المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: الثانوية العامة – فرع الاجتماع والاقتصاد نموذج رقم -2- المدّة : ساعتان

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي 2016-2017 وحتى صدور المناهج المطوّرة)

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (4 points)

The United Nations Organization has established in 2010 a statistical survey over the world population. The following table shows the result obtained by this study.

Year	1970	1980	1990	2000	2010
Rank of the year x_i	1	2	3	4	5
Population (in millions of persons) : y_i	3 023	4 438	5 290	6 115	6 908

1) Represent graphically the scatter plot of the points $(x_i ; y_i)$.

2) The percentage of increase of the world population between the years 2010 and 2013 is 3.47%. Calculate the population in 2013.

3) For each year, calculate $\ln y_i$ and complete the following table :

Year	1970	1980	1990	2000	2010
x_i	1	2	3	4	5
$z_i = \ln y_i$					

4) Determine by the least squares method, the equation of the regression line of z in terms of x.

5) Deduce from the preceding adjustment that the expression of the population y in terms of the rank x, is under the form of : $y = Ee^F$ where E and F are two real numbers to be determined.

6) Estimate the world population in 2030 ?

II- (5 points)

Part A

Consider the sequence (u_n) defined as $u_0 = 900$ and, for every natural number n, $u_{n+1} = 0.6u_n + 200$.

1) Prove that the sequence (u_n) is neither arithmetic nor geometric.

2) Consider the sequence (v_n) defined, for every natural number n, as $v_n = u_n - 500$.

a) Prove that (v_n) is a geometric sequence whose first term and common ratio are to be determined.

b) Prove that $u_n = 400\times(0.6)^n + 500$.

c) Discuss the variations of the sequence (u_n).

d) Determine the limit of the sequence (u_n).

Part B

In a certain country, two companies A and B share the communications' market.
The clients subscribe, the first of January, with either A or B, with a one year contract of which they are free to chose again A or B

The company A holds 90% of the market and the company B, which has just launched 10% of it. We estimate that, each year, 20% of the clients of A change to B, while 20% of the clients of B change to A.
Consider a population which is represented by 1 000 clients in the year 2000. Thus 900 clients are registered in A and 100 clients are registered in B.
We want to study the evolution of this population in the coming years.

1) Verify that the company A counts 740 clients in 2001.
2) Calculate the number of clients of B in 2002.
3) Denote by a_n the number of clients of A in the year (2000 + n).
  a) Establish that $a_{n+1} = 0.6a_n + 200$.
  b) Using the result in **part A**, what can you expect for the evolution of the communication market in this country?

III- (4 points)
The seats of a movie theater are fully occupied. The proposed film is a replay of a blockbuster comedy.
In this theater, men represent 25% of the spectators, women represent $\frac{2}{5}$ of the spectators and the remaining spectators are kids.

$\frac{1}{5}$ of men and 30 % of women have already seen this movies before.
At the end of the show , one spectator is interviewed randomly.
Consider the following events :
H : « the interviewed spectator is a man »
F : « the interviewed spectator is a woman »
E : « the interviewed spectator is a kid »
V : « the interviewed spectator has already seen this movie before »

1) **a)** Express, using a sentence, the event $V \cap H$.
  b) Calculate $P(V/H)$ and deduce $P(V \cap H)$.
2) The probability of the event V is equal to 0,4.
  a) Determine the probability that the interviewed spectator a kid who has already seen this movie before .
  b) Determine the probability that the interviewed spectator who has already seen this movie before knowing that he is a kid.
3) Two spectators re randomly interviewed one after another with replacement . Denote by X the random variable equal to the number of spectators who have already seen this movie before .
  a) Prove that P(X = 1) = 0.48.
  b) Determine the probability distribution of X.
4) This replay was seen by 1000 spectators on a night, and we want to choose simultaneously three spectators for interview among these 1000 persons.
  a) What is the probability of interviewing three women?
  b) What is the probability of interviewing three persons who has never seen this movie before knowing that they are men?

IV- (8 points)
Part A
Consider the function f defined over $[0;+\infty[$ as $f(x) = x - 1 - \ln(x+1)$ and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

1) Calculate f(1), f(7) and $\lim\limits_{x\to+\infty} f(x)$.

2) Prove that $f'(x) = \dfrac{x}{x+1}$. Deduce that f is strictly increasing and set up the table of variations of the function f.

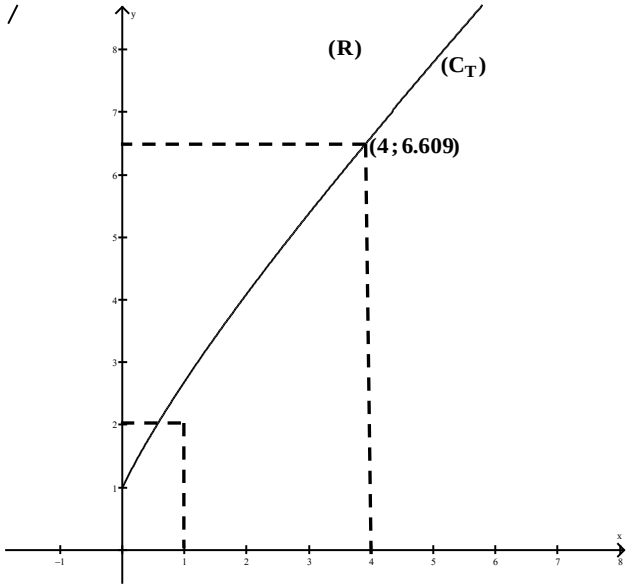
3) Write an equation of (T) the tangent to the curve (C) at the point of (C) with abscissa 1.

4) Prove that the equation f(x) = 0 has a unique solution α. Verify that $2.1 < \alpha < 2.2$.

5) Draw the tangent (T), and the curve (C).

Part B (In what follows take α = 2.15)

An enterprise produces copybooks.
The function of the profit P, in millions of L.L, is given as P(x) = f(x).
Denote by x the quantity of produced copybooks (in thousands).
The functions C_T and R of the total cost and the revenue in millions of L.L, are represented in the given figure.
($x \geq 0$)



1) Calculate the maximum loss of this enterprise.

2) Use the figure to :
- **a)** Calculate the fixed cost of the enterprise.
- **b)** Calculate the average cost of production of one copybook during the production of 4000 copybooks.

3) We admit that the function R is defined as R(x) = ax.
- **a)** Use the figure to prove that a = 2.
- **b)** Deduce that 2000 L.L is the sale price of one copybook.

4) Prove that α is the solution of the equation $R(x) = C_T(x)$.
Deduce the minimum number of copybooks to be produced for the enterprise to realize a gain.

5) Prove that the function C_T is defined as $C_T(x) = x + 1 + \ln(x+1)$.

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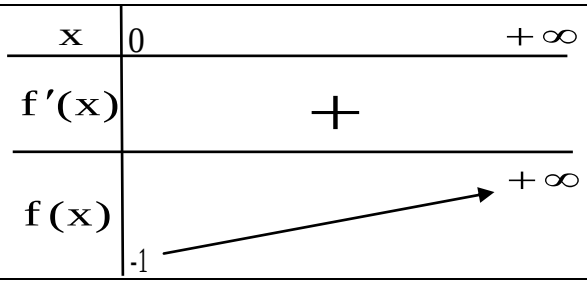
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي 2016-2017 وحتى صدور المناهج المطوّرة)

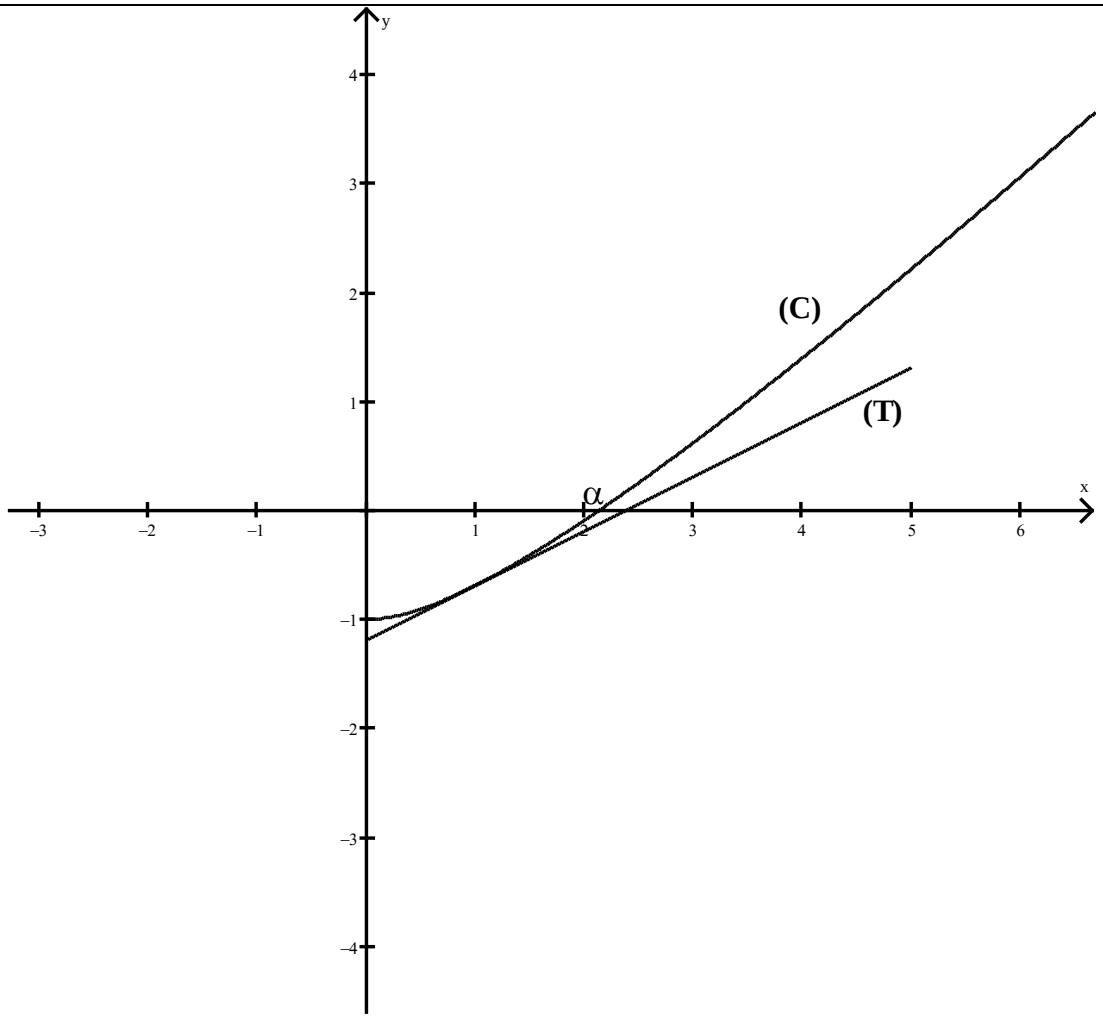
QI	Answers	Mark
1	Scatter plot	1
2	The population in 2013 is 7147 in millions of persons then it is 7147707600 persons	1.5
3	<table><tr><td>Year</td><td>1970</td><td>1980</td><td>1990</td><td>2000</td><td>2010</td></tr><tr><td>x_i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$z_i = \ln y_i$</td><td>8.014</td><td>8.397</td><td>8.573</td><td>8.718</td><td>8.840</td></tr></table>	1
4	$z = 0.1973x + 7.9165$	1/2
5	$y = e^{0.1973x+7.9165} = e^{0.1973x} \times e^{7.9165} = 2742.156e^{0.1973x}$; E = 2742.156 and F = 0.1973	1.5
6	x = 7 then y = 10911.79944 in millions of persons then it is 10911799440 persons	1.5

QII	Answers	Mark
A1	$u_1 = 740$; $u_2 = 644$; $u_2 - u_1 \neq u_3 - u_2$ $u_2 / u_1 \neq u_3 / u_2$	1
A2a	The ratio = 0.6 and the first term is $v_0 = 400$	1
A2b	$u_n = 400\times(0.6)^n + 500$	1/2
A2c	(u_n) is decreasing	1
A2d	The limit = 500	1/2
B1	The company A counts 740 clients in 2001	1/2

B2	The company B count 356 clients in 2002	1/2
B3	$a_{n+1} = 0.6a_n + 200$	1
B4	The number of clients of A decreases but remains more than 500 while the number of clients of B increases but remains less than 500 so A et B will never have the same number of clients.	1

QIII	**Answers**	**Mark**
1a	$V \cap H$ represents the interviewed spectator is a man who has seen this movie before at least once.	1/2
1b	$P(V/H) = \frac{1}{5}$; $P(V \cap H) = \frac{1}{20}$	1/2 1/2
2a	$P(V \cap E) = 0.23$	1/2
2b	$P(V/E) = \frac{23}{35}$	1/2
3a	P(X = 1) = 0.48	1
3b	We have proven P(X = 1) = 0.48 ; P(X = 0) = 0.36 and P(X = 2) = 0.16	1/2 1/2
4a	P(3F) = 0,063	1
4b	$P(3\overline{V}/H) = 0.51.$	1.5

QIV	**Answers**	**Mark**
A1	f(1) = -0.69 ; f(7) = 3.9 ; $\lim\limits_{x \to +\infty} f(x) = +\infty$	1/4 1/4 1/2
A2	$f'(x) = 1 - \frac{1}{x+1} = \frac{x}{x+1} > 0$ so f is strictly increasing 	1/2 1/2 1
A3	(T) : $y = \frac{1}{2}x - \frac{1}{2} - \ln 2$	1
A4	Over $[0;+\infty[$ the function f is defined, continuous and strictly increasing, it passes from - to + so the equation f(x) = 0 admits unique solution. f(2.1) = -0.03 < 0 and f(2.2) = 0.03 > 0.	1/2 1/2

A5		2
B1	P′(x) = 0 ; maximum loss = 1000000 L.L **or** by using the curve (C).	1
B2a	$C_T(0)$ = 1 in millions L.L thus 1000000 L.L	1/2
B2b	$C_T(4)$ = 6.609 in millions L.L so 6609000 L.L The average cost of production of a copybook during the production of 4000 copybooks = 1652,25 L.L	1/2 1
B3a	R(1) = 2 then a = 2	1/2
B3b	$R(x) = \frac{(\text{sale price}) \times x \times 100}{1000000} = 2x$; sale price = 2000 L.L	1.5
B4	R(x) = C_T(x) gives P(x) = 0 so f (x) = 0 thus x = α = 2.15. Hence 2150 copybooks. Therefore 2151 copybooks is the minimum number of copybooks to sell for the enterprise to realize a gain	1.5
B5	$C_T(x)$ = R(x) - P(x) ; $C_T(x) = x+1+\ln(x+1)$	1/2