| وزارة التربية والتعليم العالي<br>المديرية العامة للتربية<br>دائرة الامتحانات | امتحانات الشهادة الثانوية العامة<br>فرع العلوم العامة | دورة سنة ٢٠٠٥ الاستثنائية |
|---|---|---|
| عدد المسائل: ستة | مسابقة في مادة الرياضيات<br>المدة: أربع ساعات | الاسم:<br>الرقم: |

**ملاحظة:** يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

## I- ( 1.5 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of each question and give, **with justification**, the answer corresponding to it.

| Nº | Questions | Answers a | b | c | d |
|---|---|---|---|---|---|
| 1 | The particular solution of the differential equation $y' - \frac{1}{2}y = 0$, that verifies $y(-2) = 1$, is: | $y = -2e^{\frac{x}{2}}$ | $y = e^{\frac{x}{2}+1}$ | $y = 2\cos x - \sin x$ | $y = \sqrt{x^2 - 3}$ |
| 2 | $f(x) = 2\sin(\pi x + 2)$. The period of $f$ is: $T =$ | $\pi$ | 2 | $2\pi$ | $\frac{\pi}{2}$ |
| 3 | The equation $2\ln x = \ln(2x)$ has : | 2 roots | One root only | No roots | 3 roots |
| 4 | If $f(x) = \ln\lvert -3x \rvert$; then $f'(x) =$ | $\frac{3}{x}$ | $-\frac{3}{x}$ | $\frac{1}{\lvert x \rvert}$ | $\frac{1}{x}$ |
| 5 | $e^{\frac{1}{2}\ln 9} \times e^{-\ln\frac{1}{3}} =$ | $e^3$ | 6 | $e^{\frac{3}{2}}$ | 9 |
| 6 | $\cos^2(\frac{1}{2}\arccos x) =$ | $\frac{1+x}{2}$ | $1+\frac{x}{2}$ | $\frac{1}{2}x$ | $(1+x)^2$ |

## II- ( 2.5 points)

In the space referred to a direct orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ ,\ \vec{k})$, consider the point A(2 ;− 2 ; 0) , the plane (P) of equation $x + y - 2z + 2 = 0$ and the line (d) defined by :

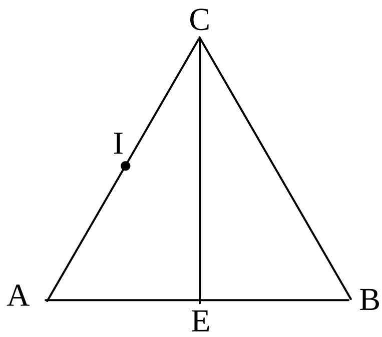
$$\begin{cases} x = t+1 \\ y = -2\,t \\ z = -t+1 \end{cases} \qquad \text{( t is a real parameter).}$$

Designate by H the orthogonal projection of the point A on the plane (P).

1) a- Determine the coordinates of B, the point of intersection of the line (d) with the plane (P).
   b- Verify that A is a point on (d).
   c- Write an equation of the plane (Q) that contains the line (d) and is perpendicular to the plane (P), and deduce a system of parametric equations of the line (BH).
   d- Calculate the distance from A to (P).

2) a- Calculate the sine of the angle $\widehat{ABH}$.
   b- Calculate the area of triangle ABH.

## III- (3 points)




Given, in an oriented plane, a direct equilateral triangle ABC of side 4 cm .
Designate by E and I the mid points of [AB] and [AC] respectively.
Let S be the direct plane similitude that transforms A onto E and E onto C.

1) a- Determine the ratio and an angle of S.
   b- Construct the image under S of each of the straight lines (AC) and (EI), and deduce the image of I under S.

2) Suppose that the plane is referred to a direct orthonormal system $(A\ ;\ \vec{u}\ ,\ \vec{v})$ where $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.
   a- Give the complex form of S.
   b- Find the affix of the point W, the center of S.
   c- Prove that W is a point on [AC] .
   d- Let J be the image of the point I under $S \circ S$; Compare WC and WJ.

**IV- (3 points)**

A purse contains exactly :

4 bills of 10 000 LL,

2 bills of 50 000 LL,

and 3 bills of 100 000 LL.

A) **3 bills** are drawn, **simultaneously** and randomly, from this purse.

1) What is the probability of the event E :
“Drawing three bills of 100 000 LL.”?

2) What is the probability of the event F :
“Drawing two bills of 10 000 LL and one bill of 50 000 LL.”?

B) In order to settle a purchase of 100 000 LL, we draw randomly bills from this purse, **one bill after the other** without replacement, just till we obtain a sum that is equal to 100 000 LL or more.
Let X be the random variable that is equal to the number of bills thus needed to be drawn from this purse.

1) a- Calculate the following probabilities : p(X = 1) and p(X = 2).
b- Justify that the maximal value of X is 6.

2) What is the probability of having to draw at least three bills in order to be able to settle the purchase of 100 000 LL?

**V- (3 points)**

In the complex plane referred to a direct orthonormal system $(O\,;\,\vec{u}\,,\vec{v})$, to every point M of affix z associate the point M' of affix z' such that :

$$z' = f(z) = z^2 - (3 - i)z + 4 - 3i \ .$$

Let $z = x + iy$ and $z' = x' + iy'$ .

1) Determine the points M such that $f(z) = 0$.
2) Calculate x' and y' , in terms of x and y.
3) a- Prove that when M' moves on the axis of ordinates then the point M moves on the curve (C) of equation $x^2 - y^2 - 3x - y + 4 = 0$.
b- Determine the nature of (C) and specify its center I .
c- Determine the vertices, the foci, the asymptotes and the directrices of (C).
d- Draw the curve (C) .
e- Write an equation of the tangent (T), and an equation of the normal (N), to the curve (C) at the point E( 2 ; 1) .

**VI- (7points)**

Let f be the function that is defined, on $\left]\frac{1}{e};+\infty\right[$, by $f(x)=\frac{x}{1+\ln x}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$; *(unit : 2 cm).*

**A-** 1) Calculate $\lim\limits_{x\to\frac{1}{e}} f(x)$ , $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty}\frac{f(x)}{x}$.

2) Calculate f '(x) and set up the table of variations of f.

3) a- Prove that the curve (C) has a point of inflection W of abscissa e.
b- Write an equation of the line (d) that is tangent to (C) at the point W.

4) Study, according to the values of x, the relative position of (C) with respect to the line (D) of equation y = x .
5) Draw (d), (D) and (C).

**B-** Consider the interval I = [ 1 ; e ] .

1) a- Prove that f(I) is included in I.
b- Study the sign of $f'(x)-\frac{1}{4}$ , and deduce that for every x in I we have $0\le f'(x)\le\frac{1}{4}$.
c- Prove that, for every x in I, we have: $|f(x)-1|\le\frac{1}{4}|x-1|$.

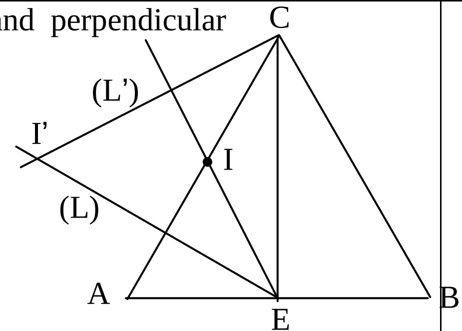
2) Consider the sequence $(U_n)$ that is defined by :
$U_0=2$ and for every $n\ge 0$ , $U_{n+1}=f(U_n)$ .
a- Prove by mathematical induction on n that $U_n$ belongs to I.
b- Prove that $|U_{n+1}-1|\le\frac{1}{4}|U_n-1|$.
c- Prove that $|U_n-1|\le\frac{1}{4^n}$ , and deduce the limit of $U_n$ as n tends to $+\infty$.

# G.S-MATHS

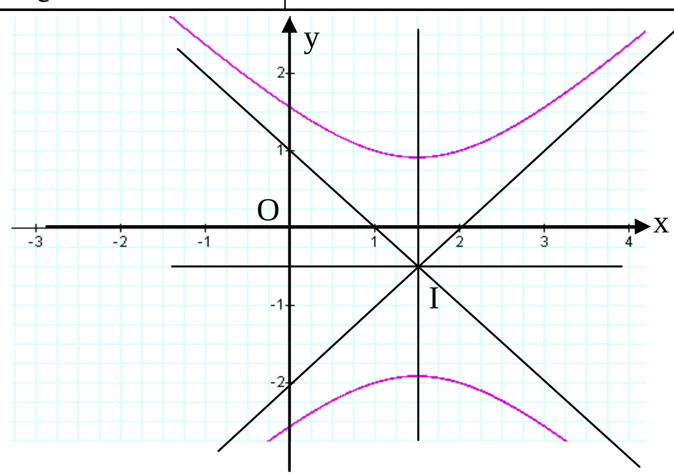
**$2^{nd}$ session 2005**

| QI | Short answers | | M |
|---|---|---|---|
| 1 | $y' - \frac{1}{2}y = 0$ ; $y = Ce^{\frac{x}{2}}$, $y(-2) = Ce^{-1} = 1$ ; $C = e$ ; $y = e^{\frac{x}{2}+1}$ | b | 3 |
| 2 | $T = \frac{2\pi}{\pi} = 2$ | b | |
| 3 | $2\ln x = \ln 2x$ ; $2\ln x = \ln 2 + \ln x$ ; $\ln x = \ln 2$ ; $x = 2$ | b | |
| 4 | $f'(x) = \frac{-3}{-3x} = \frac{1}{x}$ | d | |
| 5 | $e^{\ln\sqrt{9}} \times e^{\ln 3} = 3 \times 3 = 9$ | d | |
| 6 | $\cos^2(\frac{1}{2}\arccos x) = \frac{1+\cos(\arccos x)}{2} = \frac{1+x}{2}$. (or we take x = 0) | c | |

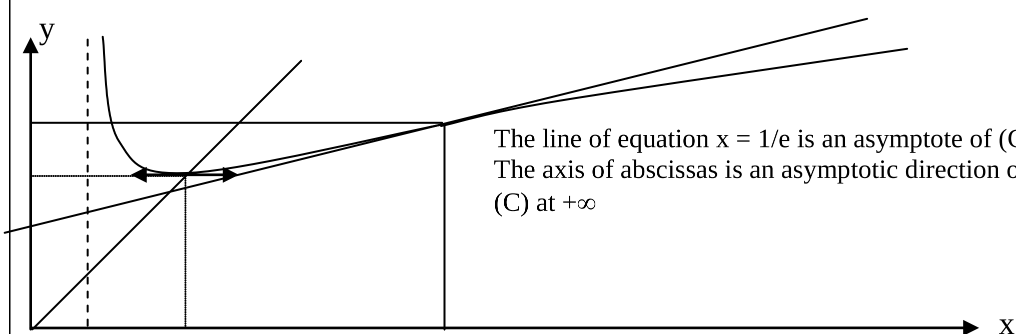
| QII | Short answers | M |
|---|---|---|
| 1.a | $t + 1 - 2t + 2t - 2 + 2 = 0$ ; $t = -1$ ; B(0 ; 2 ;2). | ½ |
| 1.b | For t = 1 ; A(2 ; – 2 ; 0) is a point of (d). | ½ |
| 1.c | An equation of (Q) is given by : $\vec{AM}.(\vec{N}_P \wedge \vec{V}_d) = 0$ ; $\begin{vmatrix} x-2 & y+2 & z \\ 1 & 1 & -2 \\ 1 & -2 & -1 \end{vmatrix} = 0$ ;<br>$5x + y + 3z - 8 = 0$.<br>(BH) = (P)$\cap$(Q) ; $\begin{cases} x+y-2z+2=0 \\ 5x+y+3z-8=0 \end{cases}$ ; $\begin{cases} x = \frac{-5}{4}m + \frac{5}{2} \\ y = \frac{13m}{4} - \frac{9}{2} \\ z = m \end{cases}$ | 1 ½ |
| 1.d | $d(A ;(P)) = \frac{\lvert 2-2+2 \rvert}{\sqrt{1+1+4}} = \frac{2}{\sqrt{6}}$ | ½ |
| 2.a | $\sin A\hat{B}H = \frac{AH}{AB} = \frac{2/\sqrt{6}}{\sqrt{24}} = \frac{1}{6}$ | 1 |
| 2.b | $Area(ABH) = \frac{1}{2} AB \times AH \sin B\hat{A}H = \frac{1}{2} AB \times AH \cos A\hat{B}H$<br>$\cos A\hat{B}H = \sqrt{1 - \frac{1}{36}} = \frac{\sqrt{35}}{6}$<br>$Area(ABH) = \frac{1}{2} \times 2\sqrt{6} \times \frac{2}{\sqrt{6}} \times \frac{\sqrt{35}}{6} = \frac{\sqrt{35}}{3}$ $u^2$<br>•or : Area (ABH) = $\frac{1}{2} \lVert \vec{AB} \wedge \vec{AH} \rVert$ , and finding the coordinates of H. | 1 |

| QIII | Short answers | N |
|---|---|---|
| 1.a | S(A) = E and S(E) = C , k = $\frac{EC}{AE}=\frac{(4\sqrt{3})/2}{4/2}=\sqrt{3}$ and $\alpha = (\overrightarrow{AE},\overrightarrow{EC}) = \frac{\pi}{2}$ | 1 |
| 1.b | The image of (AC) is the line (L) passing through E and perpendicular to (AC).<br>The image of (EI) is the line (L') passing through C and perpendicular to (EI).<br>The image of I will be the point of intersection of the two lines (L) and (L').<br><br> | 2 |
| 2.a | $z_A = 0$ and $z_E = 2$<br>$z' = i\sqrt{3}z + b$ with S(A) = E ; b = 2 hence z' = = $i\sqrt{3}z+2$<br>• or : $z_C = 2 + 2i\sqrt{3}$ ; z' = az + b with $\begin{cases} 2 = 0 + b \\ 2+2i\sqrt{3} = 2a + b \end{cases}$, b = 2 and a = $i\sqrt{3}$ | 1 |
| 2.b | $z_W = i\sqrt{3}\ z_W + 2$ ; $z_W = \frac{1}{2} + i\frac{\sqrt{3}}{2}$. | ½ |
| 2.c | $\frac{z_W - z_A}{z_C - z_A} = \frac{1}{4}$.<br>$\overrightarrow{AC} = 4\overrightarrow{AW}$ then W is a point of [AC] and W is the mid point of [AI]. | ½ |
| 2.d | S o S is the plane similitude of center W , of angle $\pi$ and of ratio 3, hence it is the homothetecy of center W and ratio – 3 .<br>$\overrightarrow{WJ} = -3\overrightarrow{WI} = 3\overrightarrow{WA}$ ; $\overrightarrow{WC} = -3\overrightarrow{WA}$ ; WJ = WC. | 1 |

| QIV | Short answers | |
|---|---|---|
| A.1 | P(E) = $\frac{C_3^3}{C_9^3} = \frac{1}{84}$ | 1 |
| A.2 | P(F) = $\frac{C_4^2 \times C_2^1}{C_9^3} = \frac{12}{84} = \frac{1}{7}$ | 1 |
| B.1.a | P(X = 1) = P(drawing one bill of 100 000) = $\frac{3}{9}$<br>P(X = 2) = P(10 000 , 100 000) + P( 50 000 , 100 000) + P(50 000 , 50 000)<br>$= \frac{4}{9}\times\frac{3}{8}+\frac{2}{9}\times\frac{1}{8}+\frac{2}{9}\times\frac{3}{8} = \frac{20}{72} = \frac{5}{18}$ | 2 |
| B.1.b | The number of draws is maximal when we obtain among the first five draws : 4 bills of 10 000 and 1 bill of 50 000 , which justifies that the maximal value of X is 6. | 1 |
| B.2 | p(drawing at least 3 bills) = 1 – [p(X=1) + p(X=2)] = 1 – [$\frac{3}{9}+\frac{5}{18}$] = $\frac{7}{18}$. | 1 |

<table>
<tr><th>QV</th><th>Short answers</th><th></th></tr>
<tr><td>1</td><td>z’ = 0 for $z^2 - (3 - i)z + 4 - 3i = 0$ ; $\Delta = -8 + 6i = (1 + 3i)^2$<br>$z_1 = \frac{3 - i + 1 + 3i}{2} = 2 + i$ and $z_2 = \frac{3 - i - 1 - 3i}{2} = 1 - 2i$</td><td>1</td></tr>
<tr><td>2</td><td>$x' + iy' = x^2 - y^2 - (3 - i)(x + iy) + 4 - 3i$ ; $x' = x^2 - y^2 - 3x - y + 4$ and $y' = 2xy - 3y + x - 3$</td><td>1</td></tr>
<tr><td>3.a</td><td>If x’ = 0 then $x^2 - y^2 - 3x - y + 4 = 0$ ;</td><td>½</td></tr>
<tr><td>3.b</td><td>$(x - \frac{3}{2})^2 - (y + \frac{1}{2})^2 = -2$ ; (C) is a rectangular hyperbola of center $I(\frac{3}{2}, -\frac{1}{2})$.</td><td>1</td></tr>
<tr><td>3.c</td><td>Under translation of vector $\vec{OI}(\frac{3}{2}, -\frac{1}{2})$ the equation becomes $Y^2 - X^2 = 2$ ; $a = b = \sqrt{2}$ ,<br>$c^2 = 2a^2 = 4$ ; c = 2.
<table>
<tr><th></th><th>In $(I, \vec{i}, \vec{j})$</th><th>In $(O, \vec{i}, \vec{j})$</th></tr>
<tr><td>Vertices</td><td>$A(0, \sqrt{2})$ ; $A'(0, -\sqrt{2})$</td><td>$A(\frac{3}{2}, \sqrt{2} - \frac{1}{2})$ ; $A'(\frac{3}{2}, -\sqrt{2} - \frac{1}{2})$</td></tr>
<tr><td>Foci</td><td>F(0 , 2) ; F’(0, – 2)</td><td>$F(\frac{3}{2}, \frac{3}{2})$ ; $F'(\frac{3}{2}, -\frac{5}{2})$</td></tr>
<tr><td>Asymptotes</td><td>Y = X or Y = –X</td><td>y = x – 2 or y = – x +1</td></tr>
<tr><td>Directrices</td><td>$Y = \frac{a^2}{c} = 1$ or Y = – 1</td><td>$y = \frac{1}{2}$ or $y = -\frac{3}{2}$</td></tr>
</table></td><td>1</td></tr>
<tr><td>3.d</td><td></td><td>½</td></tr>
<tr><td>3.e</td><td>$2x - 2yy' - 3 - y' = 0$ ; $y' = \frac{2x - 3}{1 + 2y}$ ; $y'_E = \frac{1}{3}$. (T) : $y - 1 = \frac{1}{3}(x - 2)$ or $y = \frac{1}{3}x + \frac{1}{3}$<br>(N) : y = – 3x + 7.</td><td>1</td></tr>
</table>

<table>
<tr><th>QVI</th><th>Short answers</th><th></th></tr>
<tr><td>A.1</td><td>$\lim_{x \to \frac{1}{e}} f(x) = \frac{1}{0^+} = +\infty$ ; $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{1}{\frac{1}{x}} = +\infty$ ; $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{1}{1 + \ln x} = 0$</td><td>1 ½</td></tr>
<tr><td>A.2</td><td>$f'(x) = \frac{\ln x + 1 - 1}{(1 + \ln x)^2} = \frac{\ln x}{(1 + \ln x)^2}$.
<table>
<tr><td>x</td><td>1/e</td><td></td><td>1</td><td></td><td>+∞</td></tr>
<tr><td>f’(x)</td><td></td><td>–</td><td>0</td><td>+</td><td></td></tr>
<tr><td>f(x)</td><td>+∞</td><td>↘</td><td>1</td><td>↗</td><td>+∞</td></tr>
</table></td><td>1 ½</td></tr>
</table>

| | | |
|---|---|---|
| A.3.a | $f''(x)=\frac{-\ln x+1}{x(1+\ln x)^3}$ . $f''(x)$ vanishes for x = e<br>and changes signs , then (C) has a point of inflection W(e ; $\frac{e}{2}$ ). | 1 |
| A.3.b | $y-\frac{e}{2}=f'(e)(x-e)$ ; $y=\frac{1}{4}x+\frac{e}{4}$. | 1 |
| A.4 | $f(x)-x=\frac{x}{1+\ln x}-x=\frac{-x\ln x}{1+\ln x}$. • (C) cuts (D) at the point ( 1 ; 1)<br>• (C) is above (D) for $\frac{1}{e}<x<1$ ; (C) is below (D) for $x>1$. | 1 |
| A.5 | <br>The line of equation x = 1/e is an asymptote of (C)<br>The axis of abscissas is an asymptotic direction of (C) at +∞ | 2 |
| B.1.a | f is strictly increasing on I ; f(I) = [f(1) ; f(e)] = [1; $\frac{e}{2}$ ] then $f(I)\subset I$ . | 1 |
| B.1.b | $f'(x)-\frac{1}{4}=\frac{\ln x}{(1+\ln x)^2}-\frac{1}{4}=\frac{-(1-\ln x)^2}{4(1+\ln x)^2}$ ; $f'(x)-\frac{1}{4}\leq 0$ then $f'(x)\leq\frac{1}{4}$ .<br>but $f'(x)\geq 0$ on [1 ; e] , hence $0\leq f'(x)\leq\frac{1}{4}$. | 1 |
| B.1.c | Using the mean value inequality we have, $\lvert f(x)-f(1)\rvert\leq k\lvert x-1\rvert$<br>Where K is the maximum of $\lvert f'(x)\rvert$ on [1 ; e], hence $\lvert f(x)-1\rvert\leq\frac{1}{4}\lvert x-1\rvert$. | 1 |
| B.2.a | $U_0=2$ then $U_0\in[1;e]$ ; for n > 0 if $U_n\in[1;e]$ then $f(U_n)\in[1;e]$ i .e<br>$U_{n+1}\in[1;e]$. | 1 |
| B.2.b | $\lvert f(U_n)-1\rvert\leq\frac{1}{4}\lvert U_n-1\rvert$ ; $\lvert U_{n+1}-1\rvert\leq\frac{1}{4}\lvert U_n-1\rvert$. | ½ |
| B.2.c | By mathematical induction :<br>$\lvert U_o-1\rvert=1\leq\frac{1}{4^0}$<br>Suppose that $\lvert U_n-1\rvert\leq\frac{1}{4^n}$ and prove that $\lvert U_{n+1}-1\rvert\leq\frac{1}{4^{n+1}}$ .<br>$\lvert U_{n+1}-1\rvert\leq\frac{1}{4}\lvert U_n-1\rvert\leq\frac{1}{4}\times\frac{1}{4^n}$ then $\lvert U_{n+1}-1\rvert\leq\frac{1}{4^{n+1}}$<br>$\lvert U_n-1\rvert\leq\frac{1}{4^n}$ with $\lim\frac{1}{4^n}=0$, thus $\lim U_n=1$. | 1 ½ |