نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة: ـ يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (1.5 points)

Given the number a = $\dfrac{1+\sqrt{5}}{2}$.

1) Show that: $a^2 - a - 1 = 0$.

2) Prove that: $\dfrac{1}{a} = a - 1$.

II - (3 points)

1)
   a) Verify that $x^2 + 6x + 5 = (x + 3)^2 - 4$
   b) Factorize $x^2 + 6x + 5$.

2) ABC is a triangle right at A, where AC $=2x + 2$ and its area is $x^2 + 6x + 5$, (x is a positive real number).
   a) Show that $AB = x + 5$.
   b) Calculate, in terms of x, BC^2.

3) Determine x if the area of triangle ABC equals 12 units of area.

III- (2.5 points)

A survey about the content of a website was organized. Some visitors of the website were asked to rate the content using a 5-unit mark.
The table below shows the different opinions of the visitors.

Mark	1	2	3	4	5
Frequency	4	10	14	12	10

1) Calculate the average mark of the obtained marks.

2) The owners of the website will be satisfied if at least 65% of the visitors give a mark greater than or equal to 3. Will the owners of the website be satisfied? Explain your answer.

3) Construct the bar graph of this distribution.

IV-(2.5points)

To transport goods, Mr. Jad has the opportunity to address two transport companies: A and B. The offers presented by the two companies are given below.

   Offer of Company A: 2 000 LL per kilometer.

   Offer of Company B:210 000LL down payment and 1 500 LL per kilometer.

1) Calculate the amount to be paid for a distance of 100 km by the two offers.

2) Let x be the number of kilometers traveled. Denote by y_1 the price charged by Company A and by y_2 the price charged by Company B.

   Express y_1 and y_2 in terms of x.

3) Calculate the number of kilometers traveled for which of the prices charged in the two offers arethe same. What is, then, the price to pay?

4) Jad chooses offer A and pay 500 000 LL. Does he choose the more advantageous offer?

V- (5.5 points)

ABE is an isosceles triangle of vertex B such that $BA = BE = 6$ cm and $\widehat{ABE} = 140^{\circ}$. The circle (C) with diameter [BE] and center O intersects the line (AB) at point F.LetI be the midpoint of [AE].

1) Draw a figure.

2) What is the nature of triangle BEF? Justify your answer.

3) Prove that I is on (C).

4) Calculate an approximate value of BF to the nearest millimeter.

5) **a-**Show that the triangles ABI and AEF are similar. Deduce that $AB \times AF = 2AI^{2}$.

b- Then calculate AE.

6) The two lines (BI) and (EF) intersect at H. Let (d) be the parallel through B to (AL). The line (d) intersects (AE) and (EF) at G and L respectively.

   a) Prove that (LG) is tangent to (C).
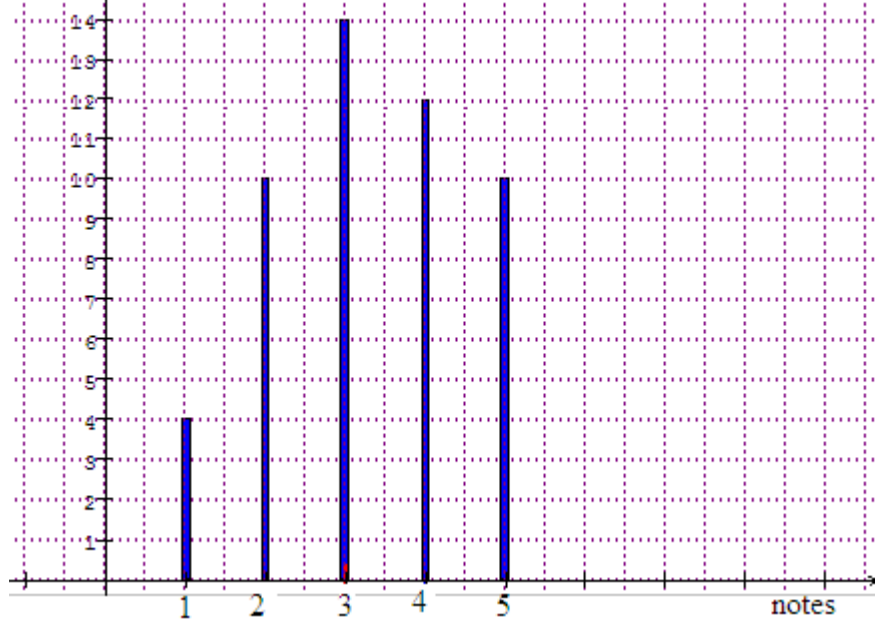
   b) Calculate the ratio $\dfrac{EL}{EH}$.

VI-(5 points)

In an orthonormal system of axes $x'Ox$ and $y'Oy$, Consider the points A(4 ; 2) and B(0 ;4),
Let (d) the line thought B and perpendicular to (OA).

   1) Plot the points A, B and draw (d).

   2) a)show that the triangle AOB is isosceles.

      b) Prove that $y = -2x + 4$ is the equation of (d).

   3) Let H be the orthocenter of the triangle OAB.

      a) Verify that $y_H = 2$.

      b) Calculate x_H.

   4) Denote by E the meeting point of (OH) and (AB), and F that of (d) and $x'Ox$.

      a) Calculate the coordinates of F.

      b)Prove that the 2 triangles OBF and EOB are similar. Calculate the similarity ratio.

   5) Let (C) be the circle circumscribed about the triangle BOF. Denote by (T) the tangent at B to (C). line (T) intersects the x-axis at G.

a) Show that (T) is parallel to (OA)

      b) Write an equation for (T).

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

	Question I	
	Answers	Note
1	$a+1=\dfrac{3+\sqrt5}{2}$; $a^2=\dfrac{3+\sqrt5}{2}$ then $a+1=a^2$ therefore $a^2-a-1=0$	1
2	$\dfrac{1}{a}=\dfrac{2}{1+\sqrt5}=\dfrac{1-\sqrt5}{-2}$ $a-1=\dfrac{1-\sqrt5}{-2}$	0.5

	Question II	
1.a	$(x+3)^2-4= x^2 + 6x +9 -4 = x^2 +6x +5$	0.5
1.b	$x^2 +6x +5= [x+3 \ -2][x+3+2]=(x+1)(x+5)$	0.5
2.a	Area of ABC $=\dfrac{AB \times AC}{2}$, $(x+1)(x+5) = \dfrac{2(x+1).AB}{2}$ then AB = x+5	0.75
2.b	$BC^2 =(2x+2)^2 +(x+5)^2 = 5x^2+18x+29$	0.5
3	$(x+3)^2-4=12$ then $(x+3)^2 =16$, x+3 = 4 or x+3=−4 then x = 1, x = -7 impossible.	0.75

	Question III	
1	$\bar{x} =\dfrac{1 \times4 + 2\times10 +3\times14+4\times12+5\times10}{50} = 3{,}28$	0.75
2	14+12+10= 36 the frequency $\geq$ 3. $\dfrac{36}{50}\times 100 = 72\%$. 72% $\geq$ 65 % is true.	0.75
3		1

	Question IV	
1	A : Sum= 2000x100 = 200 000 LL B : Sum=1500 x100 +210 000= 370 000 LL	0.5

2	y_1= 2000 x , y_2= 1500 x +210000	0.5
3	2000 x = 1500 x +210000 alors x = 420 km is the price paid = 2000x420 = 820000 L.L	0.75
4	Pour A : 500000 = 2 000 x alors x =250km If he chooses B, the sum= 1500 x 250 + 210 000= 585 000 LL. Yes, Jad made good choice.	0.75

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Question V

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1		0.5
2	BFE is a right angled triangle (inscribed in half circle).	0.5
3	I midpoint of [AE] then [BI] perpendicular bisector in ABE isosceles , then BIE is right at I, then inscribed in half circle of diameter [BE]	0.5
4	ABE isosceles then : $\widehat{BAE}$ =$\widehat{AEB}$ = (180 -140) ÷ 2 = 20° $\widehat{EBF}$ =40° (exterior). $\cos\widehat{EBF}=\dfrac{BF}{BE}$ alors BF = EB x cos (40°) = 6 x cos(40)= 4.59cm	1
5.a	$\widehat{BFE}=\widehat{BIA}=90°$ A common angle . $\dfrac{AB}{AE}=\dfrac{AI}{AF}$ then : ABx AF = AI x AE = AI x 2 AI = 2 AI².	0.5 0.75
5.b	In EBI right at B we have: AF = 6+4.59=10,59 cm AB x AF = 2 AI ²we calculate AI then AE =2AI.	
6.a	B orthocenter of AEH then [EB] third altitudethen (EB) perpendicular to(AH) then to (GL).	1.25
6.b	Thales : $\dfrac{EL}{EH}=\dfrac{EG}{EA}$ but $EG=\dfrac{EB}{\cos 20}=6.38$ then $\dfrac{EL}{EH}=\dfrac{6.38}{11.26}$	0.5

	Question VI	
1		0,5
2.a	AB = AO = $\sqrt{20}$	0,5
2.b	Slope of (OA) = $\dfrac{1}{2}$, (d) perpendicular à (OA) then Slope of(d)= -2 B is on (d) because 4 = -2x0 +4	0,5
3.a	(AH) altitude then perpendicular to (y'y) then y_H= y_A = 2	0,5
3.b	H is on (d) then x_H= 1	0,5
4.a	yF =0 and F is on (d) thenxF = 2	0,5
4.b	$\hat{O} = \hat{E} = 90$ *and* $E\hat{O}B = O\hat{B}F$ *HOB isoceles triangle* Ratio $=\dfrac{OB}{BF} = \dfrac{4}{\sqrt{20}}$	1
5.a	(T) and(OA) are parallel since they are perpendicular to (FB).	0,5
5.b	a(T) = a(OA) = $\dfrac{1}{2}$ and B on (T) then the equation is y = $\dfrac{1}{2}$ x+4.	0.5