve.



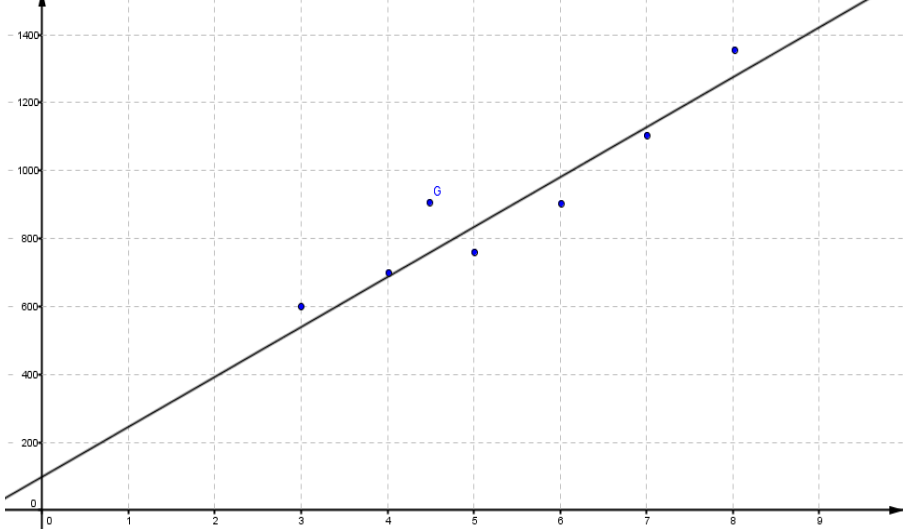
C) In all what follows, let  $\alpha = 2.94$

A factory produces a certain electronic articles. The demand, and the supply of this product in thousands of articles, are modeled by:  $D(p) = 3(1 - e^{-0.2p})$  and  $S(p) = 3 - 0.2p - \ln p$   
Where  $p$  is the unit price (price of one article) in thousands LL. ( $0.2 \leq p \leq 5$ ).

1. Calculate the supply corresponding to a unit price of 2 000 LL.
2. Calculate the unit price for a demand of 4000 items.
3. Give an economical interpretation for the value 2.94 of  $\alpha$ .  
Calculate, in this case, the total revenue.
3. a- Determine  $E(p)$ , the elasticity of the demand with respect to the price  $p$ .  
b- Calculate  $E(2.94)$ , and give an economical interpretation of the value thus obtained.

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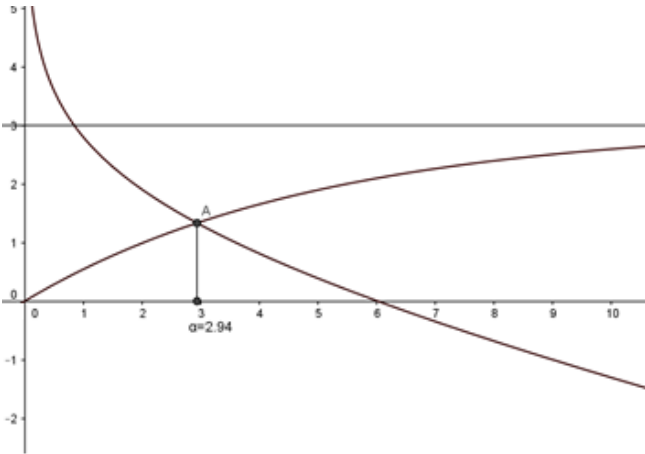
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

Question I		Mark
1	$\bar{x} = 5,5$ and $\bar{y} = 908,33$	1
2		1.5
3	$y = 147,142x + 99,047$	1.5
4	for $x = 13$ so $y = 147,142 \times 13 + 99,047 = 2011,893$ millions of LL	1

Question II		Mark															
1)	a-	$P(A \cap F) = \frac{2}{5}$ , $P(A \cap H) = \frac{1}{5}$ , $P(A \cap S) = \frac{2}{5}$ , $P(F) = P(A \cap F) + P(B \cap F) = \frac{2}{5} + \frac{5}{30} = \frac{17}{30}$	0.5 0.5 0.5 0.5														
	b-	$P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{12}{17}$	0.5														
2)	a-	<table border="1" style="width: 100%; text-align: center;"> <tr> <td><math>X = x_i</math></td> <td>5 000</td> <td>10 000</td> <td>15 000</td> <td>50 000</td> <td>80 000</td> <td>Total</td> </tr> <tr> <td><math>P(X = x_i)</math></td> <td><math>\frac{1}{15}</math></td> <td><math>\frac{3}{10}</math></td> <td><math>\frac{2}{5}</math></td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{15}</math></td> <td>1</td> </tr> </table>	$X = x_i$	5 000	10 000	15 000	50 000	80 000	Total	$P(X = x_i)$	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{6}$	$\frac{1}{15}$	1	1
	$X = x_i$	5 000	10 000	15 000	50 000	80 000	Total										
$P(X = x_i)$	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{6}$	$\frac{1}{15}$	1											
b-	$E(X) = \sum P_i \times x_i = 23$ . The average amount paid by the client is 23 000 LL.	0.5															

Question III		Mark	
1)	a-	$C_1 = 10\,000\,000 + 10\,000\,000 \times 0.08 + 400\,000 = 11\,200\,000$ $C_2 = 11\,200\,000 + 11\,200\,000 \times 0.08 + 400\,000 = 12\,496\,000$ $\frac{C_1}{C_0} \neq \frac{C_2}{C_1}$ and $C_1 - C_0 \neq C_2 - C_1$	0.25 0.25 0.25 0.25
	b-	$C_{n+1} = C_n + 0.08C_n + 400\,000 = 1.08C_n + 400\,000$	0.5
2)	a-	$U_{n+1} = 1.08(C_n + 5\,000\,000) = 1.08U_n$ ; $(U_n)$ is a geometric sequence of common ratio $r = 1.08$ and of first term $U_0 = 15\,000\,000$ .	1
	b-	$U_n = U_0 \times r^n = 15 \times 10\,000\,000 \times 1.08^n$ and $C_n = 15 \times 10\,000\,000 \times 1.08^n - 5\,000$	0.5 0.5
	c-	$C_{18} = 15\,000\,000 \times 1.08^{18} - 5\,000 = 54\,940\,000$ ; the capital of a child whose parents	0.5

	accepted the offer of this bank, after 18 years, is 54 940 000 LL	
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Question IV		note									
A	<p>1) <math>h(1)=2,8</math> donc <math>a+b=2,8</math>  <math>h'(1)=-1,2</math> alors <math>b-1=-1,2</math> alors <math>b=-0,2</math> and <math>a=3</math></p> <p>2)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>0</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>h'(x)</math></td> <td style="padding: 5px; text-align: center;">-</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>h(x)</math></td> <td style="padding: 5px;">  </td> </tr> </table>	$x$	$0$	$h'(x)$	-	$h(x)$		1,5			
$x$	$0$										
$h'(x)$	-										
$h(x)$											
B-1	$\lim_{x \rightarrow +\infty} f(x) = 3$ $y=3$ asymptote horizontale.	0,5									
B-2	$g'(x) = 0,6e^{-0,2x}$ . <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>x</math></td> <td style="padding: 5px;"><math>0</math></td> <td style="padding: 5px;"><math>+\infty</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>g'(x)</math></td> <td style="padding: 5px; text-align: center;">+</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>g(x)</math></td> <td style="padding: 5px; text-align: center;">0</td> <td style="padding: 5px; text-align: center;">3</td> </tr> </table>	$x$	$0$	$+\infty$	$g'(x)$	+		$g(x)$	0	3	1
$x$	$0$	$+\infty$									
$g'(x)$	+										
$g(x)$	0	3									
B-3	Let $L(x)=f(x)-g(x)$ $L(2,93) \times L(2,95) < 0$ Donc $2,93 < \alpha < 2,95$	0,5									
B-4		1									
C-1	$S(2)=1.906$ thousands articles										
C2	$D(p)=2$ so $e^{-0,2p} = \frac{-1}{3}$ then $p = \ln(3)/0.2$ donc $p=5.4930$ thousands LL.	0,5									
C-3	$2.94$ is the price of equilibrium $R = 2,94 \times D(2,94) = 3.921$ millions LL.	1									
C-4-a	$e(p) = \frac{0,2p e^{-0,2p}}{1 - e^{-0,2p}}$	0,5									
C-4-b	$e(2,94)=0,73$ if the price increase 1% from 2.94 in thousands of LL the demand decrease 0,73%	1									