نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة: ـ يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
ـ يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (4 points)

Remark: The four parts of this question are independent.

1) Given: $A = 3^{24} + 3^{25} + 3^{26}$. Factorize A and deduce that A is a multiple of 13.

2) Given: $a = \sqrt{14} + \sqrt{2}$ and $b = -\sqrt{\frac{7}{2}} - \sqrt{\frac{1}{2}}$.

   a) Calculate a^2 and b^2. Show your work.

   b) Compare $\frac{a}{2}$ and b.

3) What is the sign of $\frac{-2x^2}{-y}$, when $x < 0 \ and \ y > 0$?

4) A Grade 9 student wrote: "If the price of one ball decreases by 15%, then the price of 2 balls will decrease by 30%."Is he right? Justify.

II- (4 points)

Part A

Let $A(x) = (1 - x)(6x - 8) + 16 - 9x^2$.

1) Expand and reduce A(x).
2) Factorize A(x).
3) Solve the following equations: A(x) = 0; A(x) = 8.

Part B

Given $E(x) = \frac{(x-3)(-2-5x)}{(2+5x)(4-3x)}$.

1) Determine the values of x in which E(x) is defined.
2) Simplify E(x).
3) Solve the equation: $E(x) = \frac{1}{3}$.

III- (4 points)

ABC is a right triangle right at A such that AC =2 AB. Let AB = a, with $a > 0$.
[AM] is the median relative to [BC].

1) Draw a figure to be completed by the remaining parts of the problem.
2) Calculate, in terms of a, BC and AM.
3) Draw (d), the perpendicular line to (AM) at A. Then, draw the perpendicular lines to (d) passing through the two points B and C; these lines intersect (d) in B′and C′ respectively.

   a) Prove that [BA) and [CA) are the respective bisectors of angles $\widehat{CBB'}$ and $\widehat{BCC'}$.
   b) Prove that the triangles ACC' and ABB' are similar and find their ratio of similarity.

4) [AH] is the altitude in the triangle ABC.

   a) Prove that $AH = \frac{2a\sqrt{5}}{5}$.
   b) Calculate, in terms of a, the lengths B′C′, BB′and CC′.

IV- (4 points)

The plane is referred to an orthonormal system x'Ox, y'Oy.

1) Plot the three points $A(3 ; 2)$, $B(2 ; 5)$, and $C(-1 ; 4)$.
2) Prove that the triangle ABC is right isosceles.
3) Let S be the midpoint of [AC], and D be the symmetric of B with respect to S.
   a) Determine the nature of the quadrilateral ABCD.
   b) Calculate the coordinates of point D.
4) The parallel line to (AC) passing through D intersects the x-axis at a point T.
   a) Determine an equation of line (DT).
   b) Calculate the coordinates of point T.
5)
   a) Solve the equation: $x^2 = 5$.
   b) $(1; m)$ are the coordinates of a point M, where by m is a real number. Determine the values of m so that M is on the circle circumscribed about the square ABCD.

V- (4 points)

Remark: The questions 1) and 2) are independent.

1) A team of 6 girls and 4 boys did the same test in mathematics. The mean of the girls on this test is 14 and the mean of boys is 12.
   What is the mean of the team in this test?

2) Grade 7 students handle a survey on the number of mobile phones per family.

Number of mobile phones	0	1	2	3
Number of families	5	15	50	30

   a) What is the total frequency of the above distribution?
   b) What is the percent age of families having at least 2 mobile phones?
   Represent this situation in a circle graph. Write down the calculation of the different.

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

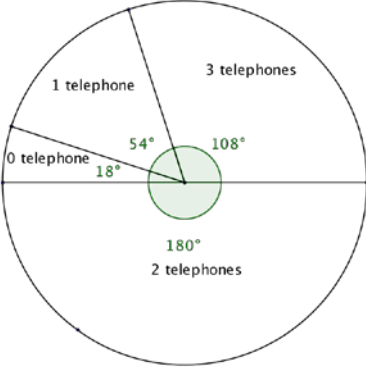|		Answer key	Mark
I-	**1)**	$A = 3^{24}(1 + 3 + 9) = 3^{24} \times 13$ A is a multiple of 13 since 3^{24} is a natural number.	0.5 0.5
	2) **a)**	$a^2 = 4(4 + \sqrt{7})$; $b^2 = \frac{7}{2} + \frac{1}{2} + 2\sqrt{\frac{7}{2}}\sqrt{\frac{1}{2}} = 4 + \sqrt{7}$; $a^2 = 4b^2$	0.25 0.5
	b)	$a = -2\,b$	0.25
	3)	for all x ; $x^2 > 0$ $then -2\,x^2 < 0$ $-y < 0$, hence $\frac{-2x^2}{-y} > 0$	0.5 0.25 0.25
	4)	False. Before reduction: x is the price of a ball and 2x is the price of 2 balls. After reduction: 1 ball: 0.85 x ; Price of 2 balls: $(0.85\,x) \times 2 = 0.85 \times (2x)$ The price of 2 balls decreased by 15% We can think in this way: if this student is right and if he buys 7 balls, then the salesman should pay him because the reduction will be 105%	1

		Part A	Mark
II-	**1)**	$A(x) = -15x^2 + 14x + 8$	0.25
	2)	$A(x) = (1 - x)(6x - 8) + 16 - 9x^2$ $A(x) = 2(1 - x)(3x - 4) + (4 - 3x)(4 + 3x)$ $A(x) = 2(1 - x)(3x - 4) - (3x - 4)(4 + 3x)$ $A(x) = (3x - 4)(-2 - 5x)$	0.25 0.25 0.25 0.25
	3)	$A(x) = 0$, for $x = \frac{4}{3}$ or $x = -\frac{2}{5}$ $A(x) = 8$ $-15x^2 + 14x + 8 = 8$ $-15x^2 + 14x = 0$ $x(-15x + 14) = 0$ $x = \frac{14}{15}$ or $x = 0$	0.25 0.25 0.25 0.25 0.25 0.5
		Part B	
	1)	$x \neq \frac{4}{3}$ and $x \neq -\frac{2}{5}$	0.5
	2)	$E(x) = \frac{x-3}{3x-4}$	0.25
	3)	$\frac{x-3}{3x-4} = \frac{1}{3}$, then $3x - 9 = 3x - 4$; the equation has no solution	0.5

III-	**1)**			0.25
	2)	Pythagoras: AC = 2a, so BC^2 = 4a^2 + a^2 ; with BC > 0, therefore BC = a$\sqrt{5}$		0.25
		AM = $\frac{a\sqrt{5}}{2}$		0.25
	3)	**a)**	BMA isosceles and M is the principal vertex, therefore $\widehat{MAB} = \widehat{MBA}$; (AM) //(BB'), $\widehat{B'BA}$ and $\widehat{MAB}$ are alternate-interior angles, thus $\widehat{B'BA} = \widehat{MAB}$ then$\widehat{B'BA} = \widehat{MBA}$ and [BA) is the bisector of $\widehat{CBB'}$. Same for: [CA) bisector of $\widehat{BCC'}$.	0.5
		b)	Right angle and$\widehat{B'BA} = \widehat{CAC'}$, same complements	0.5
			$\frac{BB'}{AC'} = \frac{B'A}{C'C} = \frac{AB}{CA} = \frac{a}{2a} = \frac{1}{2}$	0.25
	4)	**a)**	By areas of ABC: $\frac{1}{2}$AH × BC = $\frac{1}{2}$ AB × AC	0.5
			OR	0.5
			Using the similar triangles BAH and ABC. AH × BC=AB × AC and AH = $\frac{AB \times AC}{BC} = \frac{2a^2}{a\sqrt{5}} = \frac{2a}{\sqrt{5}} = \frac{2a\sqrt{5}}{5}$	
		b)	A is a point on the bisector of $\widehat{CBB'}$; therefore A is at the same distance from the sides of the angle. So AH = AB'. Same for AH = AC'.	0.25
			ThereforeB'C' = 2AH = $\frac{4a\sqrt{5}}{5}$	0.25
				0.25
			BB' = $\frac{1}{2}$ AC', using 3) b) then BB' =$\frac{a\sqrt{5}}{5}$	
			CC' = 2B'A, using 3) b). Hence CC' = 2 × $\frac{2a\sqrt{5}}{5}$, using 4) a)	0.25
			CC' = $\frac{4a\sqrt{5}}{5}$	

IV	**1)**			0.25
		although it is not requested, we have completed the drawing.		
	2)	AB^2 = $(2-3)^2 + (5-2)^2 = 1+9 = 10$;AC^2 = 16 + 4 = 20		0.5
		BC^2 = 9 + 1 = 10; AB= BC et AB^2 + BC^2 = AC^2		
		$\qquad$ Therefore ABC is right isoceles. The right angle is at B.		0.25
	3)	**a)**	ABCD is a square.	0.5
		b)	Different ways for calculation: using the midpoint S(1 ; 3) or $\overrightarrow{AD} = \overrightarrow{BC}$; D(0 ; 1)	0.5

	4)	**a)**	(DT) // (AC),slope: a $= \frac{y_A - y_C}{x_A - x_C} = \frac{-2}{4} = -\frac{1}{2}$ and D belongs to (DT), therefore $y = -\frac{1}{2}x + 1$.	**0.5** **0.25**
		b)	Ordinate of T is equal to zero. T(2 ;0)	**0.25**
	5)	**a)**	$x = \sqrt{5}$ or $x = -\sqrt{5}$	**0.25**
		b)	S(1 ;3) ; $SA^2 = 5$ $MS^2 = SA^2$; $MS^2 = 5$ $(1 - 1)^2 + (m - 3)^2 = 5$; $(m - 3)^2 = 5$; $m - 3 = \sqrt{5}$ or $m - 3 = -\sqrt{5}$; $m = 3 + \sqrt{5}$ or $m = 3 - \sqrt{5}$	**0.25** **0.25** **0.25**

V	**1)**		$[(6 \times 14) + (4 \times 12)] \div 10 = 13.2$	**1.5**
	2)	**a)**	$5 + 15 + 50 + 30 = 100$	**0.25**
		b)	$(50 + 30) \div 100 = 0.8$ or 80%	**0.5 +** **0.25**
		c)	Calculation of angles: 	**0,75**

Calculation of angles:

Number of mobile phones	0	1	2	3
Number of families	5	15	50	30
Angles	18°	54°	180°	108°

0.5 + 0.25

Circle graph