دورة سنة ٢٠٠٨ الاكمالية الاستثنائية	**امتحانات الشهادة الثانوية العامة** **الفرع : إجتماع و إقتصاد**	**وزارة التربية والتعليم العالي** **المديرية العامة للتربية** **دائرة الامتحانات**
الاسم: **الرقم:**	مسابقة في مادة الرياضيات المدة ساعتان	**عدد المسائل : اربع**

إرشادات عامة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

The fuel consumption y, of a car that travels 100 km, is expressed as a function of its speed x as shown in the following table:

x (in km /h)	80	90	120	150
y (in liters)	5	5.5	8.4	12

1) Represent the scatter plot of the points (x_i, y_i) in a rectangular system.
2) Calculate the averages $\overline{x}$ and $\overline{y}$. Place the center of gravity G in the preceding system.
3) Determine an equation of the regression line $D_{y/x}$ of y in terms of x and draw this line in the same system.
4) Estimate the fuel consumption of the car that travels 100 km with a speed of 85 km/h.
5) After which speed would the fuel consumption of the car exceeds 10 liters for covering 100 km?

II- (4 points)

A sports club offers two types of membership.

Type **A** : A membership fee of 1 million LL for the first year only, after which a yearly amount of 300 000 LL is to be paid after the first year.

Type **B** : An annual membership fee of 500 000 LL for the first year which increases each year by 10% after the first year.

1) Show that:
   a- The total sum T_n paid by a member who chose type **A** for the first n years is : $T_n = 700\,000 + 300\,000\,n$.
   b- The membership fee in the year (n+1) paid by a member who chose type **B** ,denoted by C_{n+1}, is : $C_{n+1} = 1.1\,C_n$.

2) Answer by true or false and justify your answers:
   a- For every non-zero natural integer n , the sequence (C_n) is arithmetic with common difference 1.1.
   b- For every non-zero natural integer n , $C_n = 500\,000 \times (1.1)^{n-1}$.
   c- The sum S_n paid to the club by a member who chose type **B**, for the first n years, is : $S_n = 5\,000\,000 \times [(1.1)^{n-1} - 1]$, where n is non-zero.
   d- The type **A** becomes more advantageous for the member for the first three years.

III- (4 points)

A- An urn U contains **4** red balls, **1** blue ball and **3** green balls. We draw randomly and simultaneously two balls from this urn.

1) Show that the probability of obtaining two balls having the same color is equal to $\frac{9}{28}$.
2) Calculate the probability to obtain:
   a- At least one red ball.
   b- Exactly one green ball.

B- A school administration organizes between its students the following game.
A participant in the game draws, randomly and simultaneously, from the urn U two balls then he replaces them in this urn and draws again from the urn two balls randomly and simultaneously.
If the two drawn balls, in the same draw, have the same color the participant receives 7 points, otherwise he receives 5 points.
Let X be the random variable which is equal to the sum of points received by the participant in the two drawings.

1) Determine the probability distribution of X.

2) Calculate the mean (expected value) E(X).

IV- (8 points)

Consider the function f defined on] 0; +∞ [by : $f(x) = 2x + \ln\left(\frac{2x}{2x+1}\right)$.

Let (C) be the representative curve of f in an orthonormal system $(O ; \vec{i}, \vec{j})$.

A-

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$ and prove that the line (D) of equation $y = 2x$ is an asymptote to (C).

b- Verify that $\frac{2x}{2x+1} < 1$ and deduce that (C) is below (D).

2) Calculate $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).

3) Verify that $f'(x) = 2 + \frac{1}{x(2x+1)}$ and set up the table of variations of f .

4) Draw (D) and (C).

B-

A study of the market revealed that:
The quantity of objects produced by a factory is modeled by the function f .
The quantity of objects demanded from this factory is modeled by the function g given by g(x) = 2x + 1.
That is to say, for a date x expressed in weeks ($1 \leq x \leq 10$), f (x) is the quantity of objects produced by this factory expressed in thousands and g(x) is the quantity of demanded objects expressed in thousands.

1) We say that « demand is satisfied on the date x » if $f(x) \geq g(x)$.
Show that the demand is never satisfied.

2) Suppose that the total number of objects, in thousands, whose demand is not satisfied between two dates n and m is given by $\int_n^m [g(x) - f(x)]dx$.

a- Let H be the function defined on]0; +∞ [by H(x) = x- xln2x + $\frac{1}{2}$(2x+1)ln(2x+1).
Show that H(x) is a primitive (antiderivative) of g(x) –f(x).

b- Calculate the total number of objects whose demand is not satisfied between the dates 1 and 5.

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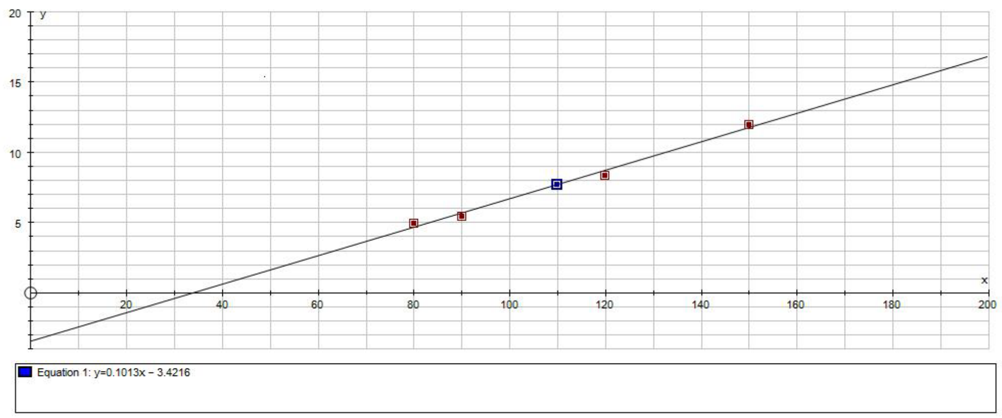
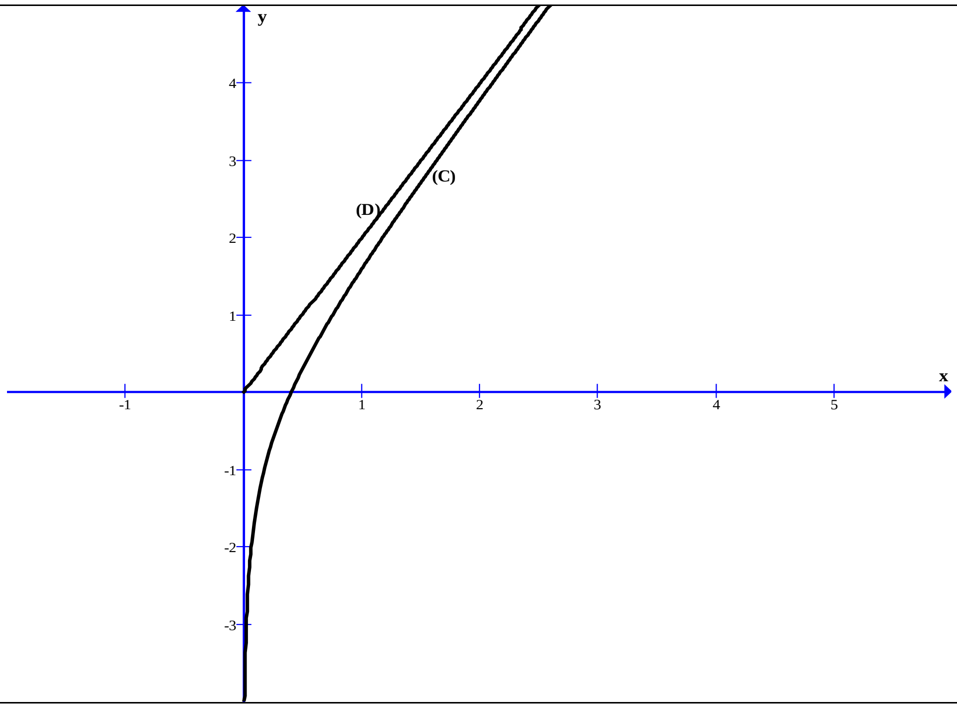
QI	Answer	Mark
1		1.5
2	$\bar{x} = 110$ and $\bar{y} = 7.725$	1
3	$y = 0.1013x - 3.4216$	1.5
4	$y = 0.1013 \times 85 - 3.4216 = 5.1889 \approx 5.19$ liters for 100 km.	1.5
5	$0.1013 \times x - 3.4216 > 10$, so $x > 132$. After 132 km/h.	1.5

QII	Answer	Mark
1a	$T_n = T_1 + 300\,000\,(n-1) = 1\,000\,000 + 300\,000\,n$.	1.5
1b	$C_{n+1} = C_n\,(1 + \frac{10}{100}) = 1.1\,C_n$.	1
2a	False since (C_n) is geometric, of ratio 1.1.	1
2b	True since $C_1 = 500\,000$ and $q = 1.1$ thus $C_n = C_1 \times q^{\,n-1}$	1
2c	False since $S_n = C_1 \times \frac{1-q^n}{1-q} = 500\,000 \times \frac{1-1.1^n}{-0.1} = 5\,000\,000\,[1.1^n - 1]$.	1
2d	True since $T_3 = 1\,600\,000$ and $S_3 = 1\,655\,000$.	1.5

QIII	Answer	Mark
A1	p(same color) = p(2 red or 2 green) = $\frac{C_4^2}{C_8^2} + \frac{C_3^2}{C_8^2} = \frac{9}{28} = 0.321$	1
.A2a	p(at least one red) = 1- p(no red)= $1 - \frac{C_4^2}{C_8^2} = \frac{11}{14} = 0.786$	1.5
A2b	p(exactly one green) = $\frac{C_3^1 C_5^1}{C_8^2} = \frac{15}{28} = 0.536$	1
B1	The values of X are10, 12 and 14 p(X=10) = P(different colors, different colors) p(X=12) = p(same color, different colors)+p(different colors, same color) p(X=14) = P(same color, same color) <table><tr><td>$X = x_i$</td><td>10</td><td>12</td><td>14</td></tr><tr><td>$p(x_i) = p_{i_i}$</td><td>$\frac{19}{28} \times \frac{19}{28} = 0.461$</td><td>$\frac{9}{28} \times \frac{19}{28} \times 2 = 0.436$</td><td>$\frac{9}{28} \times \frac{9}{28} = 0.103$</td></tr></table>	2.5
B2	$E(X) = \sum x_i p_i = 10(0.461) + 12\,(0.436) + 14\,(0.103) = 11.2$	1

Q	Answer	Mark
A1a	$\lim_{x\to+\infty}\left(\frac{2x}{2x+1}\right)=1$ and ln1 = 0. So, $\lim_{x\to+\infty}\ln\left(\frac{2x}{2x+1}\right)=0$. $\lim_{x\to+\infty} f(x)=+\infty$ since $\lim_{x\to+\infty}2x=+\infty$ $\lim_{x\to+\infty}[f(x)-2x]=\lim_{x\to+\infty}\ln\left(\frac{2x}{2x+1}\right)=0$ so the line (D) : y = 2x is an asymptote to (C).	2
A1b	$0<\frac{2x}{2x+1}<1$, for $x>0$. So $\ln\left(\frac{2x}{2x+1}\right)<0$; $f(x)-2x=\ln\left(\frac{2x}{2x+1}\right)<0$ So (C) is below (D).	1.5
A2	$\lim_{x\to0} f(x)=0-\infty=-\infty$ .The axis of ordinates is an asymptote o (C).	1
A3	$f'(x)=2+\frac{1}{x}-\frac{2}{2x+1}=2+\frac{1}{x(2x+1)}>0$ since $x>0$. Thus, f is strictly increasing on]0 ; +∞[. $\begin{array}{c\|ccc} x & 0 & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$	2
A4		2.5
B1	(C_g) is above (D) and from A1, (D) is above (C); hence g(x)>f(x) for all $x\geq1$ So the demand is never satisfied.	1.5
B2a	$H'(x)=1-\ln 2x-1+\frac{1}{2}(2\ln(2x+1)+2)=1-\ln 2x+\ln(2x+1)=g(x)-f(x)$	1.5
B2b	The number is: $[H(x)]_1^5=\left[x-x\ln 2x+\frac{1}{2}(2x+1)\ln(2x+1)\right]_1^5=4.720$ The total number of objects is 4720.	2