المادة:الرياضيات الشهادة: المتوسطة نموذج رقم -6- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

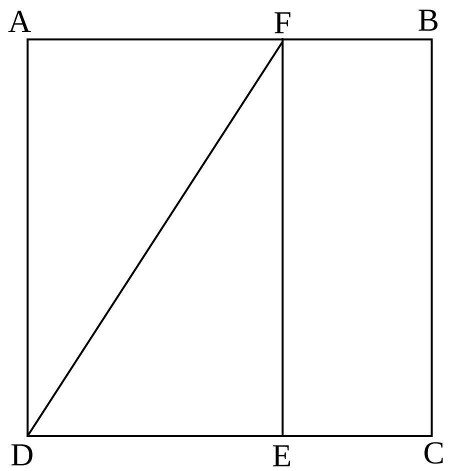
I- (2 points)

Answer "True" (T) or "False" (F) and justify your answer.

1) The solution of the inequality $\frac{-2x+3}{-3} \leq \frac{x+1}{-3}$ is $x \geq \frac{2}{3}$.
2) The price of an object becomes 90000 LL after two successive reductions of 20%. Its initial price is 150000 LL.
3) If $x^2 = \frac{\sqrt{\frac{35}{15}}}{\frac{\sqrt{14}}{\sqrt{6}}} + \frac{5}{7}\left(1-\frac{3}{10}\right)^2$, then $x = \frac{3\sqrt{15}}{10}$ or $x = -\frac{3\sqrt{15}}{10}$.
4) $\left(-\sqrt{\frac{5}{2}} - x\sqrt{\frac{1}{2}}\right)^2 = \frac{1}{2}\left(\sqrt{5}-x\right)^2$.

II- (3points)

Let x be a number that is greater than or equal to 4.
ABCD is a square.
AFED is a rectangle, where $DF^2 = 5x^2 - 10x + 10$.



1) If AF=x+1, show that the side of the square ABCD is 2x – 3.
2) Prove that F cannot be the midpoint of [AB].
3)
   a) Show that the area A of the rectangle BCEF is expressed by the relation:
   $A = (2x-3)^2 - (2x-3)(x+1)$.
   b) Factorize A.
   c) For which value of x does the area of the rectangle BCEF become one third of that of the triangle AFD?

III- (3points)

The director of a school organizes a trip for Grade 9 students at the end of the year. He decides not to make the trip if the percentage of participants is less than 70% of all grade nine students.
The table below shows the answers for each section.

Section	Total Number of Students	Answer
Gr. 9A	35 students(among them 20 girls)	$\frac{2}{5}$of the girlsand$\frac{1}{5}$of the boys will not participate.
Gr. 9B	24 students (among them 14 boys)	50% of the girls and $\frac{2}{7}$of the boys will not participate.
Gr. 9C	30 students (among them 15 boys)	60% of the girls and 80 %of the boys will participate.

1) In each section, find the number of students who will participate in this trip.
2) Will the director of the school make the trip?

IV- (2 points)

A bag contains x red balls and y blue balls.

If we replace 5 blue balls by 5 red balls, the number of red balls will be twice the number of blue balls.

If we take 3 red balls from the bag, the number of blue balls will be twice the number of red balls.

1) Choose the system that models the text given above .

$$\begin{cases} x+5=2y \\ 2(x-3)=y \end{cases} \text{ or } \begin{cases} x+5=2(y-5) \\ 2(x-3)=y \end{cases}$$

2) Calculate x and y.

V- (5 points)

In an orthonormal system of axes x′Ox and y′Oy, consider the line (D) with equation $y = -2x + 4$ and the two points I(1 ; 2) and C(4 ;4).

1) (D) intersects x′Ox at A and y′Oy at B. Calculate the coordinates of the two points A and B, then draw line (D).

2) Verify that I is the midpoint of segment [AB].

3)

a) Write an equation of the median issued from point O in triangle OAB.

b) Calculate, to the nearest one degree, the measure of the angle that line (OI) makes with the axis x′Ox.

4) Let (D′) be the perpendicular bisector of segment [BC] that intersects it at J.

a) Write an equation for (D′).

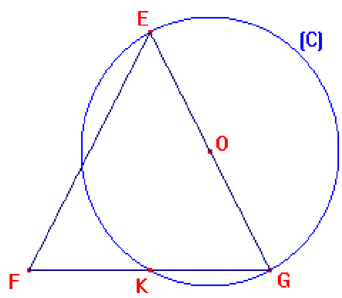
b) Deduce that AB = AC.

5) Let L be the orthogonal projection of point I on the axis x′Ox. Show that the two triangles ILA and AJC are similar. Deduce that AC = 2OI .

VI- (5points)

In the next figure ,EFG is an isosceles triangle with vertex E, where FG=5cm and EG = 6 cm.

The circle (C) with center O and diameter [EG] intersects with the segment [FG] at k.



1) Reproduce the figure in real measures.

2)

a) Show that K is the midpoint of segment [FG].

b) Calculate the value of EK to the nearest millimeter.

3) Let S be the image of K under the translation of vector $\overrightarrow{FE}$.

a) Plot the point S on the figure.

b) Prove that ESGK is a rectangle.

4) Let P be a point on segment [EG] distinct from O. The parallel through P to (FG) intersects (EF) at R. Suppose that x is the length, expressed in cm, of segment [EP].

a) What is the nature of triangle EPR? Justify your answer.

b) Prove that $PR = \dfrac{5x}{6}$ and express, in terms of x, the perimeter of triangle EPR.

c) Show that the perimeter of the trapezoid RPGF is equal to $\frac{-7x}{6} + 17$.

d) Can you find a position for point P on segment [EG] so that the triangle and the trapezoid have the same perimeter? Justify your answer.

المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة:الرياضيات الشهادة: المتوسطة نموذج رقم -6- المدّة : ساعتان

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

	Question I	Mark
1	**False** because when we multiply by a negative number, the inequality changes.	0.5
2	**False** because $\frac{90000}{0.64} = 140625$ LL.	0.5
3	**True** because $x^2 = 1 + \frac{5}{7} \times \frac{7^2}{10^2}$, then $x = \frac{3\sqrt{15}}{10}$ or $\frac{-3\sqrt{15}}{10}$.	0.5
4	**False** because it is $\frac{5}{2} + \frac{x^2}{2} + \frac{2x\sqrt{5}}{2} = \frac{1}{2}\left(\sqrt{5} + x\right)^2$.	0.5

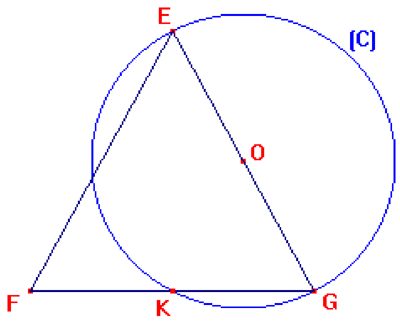
	Question II	Mark
1	Using Pythagoras: $AD^2 = DF^2 - AF^2$, alors $AD = 2x - 3$	0.5
2	AB=2AF, then 2x – 3=2x+ 2 has no solution	0.5
3.a	$A_{BCEF} = A_{ABCD} - A_{AFED} = (2x-3)^2 - (2x-3)(x+1)$	0.5
3.b	A=(2x – 3)(x – 4)	0.5
3.c	(2x-3)(x-4) = $\frac{(x+1)(2x-3)}{6}$; x = 5.	1

	Question III	Mark
1	In Gr. 9A: The number of students who will participate is $\frac{3}{5} \times 20 + \frac{4}{5} \times 15 = 24$ In Gr. 9B:The number of students who will participate is $5 + \frac{5}{7} \times 14 = 15$ In Gr. 9C: The number of students who will participate is $\frac{3}{5} \times 15 + \frac{4}{5} \times 15 = 21$	0.5 0.5 0.5
2	The number of students who will participate in Gr. 9 is 60 The percentage is $\frac{60}{89} \times 100 = 67.41\%$ and the director will not make the trip.	0.5 1

	Question IV	Mark
1	$\begin{cases} x + 5 = 2(y - 5) \\ 2(x - 3) = y \end{cases}$	1
2	x=9 and y=12	1

	Question V	Mark
1	A(2 ;0) and B(0 ; 4) (D) passes through A and B	0.75
2	$\frac{x_A + x_B}{2} = 1 = x_I$ and $\frac{y_A + y_B}{2} = 2 = y_I$	0.5
3.a	(OI):y=2x	0.5
3.b	tanα=2=$a_{(OI)}$, then $\alpha = \tan^{-1} 2 = 63.43 \approx 63°$	0.5

4.a	$y_C = y_B = 4$, then $(BC): y = 4$, $x_J = \frac{x_C + x_B}{2} = 2$. Thus (D′):x=2	**0.75**
4.b	$x_A = 2$, then (D′) passes through A. Then AB=AC.	**0.5**
5	ILA and AJC are similar because $\hat{L} = \hat{J} = 90°$ $\hat{C} = \hat{A}$ since the triangle ABC is isosceles ($\hat{C} = \hat{B}$ and $\hat{B} = \hat{A}$ (alternate interior)) Ratio of similitude: $\frac{IL}{AJ} = \frac{AL}{JC} = \frac{IA}{AC} = \frac{1}{2}$, then AC = 2AI but AI = OI.	**1** **0.5**

Question VI		Mark
1		**0.5**
2.a	EKG is a right triangle at K(inscribed in a semi-circle)and EFG is an isosceles triangle with vertex E, then [EK]is an altitude and median, then K is the midpoint of [FG]	**0.5**
2.b	By Pythagoras: $EK^2 = EG^2 - KG^2$, then EK=5.45	**0.5**
3.a	$\overrightarrow{KS} = \overrightarrow{FE}$ (by translation)	**0.5**
3.b	$\overrightarrow{ES} = \overrightarrow{FK}$ since ESFK is a parallelogram, and $\overrightarrow{KG} = \overrightarrow{FK}$ since K is the midpoint of [FG], then $\overrightarrow{ES} = \overrightarrow{KG}$ and $\hat{K} = 90°$. Thus ESGK is a rectangle.	**0.75**
4.a	EPR is an isosceles triangle since $\hat{R} = \hat{F}$ and $\hat{P} = \hat{G}$ (corresponding angles), but $\hat{G} = \hat{F}$, thus $\hat{R} = \hat{P}$.	**0.5**
4.b	By Thales: $\frac{EP}{EG} = \frac{ER}{EF} = \frac{RP}{FG} = \frac{x}{6}$, *then* $RP = \frac{5x}{6}$.	**0.75**
4.c	$\frac{5x}{6} + 6 - x + 6 - x + 5 = \frac{-7x}{6} + 17$	**0.5**
4.d	$17 = \frac{-7x}{6} + 17$, then x=0. Thus a position for P cannot be found.	**0.5**