وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٥ الاستثنائية

عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظـة : يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\ \vec{u},\ \vec{v})$, consider the points E and F of affixes $z_E = \frac{\sqrt{3}+1}{4} - \frac{\sqrt{3}-1}{4}i$ and $z_F = \frac{1}{2} + \frac{1}{2}i$.

1) a- Calculate $(z_E)^2$ and find the modulus and an argument of $(z_E)^2$.

b- Determine the modulus of z_E and verify that $-\frac{\pi}{12}$ is an argument of z_E.

c- Deduce the exact values of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.

2) Let $Z = \frac{z_E}{z_F}$.

a- Write z_E, z_F and Z in the exponential form.

b- Show that the triangle OEF is equilateral.

II- (4 points)

In the space referred to a direct orthonormal system $(O;\ \vec{i},\ \vec{j},\ \vec{k})$, consider the points A (4 ; 0 ; 0) , B (0 ; 4 ; 0) and C (0 ; 0 ; 4).

1) Write an equation of plane (ABC).

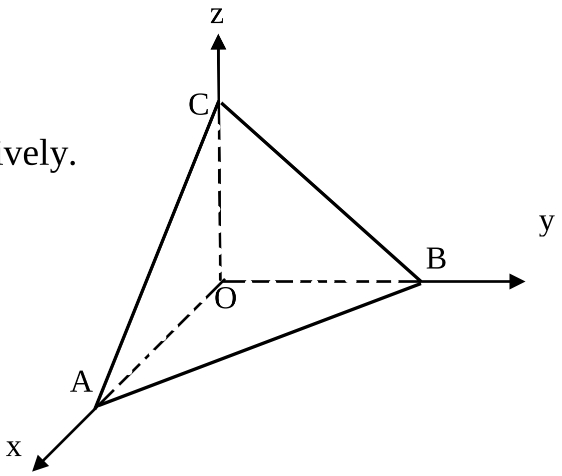
2) Calculate the area of triangle ABC.

3) Let F and G be the midpoints of [AC] and [BC] respectively.

a- Give a system of parametric equations of the straight line (FG).

b- The plane of equation z = 0 intersects the plane (OFG) along a line (d). Prove that the lines (d) and (FG) are parallel to each other.

c- Calculate the distance between the two lines (d) and (AB) .



III- (4 points)

The 80 students of the third secondary classes in a certain school are distributed into the three sections GS , LS and SE as shown in the following table :

	GS	LS	SE
Girls	8	18	10
Boys	12	14	18

The school director chooses randomly a group of 3 students, from the third secondary classes, to participate in a TV program.

1) What is the number of possible groups?
2) Designate by X the random variable that is equal to the number of boys in the chosen group. Determine the probability distribution of X .
3) Show that the probability that the chosen group contains one girl from each section is $\frac{18}{1027}$.
4) The chosen group is made up of 3 girls, What is the probability that they are from the same section ?

IV- (8 points)

Let f be the function that is defined on IR by : $f(x) = x + 2 - e^{-x}$, and (C) be its representative curve in an orthonormal system $(O ; \vec{i} , \vec{j})$.

1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x + 2$ is an asymptote of (C).

   b- Calculate $\lim\limits_{x \to -\infty} f(x)$ and give, in the decimal form, the values of f(–1.5) and f(–2).
2) Calculate f '(x) and set up the table of variations of f.
3) Write an equation of the line (T) that is tangent to (C) at the point A of abscissa 0.
4) Show that the equation f(x) = 0 has a unique root α and verify that $-0.5 < \alpha < -0.4$.
5) Draw (d), (T) and (C) .
6) Designate by g the inverse function of f, on IR.

   a- Draw, in the system $(O ; \vec{i} , \vec{j})$, the curve (G) that represents g.

   b- Designate by $A(\alpha)$ the area of the region that is bounded by the curve (C) , the axis of abscissas and the two lines of equations $x = \alpha$ and $x = 0$.

   Show that $A(\alpha) = (-\frac{\alpha^2}{2} - 3\alpha - 1)$ units of area.

   c- Deduce the area of the region that is bounded by the curve (G), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

Q1	Short Answers	M
1.a	$z_E^2 = \frac{1}{16}(3+1+2\sqrt{3}-3-1+2\sqrt{3}-4i) = \frac{1}{16}(4\sqrt{3}-4i) = \frac{\sqrt{3}}{4}-\frac{1}{4}i$ $z_E^2 = \frac{1}{2}(\frac{\sqrt{3}}{2}-\frac{1}{2}i) = \frac{1}{2}e^{-i\frac{\pi}{6}}$; $\left\|{z_E}^2\right\| = \frac{1}{2}$; $\arg({z_E}^2) = -\frac{\pi}{6}$.	1
1.b	$\left\|{z_E}^2\right\| = \frac{1}{2}$ then $\left\|z_E\right\| = \frac{1}{\sqrt{2}}$. $\arg({z_E}^2) = 2\arg(z_E) = -\frac{\pi}{6}+2k\pi$; $\arg(z_E) = -\frac{\pi}{12}+k\pi$, since $R_e(z_E) > 0$ and $Im(z_E)<0$, therefore $\arg(z_E) = -\frac{\pi}{12}$.	1
1.c	$z_E = \frac{1}{\sqrt{2}}[\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12})] = \frac{1}{\sqrt{2}}[\cos(\frac{\pi}{12})-i\sin(\frac{\pi}{12})] = \frac{\sqrt{3}+1}{4}-\frac{\sqrt{3}-1}{4}i$ $\cos(\frac{\pi}{12}) = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $\sin(\frac{\pi}{12}) = \frac{\sqrt{6}-\sqrt{2}}{4}$.	½
2.a	$z_E = \frac{1}{\sqrt{2}}e^{-i\frac{\pi}{12}}$, $z_F = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}}$, $Z = e^{i(-\frac{\pi}{12}-\frac{\pi}{4})} = e^{-i\frac{\pi}{3}}$	½
2.b	$\|Z\| = 1 = \frac{OE}{OF}$; OE = OF $\arg(Z) = \arg(z_E) - \arg(z_F) = (\vec{u},\vec{OE}) - (\vec{u},\vec{OF})\ [2\pi] = (\vec{OF},\vec{OE})\ [2\pi] = -\frac{\pi}{3}[2\pi]$. OEF is equilateral. • OR : $EF = \|z_F - z_E\| = \frac{1}{\sqrt{2}} = OE = OF$	1

Q2	Short Answers	M
1	$\vec{AM}.(\vec{AB}\wedge\vec{AC}) = 0$; $\begin{vmatrix} x-4 & y & z \\ -4 & 4 & 0 \\ -4 & 0 & 4 \end{vmatrix} = 0$; $x + y + z - 4 = 0$	1
2	$\text{Area}(ABC) = \frac{1}{2}\\|\vec{AB}\wedge\vec{AC}\\| = 2\sqrt{3}\ u^2$	½
3.a	F(2 ;0 ;2) and G(0 ; 2 ; 0) , eq. of (FG) : x = – 2t , y = 2t + 2 , z = 2.	½
3.b	The plane of eq. z = 0 is the plane (AOB) ; (FG) // (AB) then (FG) // (OAB), since (d) is the line of intersection of (OFG) and (OAB) , hence (FG) //(d). • OR :the equation of (OFG) is : x + y – z = 0 . (d) : x =m , y = – m , z = 0. $\vec{V}_d(1;-1;0)$, $\vec{FG}(-2;2;0)$ then $\vec{FG} = -2\vec{V}_d$ and the lines (d) and (FG) are distinct ; Therefore they are parallel.	1
3.c	The distance between (d) and (AB) is the distance from O to (AB) since (d) passes through O , and (d) //(AB), then $d = \frac{\\|\vec{OA}\wedge\vec{OB}\\|}{\\|\vec{AB}\\|} = 2\sqrt{2}$ u.	1

<table>
<tr><th>Q3</th><th>Short Answers</th><th>M</th></tr>
<tr><td>1</td><td>Number of possible cases : $C_{80}^{3} = 82\,160$.</td><td>½</td></tr>
<tr><td>2</td><td><table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>p_i</td><td>$\frac{C_{36}^{3}}{C_{80}^{3}} = \frac{7140}{82160}$</td><td>$\frac{C_{36}^{2} \times C_{44}^{1}}{C_{80}^{3}} = \frac{27720}{82160}$</td><td>$\frac{C_{36}^{1} \times C_{44}^{2}}{C_{80}^{3}} = \frac{34056}{82160}$</td><td>$\frac{C_{44}^{3}}{C_{80}^{3}} = \frac{13244}{82160}$</td></tr></table></td><td>1½</td></tr>
<tr><td>3</td><td>$\frac{C_{8}^{1} \times C_{18}^{1} \times C_{10}^{1}}{C_{80}^{3}} = \frac{18}{1027}$.</td><td>1</td></tr>
<tr><td>4</td><td>p(the girls are from the same section / 3 girls) = $\frac{C_{8}^{3} + C_{18}^{3} + C_{10}^{3}}{C_{36}^{3}} = \frac{248}{1785} = 0.138$</td><td>1</td></tr>
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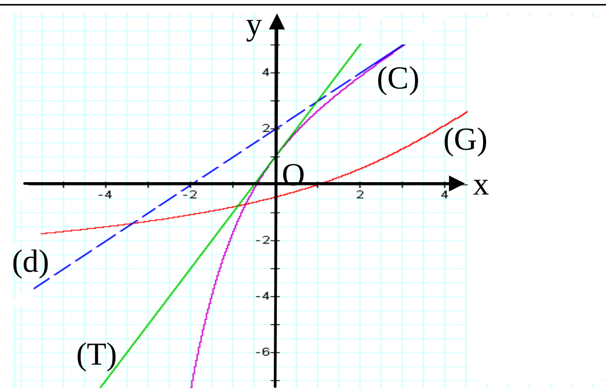
<table>
<tr><th>Q4</th><th>Short Answers</th><th>M</th></tr>
<tr><td>1.a</td><td>$\lim\limits_{x\to+\infty} f(x) = +\infty - 0 = +\infty$; $\lim\limits_{x\to+\infty} [f(x) - (x+2)] = \lim\limits_{x\to+\infty} (-e^{-x}) = 0$ then the line (d) of equation $y = x + 2$ is an asymptote of (C) .</td><td>1</td></tr>
<tr><td>1.b</td><td>$\lim\limits_{x\to-\infty} f(x) = -\infty - \infty = -\infty$; $f(-1.5) = -3.981$; $f(-2) = -7.389$.</td><td>1</td></tr>
<tr><td>2</td><td>$f'(x) = 1 + e^{-x}$
<table><tr><td>x</td><td>$-\infty$</td><td>$+\infty$</td></tr><tr><td>f '(x)</td><td colspan="2">+</td></tr><tr><td>f (x)</td><td>$-\infty$</td><td>↗ $+\infty$</td></tr></table></td><td>1</td></tr>
<tr><td>3</td><td>(T) : $y = f'(0)x + f(0)$; $y = 2x + 1$.</td><td>½</td></tr>
<tr><td>4</td><td>f is continuous, strictly increasing on IR and varies from $-\infty$ to $+\infty$, then the equation f(x) = 0 has a unique solution α.
$f(-0.5) \times f(-0.4) = -0.148 \times 0.1081 < 0$ then $-0.5 < \alpha < -0.4$.</td><td>1</td></tr>
<tr><td>5</td><td>
</td><td>1½</td></tr>
<tr><td>6.a</td><td>See the figure.</td><td>½</td></tr>
<tr><td>6.b</td><td>$A(\alpha) = \int_{\alpha}^{0} f(x)\,dx = \int_{\alpha}^{0} (x + 2 - e^{-x})\,dx = \left[\frac{x^2}{2} + 2x + e^{-x}\right]_{\alpha}^{0} = 1 - \frac{\alpha^2}{2} - 2\alpha - e^{-\alpha}$
But $f(\alpha) = 0$ i.e. $\alpha + 2 - e^{-\alpha} = 0$, therefore $e^{-\alpha} = \alpha + 2$ and $A(\alpha) = (-1 - 3\alpha - \frac{\alpha^2}{2})\,.\,u^2$</td><td>1</td></tr>
<tr><td>6.c</td><td>The region bounded by the curve (G) , the axis of abscissas and the two lines of equations x = 0 and x = 1 , is symmetric of the preceding region with respect of the line of equation y = x , therefore the required area is equal to A(α).</td><td>½</td></tr>
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