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## This exam is formed of three exercises in 3 pages numbered from 1 to 3 The use of non-programmable calculators is allowed

## First exercise ( $7 \mathbf{p t s}$ ) Study of a mechanical oscillator

The object of this exercise is to determine the stiffness of the spring of a horizontal mechanical oscillator. This oscillator is formed of a solid ( $\mathrm{S}_{1}$ ) of mass $\mathrm{M}=400 \mathrm{~g}$ and a spring of negligible mass and of stiffness k . The center of mass $G$ of $\left(\mathrm{S}_{1}\right)$ may move on a horizontal straight axis x'Ox. The position of G is defined, at any instant $t$, by its abscissa $x=\overline{\mathrm{OG}}$, O corresponding to the equilibrium position $\mathrm{G}_{0}$ of G (figure).


## A - Setting the oscillator in motion

$\left(S_{1}\right)$ is initially at rest and $G$ is at $O$. To set $\left(S_{1}\right)$ in motion, a solid $\left(S_{2}\right)$, of mass $m=\frac{M}{2}$, is launched towards ( $\mathrm{S}_{1}$ ) along the axis x'Ox . Just before collision, ( $\mathrm{S}_{2}$ ) was moving with the velocity $\overrightarrow{\mathrm{V}}_{2}=\mathrm{V}_{2} \vec{i}$ ( $\mathrm{V}_{2}=0.75 \mathrm{~m} / \mathrm{s}$ ). The collision between $\left(\mathrm{S}_{1}\right)$ and $\left(\mathrm{S}_{2}\right)$ being elastic, $\left(\mathrm{S}_{2}\right)$ rebounds along x'Ox. Just after collision, $\left(\mathrm{S}_{1}\right)$ acquires the velocity $\overrightarrow{\mathrm{V}_{0}}=\mathrm{V}_{0} \vec{i}$.

1) What are the two physical quantities that remain conserved during this collision?
2) Write the equations that express the preceding conservations.
3) Deduce that $V_{0}=0.5 \mathrm{~m} / \mathrm{s}$.

## B- Energetic study of the oscillator

The graphical recordings show that the time equation of motion of G , after collision, may be written in the form:

$$
\mathrm{x}=\mathrm{X}_{\mathrm{m}} \sin \left(\sqrt{\frac{\mathrm{k}}{\mathrm{M}}}\right) \mathrm{t} \quad\left(\mathrm{x} \text { in } \mathrm{m} ; \mathrm{t} \text { in } \mathrm{s} \text { ) where } \mathrm{X}_{\mathrm{m}}\right. \text { is a positive constant. }
$$

The horizontal plane passing through G is taken as a gravitational potential energy reference.

1) a- Write the expression of the elastic potential energy $P E_{e}$ of the oscillator in terms of $k, X_{m}, M$, and $t$.
b- Determine the expression of the kinetic energy KE of the oscillator in terms of $\mathrm{k}, \mathrm{M}, \mathrm{X}_{\mathrm{m}}$, and t .
c- Find the expression of the mechanical energy ME of the system (oscillator, Earth) in terms of k and $\mathrm{X}_{\mathrm{m}}$.
d- Deduce that $\left(\mathrm{S}_{1}\right)$ is not subjected to any force of friction during its motion.
2) a- Determine the value of ME.
b- During the motion of $\left(\mathrm{S}_{1}\right)$, G oscillates between two extreme positions A and $\mathrm{B}, 20 \mathrm{~cm}$ apart.
Determine the value of k .

## Second exercise

## ( $61 / 2 \mathrm{pts}$ ) Charge of a capacitor

The object of this exercise is to determine the capacitance of a capacitor and study the effect of certain physical quantities on the duration of its charging.
The circuit of figure (1) is formed of:

- an ideal generator delivering across its terminals an adjustable DC voltage $\mathrm{u}_{\mathrm{MN}}=\mathrm{u}_{\mathrm{g}}=\mathrm{E}$;
- a resistor of adjustable resistance R ;
- a capacitor of capacitance C ;
- a switch K.

I- The value of E is adjusted at $\mathrm{E}=10 \mathrm{~V}$ and that of $R$ at $R=2 \mathrm{k} \Omega$.
The capacitor being initially neutral, we close the switch at the instant $\mathrm{t}_{0}=0$.


Figure 1


Figure 2

1) a -Derive the differential equation giving the variation of the voltage $u_{D F}=u_{C}$ across the capacitor as a function of time.
b- Verify that the solution of this differential equation is $u_{C}=E\left(1-e^{\frac{-t}{\mathrm{RC}}}\right)$.
2) The voltages $\mathrm{u}_{\mathrm{C}}$ and $\mathrm{u}_{\mathrm{g}}$ are displayed using an oscilloscope (figure 2 ).
a- Redraw the circuit of figure (1) showing the connections of the oscilloscope.
b- Give the maximum value of $u_{C}$.
3) One method to determine the value of $C$ consists of determining the duration $t_{1}$ at the end of which the voltage $u_{C}$ attains $63 \%$ of its maximum value.
a- Show that $t_{1}$ is very close to the value of RC.
b- Using figure (2), determine the value of the capacitance C.
4) Another method allows us to determine $C$ starting from the tangent at $O$ to the curve $u_{C}=f(t)$ (fig.2)
a- Find the expression of $\frac{\mathrm{du}_{\mathrm{C}}}{\mathrm{dt}}$,at O , in terms of $\mathrm{E}, \mathrm{R}$ and C .
b- Show that the equation of this tangent to the curve is $u=\frac{E}{R C} t$.
c- Verify that this tangent intersects the asymptote to the curve at the point of abscissa $t_{1}=R C$.
d- Determine then the value of the capacitance C of the capacitor.
II - The value of R is adjusted at $\mathrm{R}=1 \mathrm{k} \Omega$.
5) Trace, on the same system of axes, the shape of the curve $u_{C}$ in the two following cases :
case (1) : $\mathrm{E}=10 \mathrm{~V}, \mathrm{C}=2 \times 10^{-6} \mathrm{~F}$ (curve 1)
case (2): $\mathrm{E}=5 \mathrm{~V}, \mathrm{C}=2 \times 10^{-6} \mathrm{~F}$ (curve 2)
Scale : on the axis of abscissas:1div $\leftrightarrow 4 \mathrm{~ms}$; on the axis of ordinates : 1 div $\leftrightarrow 1 \mathrm{~V}$.
6) Specify, with justification, which of the two physical quantities $E$ or $R$ affects the duration of charging of the capacitor.

## Third exercise ( $61 / 2 \mathrm{pts}$ ) Interaction radiation-matter

I- At the beginning of the 1880's, Balmer identified, in the emission spectrum of hydrogen, the four visible rays denoted by $\mathrm{H}_{\alpha}, \mathrm{H}_{\beta}, \mathrm{H}_{\gamma}$ and $\mathrm{H}_{\delta}$.
In 1913, Bohr elaborated a theory about the structure of the atom and showed that we can associate , to the hydrogen atom, energy levels given by the formula:
$E_{n}=-\frac{E_{0}}{n^{2}}$ where $E_{0}$ is a positive constant expressed in $e V$ and $n$ is a whole non-zero number .
According to Bohr, each of the rays of Balmer series is characterized by its wavelength $\lambda$ in air and the corresponding downward transition :
$\mathrm{H}_{\alpha}\left(\lambda_{\alpha}=658 \mathrm{~nm}\right.$; transition from $\mathrm{n}=3$ to $\mathrm{n}=2$ );
$\mathrm{H}_{\beta}\left(\lambda_{\beta}=487 \mathrm{~nm}\right.$; transition from $\mathrm{n}=4$ to $\left.\mathrm{n}=2\right)$;
$\mathrm{H}_{\gamma}\left(\lambda_{\gamma}=435 \mathrm{~nm}\right.$; transition from $\mathrm{n}=5$ to $\left.\mathrm{n}=2\right)$;
$\mathrm{H}_{\delta}\left(\lambda_{\delta}=412 \mathrm{~nm}\right.$; transition from $\mathrm{n}=6$ to $\mathrm{n}=2$ ).
The corresponding energy diagram of this series is represented in the adjacent figure.

1) Determine, using the diagram, the value of $E_{0}$ in $e V$.
2) a) Emission Spectrum
i) Show, starting from the diagram, that the ray $\mathrm{H}_{\beta}$ corresponds to the emission of a photon of energy 2.55 eV .

ii) Verify that the value of the wavelength of the ray $\mathrm{H}_{\beta}$ is around 487 nm
b) Absorption spectrum

In order to obtain the absorption spectrum of the hydrogen atom, we illuminate hydrogen gas with white light. What do we observe in the absorption spectrum?
3) The hydrogen atom, being in its first excited state ( $n=2$ ), collides with a photon of energy 2.26 eV . This photon is not absorbed. Why?

II-A hydrogen lamp illuminates now a photosensitive metallic surface of threshold wavelength $\lambda_{0}=500 \mathrm{~nm}$.

1) What are the visible radiations that may provoke photoelectric emission? Why?
2) a) Determine the radiation that is able to extract an electron having the highest possible kinetic energy KE.
b) Calculate then KE .

III- The atomic line spectra and the phenomenon of photoelectric effect show evidence of a characteristic concerning the energy of an electromagnetic wave and the energetic exchange between matter and electromagnetic waves. Specify this characteristic.

## Given:

- speed of light in vacuum c $=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$;
- Planck's constant $\mathrm{h}=6.63 \times 10^{-34} \mathrm{~J} . \mathrm{s}$;
- $\quad 1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$;
- $\quad 1 \mathrm{~nm}=10^{-9} \mathrm{~m}$.


## First Exercise (7 pts)

A- 1) The linear momentum and the kinetic energy of the system ( $S_{1}, S_{2}$ ) are conserved ( $\mathbf{1} / \mathbf{2} \mathbf{~ p t}$ )

$$
\text { 2) } \begin{align*}
& m \vec{V}_{2}=m \vec{V}_{3}+M{\overrightarrow{V_{0}}}_{(\mathbf{1} / 2 p t)} \\
& \frac{1}{2} m V_{2}^{2}=\frac{1}{2} m V_{3}^{2}+\frac{1}{2} M V_{0}^{2} \quad \text { equ .. }
\end{align*}
$$

3) The velocities are collinear, the vector expression can be written algebraically: $m V_{2}=m V_{3}+M V_{0}$ or $m\left(V_{2}-V_{3}\right)=M V_{0}$ equ........(2).
The equation (1) can be written as: $m\left(V_{2}^{2}-V_{3}^{2}\right)=M V_{0}{ }^{2}$ equ........(3)
The system of the two equations (2) and (3) give : $\mathrm{V}_{0}=\frac{2 m}{m+M} \mathrm{~V}_{2}=\frac{2}{3} \times 0.75=0.5 \mathrm{~m} / \mathrm{s} ;\left(\mathbf{1}^{1 / 2} \mathbf{~ p t}\right)$
B- 1) a-P.E $E_{e}=\frac{1}{2} k x^{2}=\frac{1}{2} k x_{m}^{2} \sin ^{2}\left(\sqrt{\frac{k}{M}} \mathrm{t}\right) .(\mathbf{1 / 2} \mathbf{~ p t})$
b- $\mathrm{K} . \mathrm{E}=\frac{1}{2} \mathrm{MV}^{2}$; with $\mathrm{V}=\mathrm{x}^{\prime}=\mathrm{X}_{\mathrm{m}} \sqrt{\frac{k}{M}} \cos \sqrt{\frac{k}{M}} \mathrm{t}$, we can write:

$$
\text { K.E }=\frac{1}{2} \mathrm{MX}_{m}^{2} \frac{k}{M} \cos ^{2}\left(\sqrt{\frac{k}{M}} \mathrm{t}\right)=\frac{1}{2} \mathrm{X}_{m}^{2} \mathrm{k} \cos ^{2}\left(\sqrt{\frac{k}{M}} \mathrm{t}\right)(\mathbf{3} / 4 \mathrm{pt})
$$

c- M.E $=$ P.E $E_{e}+K . E=\frac{1}{2} k x_{m}^{2} \sin ^{2}\left(\sqrt{\frac{k}{M}} t\right)+\frac{1}{2} X_{m}^{2} k \cos ^{2}\left(\sqrt{\frac{k}{M}}\right.$ t)

$$
\text { M.E }=\frac{1}{2} \mathrm{kX}_{m}^{2}\left(\sin ^{2} \sqrt{\frac{k}{M}} \mathrm{t}+\cos ^{2} \sqrt{\frac{k}{M}} \mathrm{t}\right)=\frac{1}{2} \mathrm{kX}_{m}^{2} .
$$

d- k and $\mathrm{X}_{\mathrm{m}}$ are constants $=>$ M.E = constant $=>$ the mechanical energy is conserved at any time; Therefore the motion takes place without friction.
(1/2pt)
2) $\mathrm{a}-\mathrm{M} \cdot \mathrm{E}(\mathrm{t}=0)=\mathrm{M} \cdot E(\mathrm{t})=>\frac{1}{2} \mathrm{MV}_{0}{ }^{2}=\frac{1}{2} \mathrm{kX}_{m}^{2}=\frac{1}{2}(0.4)(0.5)^{2}=0.05 \mathrm{~J}$.
(1/2pt)
b- $\mathrm{AB}=2 \mathrm{X}_{\mathrm{m}} \Rightarrow \mathrm{X}_{\mathrm{m}}=0.1 \mathrm{~m}, \mathrm{k}=\frac{2 E_{m}}{\left(X_{m}\right)^{2}}=\frac{2 \times 0.05}{0.01}=10 \mathrm{~N} / \mathrm{m}$. ( or $\mathrm{k}=\frac{M V_{0}^{2}}{X_{m}^{2}}$ )
(1pt)

## Second Exercise ( $61 / 2 \mathrm{pts}$ )

$\mathbf{I}-1)$ a- $\mathrm{E}=\mathrm{Ri}+\mathrm{u}_{\mathrm{C}}=\mathrm{RC} \frac{d u_{C}}{d t}+\mathrm{u}_{\mathrm{C}}$
(1pt)
b- $\frac{d u_{C}}{d t}=\frac{E}{R C} e^{\frac{-t}{R C}}$,
then : $\mathrm{E}=\mathrm{RC} \frac{E}{R C} e^{\frac{-t}{R C}}+\mathrm{E}\left(1-e^{\frac{-t}{R C}}\right)=\mathrm{E}(\mathbf{1} / 2 \mathbf{p t})$
2) a-Connection
(1/4pt)
b- $\left(u_{C}\right)_{\max }=E=10 \mathrm{~V}$
(1/4pt)

3) a) $\mathrm{u}_{\mathrm{C}}=0.63 \mathrm{E}=\mathrm{E}\left(1-e^{\frac{-t_{1}}{R C}}\right) \Rightarrow e^{\frac{-t_{1}}{R C}}=0.37 \Rightarrow \ln 0.37=-\frac{t_{1}}{R C}=>\mathrm{t}_{1}=R C .(\mathbf{1} / \mathbf{2 p t})$
b) From the graph of figure (2), we can find that the voltage $u_{C}=0.63 \times 10=6.3 \mathrm{~V}$ and this value is attained after a time of 4 ms ; this time equal RC. Then $\mathrm{C}=2 \times 10^{-6} \mathrm{~F}$. (1/2pt)
4) а) $\left.\frac{d u_{C}}{d t}=\frac{E}{R C} e^{\frac{-t}{R C}} \Rightarrow \frac{d u_{C}}{d t}\right)_{t=0}=\frac{E}{R C}(\mathbf{1 / 2 p t})$
b) The equation of the tangent to the curve through the origin is : $\mathrm{u}=\mathrm{a}_{0} \mathrm{t}=\frac{E}{R C} \mathrm{t} \quad(\mathbf{1} / \mathbf{2 p t})$.
c) For $\mathrm{u}=\mathrm{E}$, we have : $\mathrm{E}=\frac{E}{R C} \mathrm{t}_{1}$, then $\mathrm{t}_{1}=R C$. $(\mathbf{1} / \mathbf{2} \mathbf{~ p t})$
d) The abscissa of the point of intersection of the tangent with the asymptote is $\mathrm{RC}=4 \mathrm{~ms}$. ( $\mathbf{1} / \mathbf{2} \mathbf{~ p t )}$

$$
\text { Thus : } \mathrm{C}=\frac{4 \times 10^{-3}}{2000}=2 \times 10^{-6} \mathrm{~F} .(1 / 2 \mathbf{p t})
$$

II - 1) Trace ( $\mathbf{1 / 2} \mathbf{~ p t )}$

2) The duration of charging is $t=5 R C \Rightarrow t$ depends on $R$ and not on $E$. (1/2 pt)

## Third exercise ( 6 ½ pts)

$\mathbf{I} \mathbf{- 1}$ - For $\mathrm{n}=2, \mathrm{E}_{2}=-3.4 \mathrm{eV}$; We can write : $-3.4=-\frac{E_{0}}{4} \Rightarrow \mathrm{E}_{0}=13.6 \mathrm{eV} .(\mathbf{1} / \mathbf{2} \mathbf{~ p t})$
2-a-i) E (photon) $=\mathrm{E}_{4}-\mathrm{E}_{2}=-0.85-(-3.4)=2.55 \mathrm{eV} \quad(\mathbf{1} / \mathbf{4} \mathbf{~ p t})$
ii) $\mathrm{E}($ photon $)=h v=\frac{h c}{\lambda}=>\lambda=\frac{h c}{E(\text { photon })}=\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2.55 \times 1.6 \times 10^{-19}}=4875 \mathrm{~nm} . \quad$ (3/4pt)
b) We observe 4 black lines. $\qquad$
3) To absorb the photon we must have : E (photon) $=\mathrm{E}_{\mathrm{n}}-\mathrm{E}_{2}$
$2.26=-\frac{13.6}{n^{2}}-(-3.4)=>n=3.45 ; n$ is not a whole number $\Rightarrow>$ the photon is not absorbed $\qquad$

Or : To pass from level $\mathrm{n}=2$ to level $\mathrm{n}=3$, the atom absorbs a photon of energy $\mathrm{E}_{2 \ldots-3}=-1.51-(-3.4)=1.89 \mathrm{eV}$ To pass from level $n=2$ to level $n=4$, the atom absorbs a photon of energy $E_{2 \ldots>4}=-0.85-(-3.4)=2.55 \mathrm{eV}$. $1.89<2.26<2.55=>$ the atom does not absorb the photon.

II-1) The photo electric emission takes place when the wavelength of the incident radiation is less or equal to the threshold wavelength of the metal.
We have $\lambda(\alpha)>500 \mathrm{~nm}=>$ therefore, no photo electric emission for this radiation.
The other radiations of wavelengths $<500 \mathrm{~nm}=>$ allows the photo electric emission to take place........(1pt)
2- a) The energy of the photon is inversely proportional to the wavelength and the relation of Einstein gives :
$E($ photon $)=W_{0}+K . E=>K . E$ increases with the energy of the photon. Thus the required radiation is the one of the shortest wavelength ( $H_{\delta}$ ) extracts the most energetic electron. (1pt)
b) $\mathrm{K} . \mathrm{E}=\mathrm{E}($ photon $)-\frac{\mathrm{hc}}{\lambda_{0}}=\frac{h c}{\lambda}-\frac{\mathrm{hc}}{\lambda_{0}}=\mathrm{hc}\left(\frac{1}{\lambda}-\frac{1}{\lambda_{0}}\right) \Rightarrow \mathrm{KE}=8.5 \times 10^{-20} \mathrm{~J}$.

III- The exchange of energy is quantized ; the energy of the electromagnetic wave is quantized.

