امتحانات الشهادة الثانوية العامة
الفرع : الاجتماع واقتصاد

دورة العام ٢٠١٦ العاديّة
الاثنين ١٣ حزيران ٢٠١٦

عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة : ساعتان	الاسم: الرقم:

ملاحظة : يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المُرشّح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

An insurance company offers its employees a special fee for an annual life insurance policy. The following table represents this offer:

Age of the employee in years	25	26	27	28	29
Rank x_i	0	1	2	3	4
Annual fee y_i in hundred thousand LL	2	2.5	3.25	3.75	4

1) a- Calculate $\bar{x}$ and $\bar{y}$, the respective means (averages) of the two variables x and y.
   b- Write an equation of (D), the regression line of y in terms of x.
2) Draw, in a rectangular system, the scatter plot of the points associated to the distribution $(x_i ; y_i)$ as well as the center of gravity G and the line (D).
   (Unit on the x-axis = 2 cm , unit on the y-axis = 4 cm.).
3) Assume that the above pattern remains valid till the age of 35. Estimate the annual fee for an employee whose age is 31 years.
4) What is the percentage of increase in the insurance annual fee between a 25 year old employee and a 31 year old employee?

II- (4 points)

The employees of a school are distributed into three categories: Instructors, administrators and technicians.

- 80% of the employees are instructors of which 70% are women.
- 10% of the employees are administrators of which 80% are women.
- 65% of the employees are women.

One employee is randomly selected from the school. Consider the following events:

I: « The employee selected is an instructor »
A: « The employee selected is an administrator »
T: « The employee selected is a technician »
M: « The employee selected is a man »
W: « The employee selected is a woman ».

1) Calculate the probability P(M).
2) Calculate $P(I \cap W)$; $P(A \cap W)$ and verify that $P(W / T) = 0.1$.
3) Knowing that the selected employee is a man, what is the probability that he is an instructor?
4) In this part, suppose that the number of employees in this school is 200. The names of these employees are written on cards and these cards are placed in a box.
   Three cards are randomly and simultaneously selected from this box.
   What is the probability of the event B: « None of the three cards holds the name of a technician, at least one card holds the name of an administrator and at least one card holds the name of an instructor»?

III- (4 points)

At the end of July 2015, a retiree deposits an amount of 220 million LL in a bank at an annual interest rate of 6% compounded monthly. At the end of each month and after the compounding of interest, this person decides to withdraw 2 million LL for his expenses.

Denote by the natural number n the rank of the month after the deposit and by U_n the amount, in millions LL, that this person has in his account in the nth month after the withdrawal of the 2 million LL.

Thus $U_0 = 220$.

1) Verify that $U_1 = 219.1$ and calculate U_2.
2) For all natural numbers n, show that $U_{n+1} = 1.005U_n - 2$.
3) For all natural numbers n, let $V_n = U_n - 400$.
   a- Prove that (V_n) is a geometric sequence. Specify its ratio and first term.
   b- Show that $U_n = -180 \times (1.005)^n + 400$.
   c- Prove that the sequence (U_n) is decreasing.
   d- In how many years would the amount in the bank account become less than half the original amount for the first time?

IV- (8 points)

Part A

Consider the function f defined over $[0;+\infty[$ as $f(x) = -x^2e^{-x} + 3$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim_{x\to+\infty} f(x)$. Deduce an asymptote (d) to (C).
2) Show that $f'(x) = x(x-2)e^{-x}$ and set up the table of variations of the function f.
3) Draw (d) and (C).
4) Let F be the function defined over $[0;+\infty[$ as $F(x) = (x^2 + 2x + 2)e^{-x} + 3x$.
   a- Prove that F is an antidervative of the function f.
   b- Deduce the area of the region bounded by the curve (C), the x-axis, the y-axis and the line with equation $x = 2$.

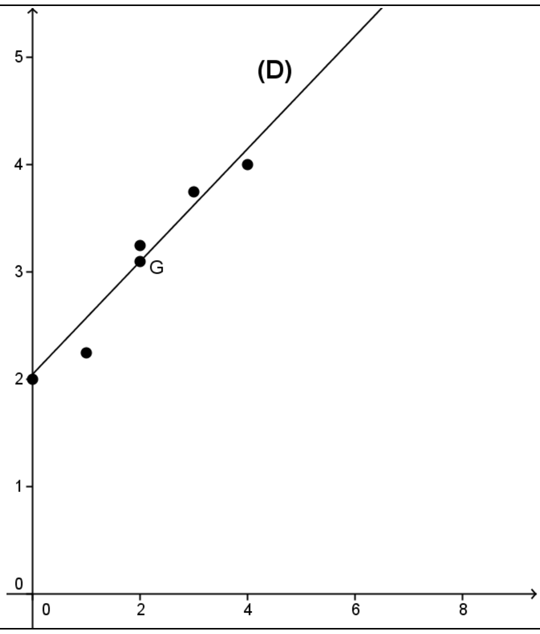
Part B

A factory produces paint. All the production is sold. The average cost of production in hundreds of thousands LL is given as $f(x) = -x^2e^{-x} + 3$, where x is expressed in hundreds of liters. $x \in [0.2 ; 9]$.

1) Determine, in LL, the average cost for the production of 600 liters of paint.
2) a- How many liters of paint should the factory produce so that the average cost of production is minimal? What is then the average cost in LL?
   b- If the sale price of 100 liters of paint is 230 000 LL, does the factory make profit ? Explain.
3) a- Express the total cost $C_T(x)$ in terms of x.
   b- How many liters of paint should the factory produce so that the average cost of production is equal to the marginal cost?

مشروع معيار التصحيح	مسابقة في مادة الرياضيات المدة: ساعتان

I- (7 points)

Q	Correction	Grade
1-a	$\overline{x}=2$; $\overline{y}=3.1$	1
1-b	$(D): y=0.525x+2.05$	1
2	$G(2,3,1)$ 	2
3	age of 31 $\rightarrow x=6$ then $y=0.525(6)+2.05=5.2$ therefore 520 000 LL	1.5
4	$\frac{5.2-2}{2}=1.6$ then 160% is the increasing percentage	1.5

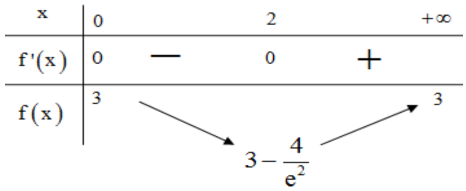
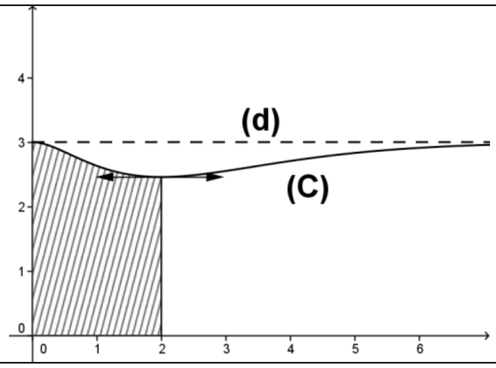
II- (7 points)

Q	Correction	Grade
1	$P(M)=1-P(W)=1-0.65=0.35$	1
2	$P(I\cap W)=P\left(W/_{I}\right)\times P(I)=0.7\times 0.8=0.56$; $P(A\cap W)=P\left(W/_{A}\right)\times P(A)=0.8\times 0.1=0.08$; $P(W\cap T)=P(W)-P(W\cap I)-P(W\cap A)=0.65-0.56-0.08=0.01$ $P\left(W/_{T}\right)=\frac{P(W\cap T)}{P(T)}=\frac{0.01}{0.1}=0.1.$	3
3	$P\left(I/_{M}\right)=\frac{P(I\cap M)}{P(M)}=\frac{0.8\times 0.3}{0.35}=0.68.$	1.5
4	$P(B)=\frac{C_{160}^{2}\times C_{20}^{1}+C_{160}^{1}\times C_{20}^{2}}{C_{200}^{3}}=...=\frac{1424}{6567}\approx 0.216.$	1.5

III- (7 points)

Q	Correction	Grade
1	$U_1 = \left(1+\frac{0,06}{12}\right)\times 200 = 219{,}1$ et $U_2 = \left(1+\frac{0.06}{12}\right)\times 219.1 = 218.1955.$	**1**
2	$U_{n+1} = \left(1+\frac{0.06}{12}\right)\times U_n - 2 = 1.005\,U_n - 2$	**1**
3-a	$V_{n+1} = U_{n+1} - 400 = 1.005\,U_n - 402 = 1.005\left(U_n - 400\right) = 1.005\,V_n$ $q = 1.005$ and $V_0 = U_0 - 400 = 220 - 400 = -180$	**1.5**
3-b	$V_n = V_0 \times q^n = -180\times\left(1.005\right)^n$ and $U_n = V_n + 400 = -180\times\left(1.005\right)^n + 400$	**1**
3-c	$U_{n+1} - U_n = -180\times\left(1.005\right)^{n+1} + 400 + 180\times\left(1.005\right)^n - 400$ $= -180\times\left(1.005\right)^n\left(1.005-1\right) = -0.9\times\left(1.005\right)^n < 0$	**1**
3-d	$U_n < \frac{U_0}{2}$; $-180\times\left(1.005\right)^n + 400 < 110$; $\left(1.005\right)^n > \frac{29}{18}$; $n > \frac{\ln\left(29/18\right)}{\ln(1.005)}$; $n > 95.6$; $n = 96$ then after 8 years.	**1.5**

IV- (14 points)

Q	Correction	Grade
A-1	$\lim_{x\to+\infty} f\left(x\right) = 3$ then (d) : y = 3 is a horizontal asymptote at $+\infty$	**1**
A-2	$f'(x) = x(x-2)e^{-x}$ same sign as $x-2$ then: 	**1.5**
A-3		**1.5**
A-4-a	$F'(x) = (2x+2)e^{-x} - e^{-x}(x^2+2x+2) + 3 = f(x)$	**1.5**
A-4-b	$A = \int_0^2 f(x)\,dx = \left[F(x)\right]_0^2 = F(2) - F(0) = \frac{10}{e^2} + 6 - 2 = \left(4 + \frac{10}{e^2}\right)u^2$	**1.5**
B-1	$f\left(6\right) = -36e^{-6} + 3 = 2.910765$ the average cost is: 2 910 765 LL.	**1.5**
B-2-a	According to the table of variations the average cost production is minimal for $x = 2$ then 200 liters. The minimal average cost is $f(2) = 2.45866$ then 245 866 LL	**1.5**
B-2-b	y = 2,3 < f(2) (The minimum of f) then no gain.	**1.5**
B-3-a	$C_T\left(x\right) = x\,f(x) = -x^3e^{-x} + 3x.$	**1**
B-3-b	$C_m\left(x\right) = \left(x^3 - 3x^2\right)e^{-x} + 3$; $C_M\left(x\right) = C_m\left(x\right)$; $\left(x^3 - 3x^2\right)e^{-x} + 3 = -x^2e^{-x} + 3$; $x^3 - 3x^2 = -x^2$; $x^2\left(x-2\right) = 0$; $x = 0$ or $x = 2$ but $0 \notin \left[0.2\ ;\ 9\right]$ then $x = 2$ therefore for a production of 200 liters.	**1.5**