المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: الثانوية العامة الفرع: العلوم العامة نموذج رقم -١- المدّة : أربع ساعات

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2points)

Answer true or false and justify.

1) The complex number $(-1+i)^{10}$ is real .

2) The function f ′ defined as $f'(x)=\int_{0}^{x^2}\sqrt{t^2+4}dt$ is the derivative function of a function f. then the function f doesn't have an inflection point over ℝ.

3) If $f(x)=x^2e^x$, then the nth derivative is $f^{(n)}(x)=\left(x^2+2nx+(n-1)\right)e^x$ for all n ∈ ℕ*.

4) $1+i+...+i^{19}=0$ (i is imaginary).

II- (2points).

In the plane referred to an orthonormal system (O ; i , j , k) , consider the plane (P) with equation 3x+y-5=0 , and the lines (D) and (D') defined as :

$$(D):\begin{cases}x=t\\y=-3t+5\\z=t-4\end{cases}\quad (D'):\begin{cases}x=m+1\\y=-2m-1\\z=-m+3\end{cases}$$

1)
   - a) Verify that (P) is perpendicular to the plane (XOY) .
   - b) Prove that (D) is included in (P).

2) Prove that (D) and (D') intersect at a point A with coordinates to be determined .

   In what follows , given the point B(0,1,4).

3) Consider in the plane (Q) formed by (D) and (D') the circle (C) with center A and radius AB.
   - a) Write an equation of (Q).
   - b) Write a system of parametric equations of the line (Δ), tangent at B to the circle (C).

4) Calculate the coordinates of E and F, the intersection points of the circle (C) with the line (D).

III- (3points)

Consider two boxes U and V .

- The box U contains 10 cards where three of them have the letter A, 5 have the letter B and 2 the letter C.
- The box V contains 6 balls where two of them are red and four are black .

The rule of game is the following :

One card is randomly selected from the box U.

- If the player selects a card A then he draws two balls from the box V , one after another with replacement
- If the player selects a card B, then he draws two balls from the box V , one after another without replacement .
- The game stops when the player selects a card C or he selects one black ball .

Consider the following events :

A : « Select a card named A»
B : « Select a card named B »
C : « Slect a card named C »
G : « The player wins »

The player wins only if he selects 2 red balls one after another or he selects a card C .

1) Calculate P(G/A) and show that $P(G \cap A) = \frac{1}{30}$.
2) Calculate $P(G \cap B)$, then P(G).
3) To participate in this game , the player should pay 2000 LL. He wins 5000 LL if he selects a card named C and 3000 LL if he selects 2 red balls .

Let X be the random variable that is equal to the amount got by the player .

- **a)** Show that the three values of X are -2000, 1000 and 3000.
- **b)** Determine the probability distribution of X .
- **c)** Estimate the amount got by the organizer if 100 players participate to this game.

IV- (3 pts)

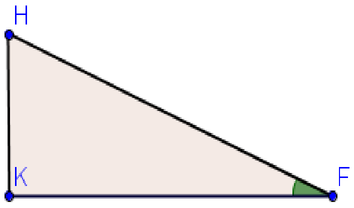
In the oriented plane , given a rectangle ABCD with center O so that $AB = 4cm$ **,**

and $\left(\overrightarrow{AB}, \overrightarrow{AC}\right) = \frac{\pi}{6}(2\pi)$.

Let E be the symmetric of A wit respect to D . Denote by S the similitude that maps E onto O and A onto B . .

1. Verify that the ratio of the similitude is $k = \frac{1}{2}$ and determine a measure of the angle α of *S.*
2. Determine the image of D under S . Show that C is the center of S .
3. Let I be the point of $[EO]$, distinct from E and O ; and (Γ) the circle with center I and passing through A . (Γ) intersects (AD) and (AB) respectively at M and P.
   a. Draw (Γ) and plot the points M and P .
   b. Justify that $C \in (\Gamma)$.
4. Let N be the orthogonal projection of C on (MP).
   a. Show that $\left(\overrightarrow{MP}, \overrightarrow{MC}\right) = \frac{\pi}{6}(2\pi)$.
   b. Deduce that $S(M) = N$.
5. Prove that B , N and D are colinear .
6. The plane is referred to an orthonormal direct system $(A, \vec{u}, \vec{v})$, with $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.
   a. Determine the affixes of the points B and *C.*
   b. Give the complex form of *S.*

V-(3points)



In the next figure , FKH is a right triangle at K with FK=3cm and KH=$\sqrt{3}$ cm.

Let A be a point on [FK] so that AK=1cm and A'is the symmetric of F With respect to K .

Denote by (H) the hyperbola with focus F,and directrice (KH) and eccentricity 2 .

1.a Determine the focal axis of (H) .

b. Prove that A and A' are the vertices of (H) .

2.a.Determine the center O of (H) and the second focus F'.

b. Show that (OH) is an asymptote of (H) then find the second asymptote .

c.Draw (H).

3.Let G be a point so that $\overrightarrow{FG} = 2\sqrt{3}\overrightarrow{KH}$, prove that G is a point on (H).

4.The plane is referred to an orthonormal system $(O, \vec{u}, \vec{v})$, with $\vec{u} = \overrightarrow{OK}$.

a. a-Verify that an equation of (H) is : $\frac{x^2}{4} - \frac{y^2}{12} = 1$

b. b- Prove that (GK) is tangent to (H).

VI-. (7pts)

The plane is referred to the orthonormal system $(O, \vec{\imath}, \vec{\jmath})$.

(Part A)

Consider the differential equation (E) defined as : $y' + y = 1 - 2e^{-x+1}$.

1. Determine a and b so that $Y = a + bxe^{-x}$ is a particular solution of (E).
2. Solve (E). Deduce the particular solution of (E) so that $y(1) = 0$.

(Part B)

Let g be a function defined over , as $g(x) = 1 + (1-2x)e^{-x+1}$. (C) is its representative curve .

1. Determine $\lim\limits_{x \to +\infty} g(x)$. Give a geometric interpretation.
2. Calculate $g'(x)$ the derivative of $g(x)$. Set up the table of variations of g .
3. Prove that $g(x) = 0$ has two roots 1 and α so that $\alpha \in [2.25, 2.26]$.

   Verify that $e^{\alpha-1} = 2\alpha - 1$.
4. Solve $g(x) \leq 0$. Deduce the solutions of the inequality $g(x^2) \leq 0$.
5. Draw (C).
6. Calculate the area of the region bounded by (C), the line (Δ) with equation $y = 1$and the two lines with equations x=1 and x =2.

(Part C)

Let f be the function f defined over $\mathbb{R}$ as $f(x) = 1 + x + xe^{-x^2+1}$. (Γ)its representative curve

and (d) the line with equation $y = x + 1$.

1. Calculate $f(-x) + f(x)$. Make a conclusion .
2. Determine $\lim\limits_{x \to +\infty} f(x)$. Show that (d) is an asymptote when x tends to $+\infty$.

Discuss the relative position of (Γ) and (d).

3. Prove that $f'(x) = g(x^2)$ for all x $\in [0, +\infty[$. Verify that $f(\sqrt{\alpha}) = 1 + \frac{2\alpha\sqrt{\alpha}}{2\alpha - 1}$.
4. Set up the table of variations of f .
5. Draw (Γ).

6. Let $(U_n)_{n\geq 0}$ be the sequence defined as : $U_n = \int_0^1 [f(nx) - nx]dx$.

   a. Calculate U_0 .

   b. Write U_n in terms of n. Determine $\lim_{n\to+\infty} U_n$.

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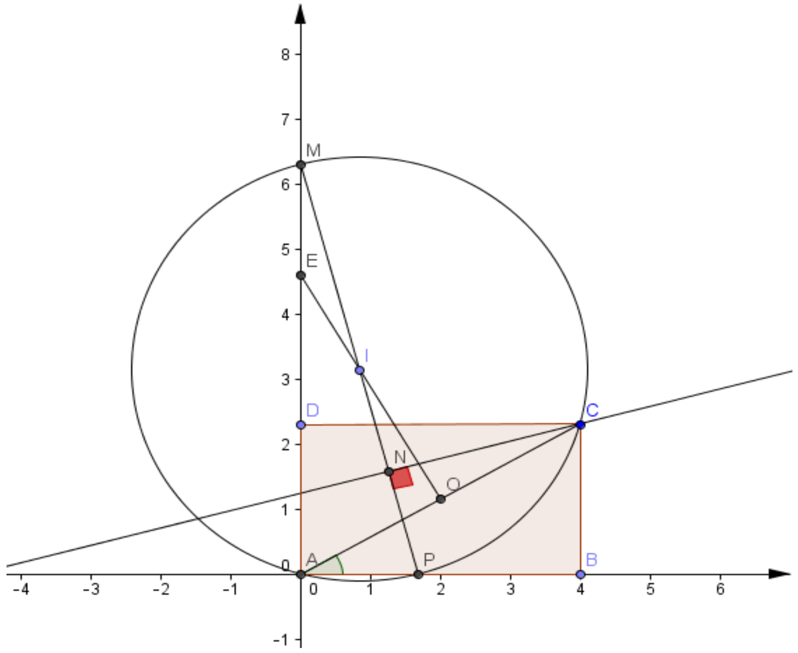
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

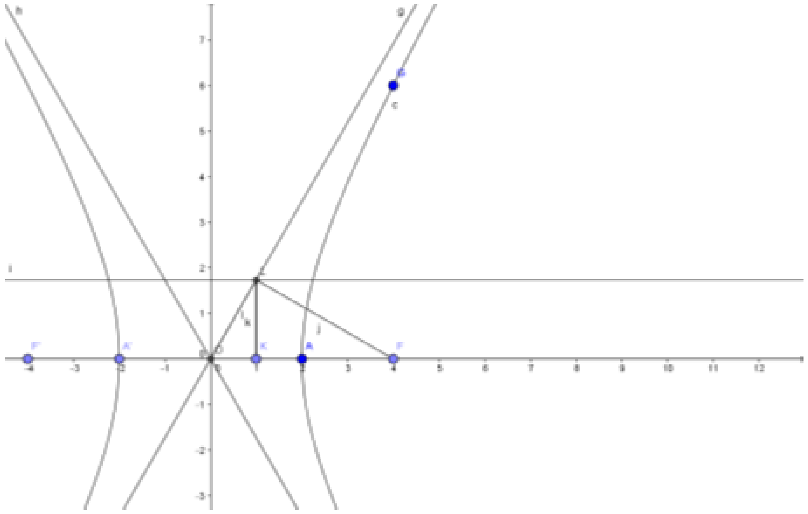
QI		Notes
1	False : on the y – axis .	1
2	False :its has at x=0 an inflection point .	1
3	False : for n=2	1
4	True : its a sum of a geometric sequence with first term 1 and ratio q=i	1

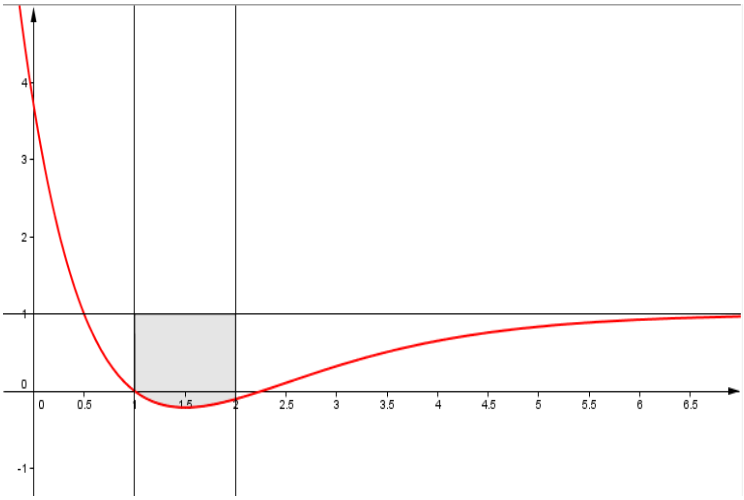
QII		Notes
1.a	$(3,1,0).(0,0,1)=0$ Then (P) is perpendicular to the plane XOY	0,5
1.b	3t-3t+5-5=0 $\Rightarrow (D)\subset(P)$	0,5
2	A(4,7,0) for m=3 et t=4	0,5
3.a	(Q) :5x+2y+z-6=0	0,5
3.b	$\vec{u}=\overrightarrow{AB}\wedge\vec{n}$ with $\vec{u}$ direction vector of the tangent and $\vec{n}$ normal vector of (Q). $(\Delta):\begin{cases}x=0\\y=6\lambda+1\\z=-12\lambda+4\end{cases}$	1
4	$t=4+\dfrac{4\sqrt{66}}{11}$ et $t=4-\dfrac{4\sqrt{66}}{11}$	1

<table>
<tr><th>QIII</th><th></th><th>Notes</th></tr>
<tr><td>1</td><td>$\text{P}(\text{G}/\text{A})=\frac{2}{6}\times\frac{2}{6}=\frac{1}{9}$. $\text{P}(\text{G}\cap\text{A})=\text{P}(\text{G}/\text{A})\times\text{P}(\text{A})=\frac{1}{9}\times\frac{3}{10}=\frac{1}{30}$</td><td>1</td></tr>
<tr><td>2</td><td>$\text{P}(\text{G}\cap\text{B})=\text{P}(\text{G}/\text{B})\times\text{P}(\text{B})=\frac{2}{6}\times\frac{1}{5}\times\frac{1}{2}=\frac{1}{30}$
P(G)=P($\text{G}\cap\text{A})+\text{P}(\text{G}\cap\text{B})+\text{P}(\text{C})=\frac{4}{15}$</td><td>1
1</td></tr>
<tr><td>3.a</td><td>–2000($\overline{\text{G}}$), 1000($\text{G}\cap\text{A}$ ou $\text{G}\cap\text{B}$),3000(C).</td><td>1</td></tr>
<tr><td>3.b</td><td><table><tr><td>x_i</td><td>–2000</td><td>1000</td><td>3000</td></tr><tr><td>P(X=x_i)</td><td>$\frac{11}{15}$</td><td>$\frac{1}{15}$</td><td>$\frac{1}{5}$</td></tr></table></td><td>1</td></tr>
<tr><td>3.c</td><td>E(x) = –800
Then the gain of the organizer is 800 × 100 = 80 000 LL.</td><td>1</td></tr>
</table>

Q4		
1.	$k=\dfrac{OB}{EA}$, using the equilateral triangle OBC: $k=\dfrac{BC}{2BC}$	
	$k=\dfrac{1}{2}$ **et** $\alpha=(\overrightarrow{AE},\overrightarrow{BO})=(BC,BO)=\dfrac{\pi}{3}\ (2\pi)$	**1**

2	The image D under S , $D'$is the midpoint of $\left[BO\right]$. EAC is an equilateral triangle . The image of the triangle EAC under S is the equilateral OBC ,then $S(C)=C$. Then, C is the center of S .	0,5
3.a		0.5
3.b	(OE) is the perpendicular bisector of [AC]. I $\in$(OE) then IC = IA , C $\in(\Gamma)$.	0.5
.4.a	$\left(\overrightarrow{MP},\overrightarrow{MC}\right)=\left(\overrightarrow{AP},\overrightarrow{AC}\right)=\frac{\pi}{6}\quad(2\pi)$.	0.5
.4.b	The triangle MNC is equilateral , then $\left(\overrightarrow{CM},\overrightarrow{CN}\right)=\frac{\pi}{3}\quad(2\pi)$. and $CN=\frac{1}{2}CM$. Then , $S(M)=N$.	1
5	$D\in(OB)$. $M\in(EA) then N\in(OB)$ B, N and D are collinear .	1
6.a	$Z_B=4\ et\ Z_C=4+4\frac{\sqrt{3}}{3}i$	0.5
.6.b	$z'=\frac{1}{2}e^{i\frac{\pi}{3}}z+(1-\frac{1}{2}e^{i\frac{\pi}{3}})(4+4\frac{\sqrt{3}}{3}i)$ $z'=\frac{1}{2}e^{i\frac{\pi}{3}}z+4$.	0.5
Q5		
1.a	The focal axis is (FK)	0.5
1.b	$\frac{AF}{AK}=2\quad\frac{A'F}{A'K}=2$ with A and A'on (FK)	1
2.a	O is the midpoint of [AA'] F' symetric of F with respect to O.	0.5

<table>
<tr><td>2.b</td><td>The tangent of the angle formed by (OH) and the focal axis is equal to $\sqrt{3} = \frac{b}{a}$ with a=OA=2 C=OF=4 et $c^2 = a^2 + b^2$
The second asymptote is symetric of (OH) with respect the perpendicular line at O.</td><td>1
0.5</td></tr>
<tr><td>2.c</td><td></td><td>0.5</td></tr>
<tr><td>3.</td><td>$\frac{GF}{d\left(G/_{(HK)}\right)} = 2$ then G is on (H).</td><td>0.5</td></tr>
<tr><td>4.a</td><td>Since a=2 and b = $2\sqrt{3}$,centre O and the focal axe is x’ox.</td><td>0.5</td></tr>
<tr><td>4.b</td><td>G(4 ;6) and K(1,0) and (GK) : y=2x-2 is the tangent at G to (H).</td><td>1</td></tr>
<tr><td>Q.6</td><td></td><td></td></tr>
<tr><td>part A</td><td></td><td></td></tr>
<tr><td>1</td><td>Y=a +bxe $^{-x}$, Y verify (E), then a =1 and b= -2e</td><td>.0.5</td></tr>
<tr><td>.2</td><td>The general solution is : y=ce $^{-x}$ + Y = Ce $^{-x}$ +1 – 2xe $^{-x+1}$
y(1)= 0 then C = e .
Particular solution : y = 1 + (1-2x)e $^{-x+1}$</td><td>1</td></tr>
<tr><td>Part B</td><td></td><td></td></tr>
<tr><td>1</td><td>$\lim\limits_{x\to-\infty} g(x) = 1$ then y = 1 is an horizontal asymptote .</td><td>0.5</td></tr>
<tr><td>2</td><td>$g'(x) = (2x-3)e^{-x+1}$
<table>
<tr><td>x</td><td>0</td><td></td><td>$\frac{3}{2}$</td><td></td><td>$+\infty$</td></tr>
<tr><td>$g'(x)$</td><td></td><td>−</td><td>0</td><td>+</td><td></td></tr>
<tr><td>$g(x)$</td><td>1+e</td><td>↘</td><td>-0.2</td><td>↗</td><td>1</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>3</td><td>g(1) = 0 then 1 is a root .
$g(2.25) = -2.77 \bullet 10^{-3}$</td><td>1</td></tr>
</table>

	$g(2.26)=1.54\bullet 10^{-3}$ g is continuous and strictly decreasing on [2,25 ; 2,26] then α is a root. g(α) = 0 if and only if $e^{\alpha-1} = 2\alpha - 1$.	
4	g(α) $\leq$ 0 if and only if x $\in$[1, α]. $g(x^2) \leq 0$ if and only if $x^2 \in [1,\alpha]$, if and only if x $\in$[1,α].	1
5		1.5
6	$A=\int_1^2\left[y_{(\Delta)}-g(x)\right]dx=\int_1^2\left[-(1-2x)e^{-x+1}\right]dx$ With integration by part : $\int_1^2 -(1-2x)\,e^{-x+1}\,dx = 3-\frac{5}{e}\approx 1.1606$ $A=1{,}16u^2$.	1
partie C		
1	f (-x) + f (x) = 2 , and f is centered at 0. I(0,1) is center of symmetry of the curve .	0.5
2	$\lim_{x\to+\infty} f(x)=+\infty$ et $\lim_{x\to+\infty}[f(x)-y_{(d)}]=0$ $f(x)-y_{(d)} = x\,e^{-x^2+1}$. $x\,e^{-x^2+1} > 0$ if x > 0 then (Γ) is above (d). $x\,e^{-x^2+1} = 0$ if x = 0 then (Γ) intersects (d). $x\,e^{-x^2+1} <$ 0 then (Γ) is below (d).	1
3	$f'(x)=g(x^2)$ $f(\sqrt{\alpha})=1+\frac{2\alpha\sqrt{\alpha}}{2\alpha-1}$.	0.5

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<tr><td>x</td><td>0</td><td></td><td>1</td><td></td><td>$\sqrt{\alpha}$</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td></td><td>+</td><td>0</td><td>−</td><td>0</td><td>+</td><td></td></tr>
<tr><td>$f(x)$</td><td>1</td><td>↗</td><td>3</td><td>↘</td><td>$f(\sqrt{\alpha})$</td><td>↗</td><td>$+\infty$</td></tr>
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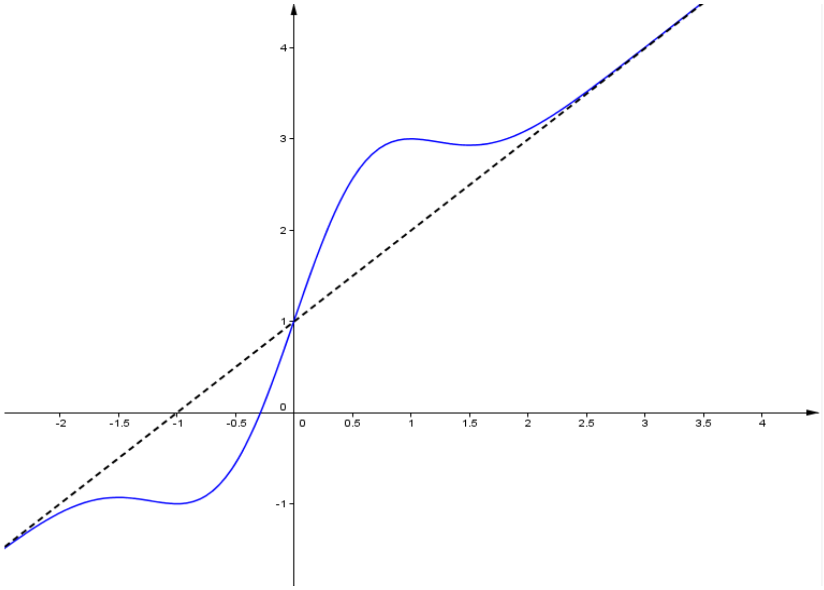

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<td>1.5</td>
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<tr>
<td>6.a</td>
<td>$U_0 = \int_0^1 1dx = 1.$</td>
<td>0.5</td>
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<tr>
<td>6.b</td>
<td>
$$U_n = \int_0^1 (1+nxe^{-(nx)^2+1})dx = 1+\int_0^1 (nxe^{-(nx)^2+1})dx$$

Let v = -(nx)^2 + 1 , then dv = -2n^{2}x dx , nxdx = - $\frac{dv}{2n}$.

$$U_n = 1+\int_1^{-n^2+1} -e^v \frac{dv}{2n} = 1-\frac{1}{2n}\left(e^{-n^2+1}-e\right).$$

$$\lim_{n\to+\infty} U_n = 1$$
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