المادة: الرياضيات الشهادة: الثانوية العامة – فرع الاجتماع والاقتصاد نموذج رقم -3- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي 2016-2017 وحتى صدور المناهج المطوّرة)

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (4 points)

An hybrid automobile is a vehicle having two types of motorization : A thermic engine and an electric engine, to limit the consummation of fuel .We propose to study the distribution of sales of hybrid vehicles these last years.
The following table represents a statistical survey concerning the sales of hybrid models during the years [2009-2014] in a certain company .

Year	2009	2010	2011	2012	2013	2014
Rank of the year x_i	1	2	3	4	5	6
Number of hybrid models sales y_i	18	32	65	84	105	123

1) Represent in an orthonormal system the scatter plot of points $(x_i ; y_i)$.
2) Determine an equation of the regression line $(D_{y/x})$. Draw this line in the same system .
3) Calculate the coefficient of correlation and interpret its value.
4) Calculate the percentage of increase in the number of hybrid models sales from 2012 to 2014.
5) we suppose that the evolution of sales continues till 2015. the previous adjustment will be used in order to predict the sales in 2015.Estimate the number of vehicles sales during 2015.
6) The target of the company is to make te sales in 2015 around 15% more than those of 2014.
- **a)** Calculate the number of vehicles that should be sold at 2015 to reach this target Round the result the nearest unit.
- **b)** Is the estimation of vehicles sales at 2015 compatible with the target of company?

II- (4 points)

An association decides to open a care center for wild birds victims of the pollution. They want to take care of birds and release them once they are cured.
The center was opened the first of january 2013 with 115 birds.
The specialists expect that 40% of the birds that exist in the center in the first of january of a year still exist in the first of next january and that 120 new birds are welcomed in the center each year.
We are interested to the number of birds that exist in the center in the first of january of next years.
Denote by (u_n) the number of birds that exist in the center in the year $(2013 + n)$.
Then $u_0 = 115$.
1) Calculate the number of birds existing in the center in 2014.
2) For all natural numbers n , show that $u_{n+1} = 0.4u_n + 120$.
3) For all natural numbers n , consider the sequence (v_n) defined as $v_n = u_n - 200$.
Show that (v_n) is a geometric sequence with common ratio 0.4
4) Express v_n in terms of n and deduce that $u_n = 200 - 85(0.4)^n$.
5) The capacity of center reception is about 200 birds. Is it suffisant ? Justify your answer.
6) Each year, the center gets an amount of 20 dollars for each bird received the first of January. Calculate the total amount obtained by the center between the first of january 2013 and the 31

december 2018 if we suppose that the evolution of number of birds continues in the same procedure during this period.

III- (4 points)

A factory produces plasma screens . Before be proposed to sale , each screen should be tested. If the test is positive,that the screen is well functioning and it will be sold.
If the test is negative, the sceen will be repaired before tested again.
If the second test is positive , it will be sold. Otherwise ,it will be destroyed.
Assume that :

* for 70% of screens , the first test is positive .
* for 65% of repaired screens , the second test is positive.

Consider the following events :
T_1 : ′ the first test is positive ′
C : ′ the screen is sold ′.

1) we choose randomly one screen in the output of the production line. Determine the probability of the events T_1 and C.

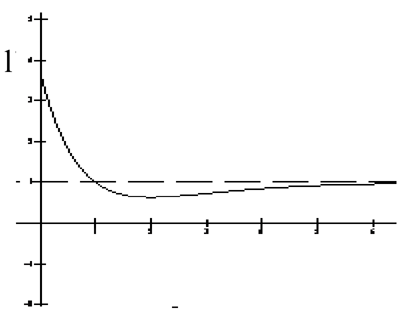
2) the screen is proposed to the sale. What is the probability that the first test is positive ?

3) the cost of production of a screen is **1000$** with a supplement of **50$** if it needs repair . Each screen is sold at **′ d $ ′** with **d** is a positive number. Let X be the random variable that is equal to the algebraic gain (positive , negative or zero) realised by the factory by selling one screen .

a- Verify that the three possible values of X are : d - 1000 ; d - 1050 and -1050.
b- Determine the probability distribution of X.
c- Prove that the expected value of X is E(X) = 0.895d – 1015.
d- Starting which value of d, the factory can make a profit ?

IV- (8 points)

Part A



Let g be the function defined over $[0;+\infty[$ as $g(x) = a + (b - x)e^{-x+1}$ and l representative curve in the next figure .
The minimum of g is realised for x = 2.

1) Determine graphically $\lim\limits_{x\to+\infty} g(x)$. Deduce that a = 1.

2) Prove that $g'(x) = (x - 1 - b)e^{-x+1}$. Deduce that b = 1.

3) For all $x \in [0;+\infty[$, prove that g(x) > 0 .

Part B

Let f be the function defined over $[0;+\infty[$ as $f(x) = x + 1 + xe^{-x+1}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}\,; \vec{j})$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$.

2) Let (d) be the line with equation y = x + 1.
a) Prove that (d) is an asymptote to (C).
b) Discuss, according to x, the relative position of (C) and (d).

3) Verify that $f'(x) = g(x)$ and set up the table of variations of f.

4) Determine an equation of (T) the tangent to (C) at the point with abscissa 1.

5) Draw (C), (d) and (T).

6) The line with equation y = 2x intersects the curve (C) at the point with x = α.

Verify that 1.8 < α < 1.9.

Part C

In that follows ,suppose that $\alpha = 1.85$.

A company produces watches. The average cost function $\overline{C}$ is given , in millions of L.L by $\overline{C}(x) = 1 + \frac{1}{x} + e^{-x+1}$.

x is the number of hundred of watches produced by the company. $(0 < x \leq 10)$

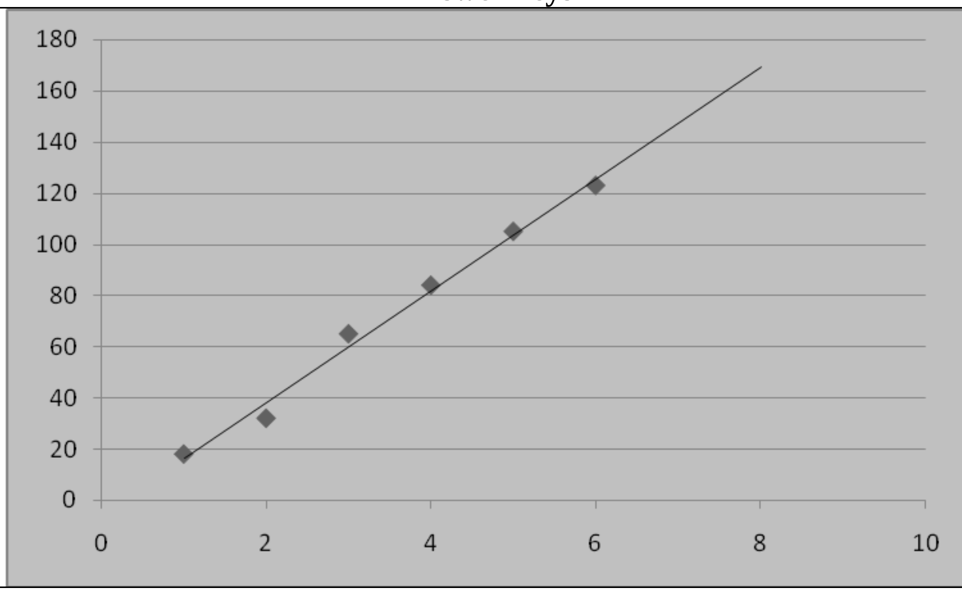
1) Calculate $\overline{C}(3)$. Give an economic interpretation to the value obtained.

2) For all $0 \leq x \leq 10$, Verify that the total cost C_T is given by $C_T(x) = x + 1 + xe^{-x+1}$.
Deduce the fixed cost to the nearest one LL.

3) Knowing that 20% of the produced watches are defective . Each defective watch is sold at 10 000 L.L and each non defective watch is sold 22 500 L.L and all the produced watches are sold.
a) Prove that the revenue function is given by R(x) = 2x.
b)Determine the minimal quantity of watches that should be sold so that the company make a profit.

المادة: الرياضيات الشهادة: الثانوية العامة - فرع الاجتماع والاقتصاد نموذج رقم -3- المدّة :	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

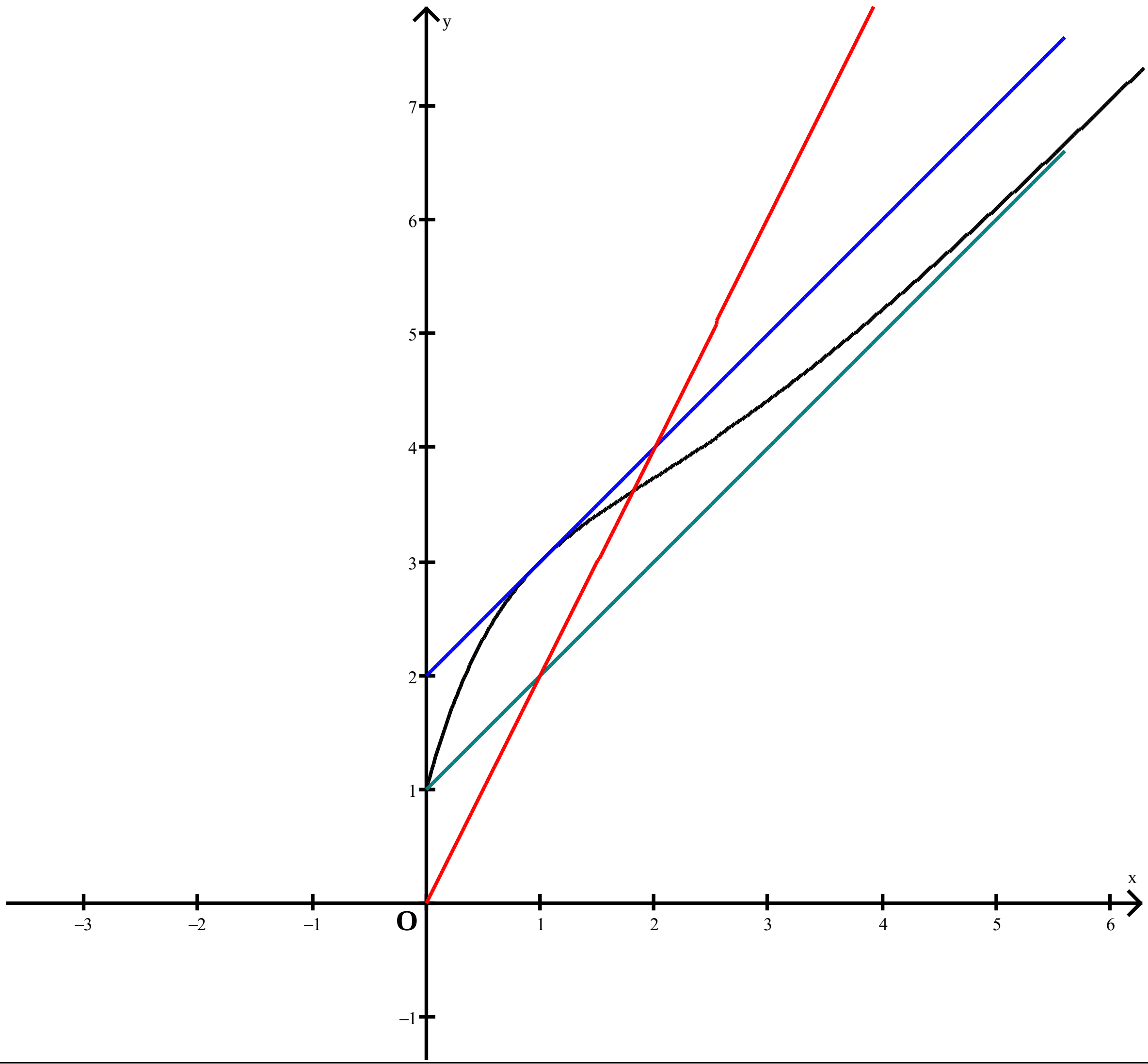
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي 2016-2017 وحتى صدور المناهج المطوّرة)

QI	Answer keys	Note
1		1/2
2	$(D_{y/x})$: y = 2.8x – 5.133. See the figure. The line should be through the average point G(3.5 ; 71.166)	1/2 1/2
3	r = 0.995. strong linear positive correlation between x and y (r is close to 1).	1/4 3/4
4	$Percentage = \left(\frac{123-84}{84}\right)(100\%) = 46.328\%$	1.5
5	y = 21.8(7) – 5.133 = 147.467 ≈ 148 hybrid automobiles	1
6a	123(1 + 0.15) = 141.45 ≈ 142 hybrid automobiles	1.5
6b	Yes since 148 > 142	1/2

QII	Answers key	Note
1	$u_1 = u_0(40\%) + 120 = 166$	3/4
2	$u_{n+1} = 0.4u_n + 120$	3/4
3	$v_{n+1} = 0.4v_n$	1
4	$v_n = -85(0.4)^n$; $u_n = 200 - 85(0.4)^n$.	1/2 1/2
5	$u_n \leq 200$; $-85(0.4)^n \leq 0$ true for all natural number n. **Or study the variations of** u_n and determine its limit when n tends to $+\infty$	1.5
6	$S = 20(u_0 + u_1 + ... + u_5) \approx 21180\$$ in approximation of number of birds.	2

QIII	Answers key	Note
1	$P(T_1)=0.7$ $P(C)=0.7+0.3\times 0.65=0.895$	1/4 1.25
2	$P(T_1/C)=\dfrac{P(T_1\cap C)}{P(C)}=\dfrac{P(T_1)}{P(C)}=\dfrac{0,7}{0,895}=\dfrac{140}{179}$	1
3a	Verify tat the three possible values of X are : d - 1000 ; d - 1050 and -1050	1
3b	$P(X=d-1000)=0.7$ $P(X=d-1050)=0.195$ $P(X=-1050)=0.105$	1/2 1/2 1/2
3c	$EX=0.7(d-1000)+0.195(d-1050)+0.105(-1050)=0.895d-1015$	1
3d	$\text{EX}>0$; $0.895d-1015>0$; from 1134.07$	1

QIV	Answer keys	Note
A1	$\lim\limits_{x\to+\infty} g(x)=a$ Verify that a = 1	1/2 1/2
A2	$g'(x)=(x-1-b)e^{-x+1}$ $g'(2)=(2-1-b)e^{-2+1}=0;\, b=1$	1/2 1/2
A3	The curve C_g is strictly above of x´Ox then g(x) > 0	1/2
B1	$\lim\limits_{x\to+\infty} f(x)=+\infty$	1/2
B2a	$\lim\limits_{x\to+\infty} (f(x)-y)=0$	1/2
B2b	$f(x)-y>0\ \text{si}\ x>0$ (0;1) point of intersection of (C) and (d)	1
B3	$f'(x)=g(x)>0$ Then f is strictly increasing. $\begin{array}{c\|ccc} x & 0 & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & 1 & \nearrow & +\infty \end{array}$	1 1
B4	$(T):\ y=x+2$	1/2

B5		2
B6	f(1.8) = 3.608 > 3.6 f(1.9) = 3.67 < 3.8	1/2
C1	$\overline{C}(3) = 1.468668$ The moyen cost of production of a watch from the 300 first watches produced is $\frac{1.468668}{100} \times 1000000 = 14686.686 L.L$	1/2 1
C2	$C_T(x) = x\overline{C}(x) = x + 1 + xe^{-x+1}$ $C_T(0) = 1$ *in* million L.L; then 1000 000L.L	1/2 1/2
C3a	$R(x) = \frac{(10000)(0.2x)(100)}{1000000} + \frac{(22500)(0.8x)(100)}{1000000} = 2x$	1
C3b	P(x) > 0 ; R(x) > CT(x) ; 2x > f(x) then x = α = 1.85. therefore 185 watches	1