عدد المسائل : اربع

مسابقة في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

ملاحظـة : يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (2 points)

The production of shirts, in a certain factory during the last six years, is distributed as shown in the following table:

Rank of the year x_i	1	2	3	4	5	6
Production y_i (in thousands)	34.6	35.8	38.8	40.5	41.5	46.1

The line $D_{y/x}$ of regression, of y in terms of x, has the equation $y = 2.18x + b$.

1) Determine the coordinates of the point G, the center of gravity (mean point) of the scatter plot associated to the given distribution $(x_i ; y_i)$, and deduce the value of b .
2) Estimate the production of this factory for the year of rank 10.

II- (4 points)

In order to examine his students, a mathematics teacher placed 30 identical cards in a bag : 18 of these cards each carries one question of statistics, while each of the remaining cards carries an algebra question.

A student draws randomly one card from this bag and answers the question written on this card.

The probability that the student answers correctly a statistics question is 0.7, and the probability that he answers correctly an algebra question is 0.5 .

Consider the following events:

S : « The drawn card carries a statistics question ».
A : « The drawn card carries an algebra question ».
C : « The student answers the drawn question correctly ».

1) Calculate the following probabilities : $P(S \cap C)$, $P(A \cap C)$ and $P(\overline{C})$.
2) The student answered the chosen question correctly, what is the probability that this question is an algebra question ?
3) The teacher assigns marks as follows :
   5 for a correct answer in statistics.
   n for a correct answer in algebra .
   -2 for an incorrect answer.

Let X be the random variable that designates the mark obtained by the student.

a- Determine the probability distribution of X.
b- Calculate E(X), the expected value of X, in terms of n .
c- Find the value of n for which $E(X) = 2.54$.

III- (4 points)

A merchant borrows a loan of 20 000 000 LL from a certain bank.
The annual rate of interest charged is **6 %** , compounded **monthly**.
To pay back this loan, he decides to pay 500 000 LL to the bank at the end of every month.
Designate by U_n the amount of the debt at the end of the n^{th} month .

1) Verify that $U_1 = 19\,600\,000$.

2) Establish that $U_{n+1} = 1.005\,U_n - 500\,000$.

3) Consider the sequence (V_n) that is defined by $V_n = U_n - 100\,000\,000$.

a- Prove that (V_n) is a geometric sequence of ratio 1.005 and determine V_1.

b- Express V_n in terms of n, and deduce U_n in terms of n.

c- Prove that this debt is paid back at the end of 45 months.

d- Determine the value of the last amount that is to be paid by the merchant at the end of the 45^{th} month.

IV- (10 points)

Let f be the function that is defined, on $[0 ; +\infty[$, by : $f(x) = x + \frac{1}{2} + e^{1-x}$

and designate by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Part A

1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$.

b- Prove that the line (d) of equation $y = x + \frac{1}{2}$ is an asymptote of (C).

2) Calculate f '(x) and set up the table of variations of f.
3) Draw (d) and (C).
4) Calculate the area of the region that is bounded by the curve (C), its asymptote (d) and the two lines of equations x = 0 and x = 1.

Part B

A factory manufactures batteries and the total cost of production, in millions LL, is expressed by $C(x) = x + \frac{1}{2} + e^{1-x}$ where x is the number, in hundreds, of batteries produced $(0 \le x \le 5)$.

1) Calculate the fixed costs.
2) Calculate the total cost of manufacturing 500 batteries.
3) Each battery is sold for 20 000 LL, but only 90 % of the production is sold.

a- Show that the revenue function is expressed by R(x) = 1.8x .

b- Represent graphically the function R, in the system $(O ; \vec{i}, \vec{j})$.

c- Justify graphically that the equation R(x) = C(x) has a unique solution α and verify that $1.43 < \alpha < 1.44$.

d- What does α represent to the factory ?

e- Indicate the minimal number of batteries that should be manufactured in order that the factory achieves a profit.

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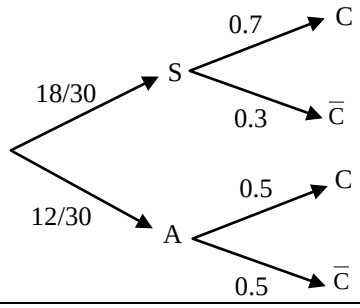
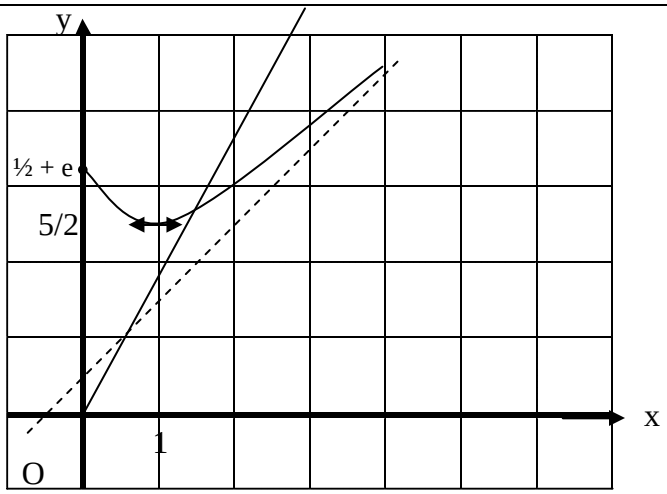
Q1	Short Answers	M
1	$\overline{X} = 3.5$, $\overline{Y} = 39.55$; the center of gravity is G(3.5 ; 39.55) y = 2.18x + b , the regression line passes through G ; 39.55 = 2.18× 3.5 + b ; b = 31.92	2
2	y = 2.18 × 10 + 31.92 = 53.72 ie 53720 shirts.	1 ½

Q2	Short Answers		M
1		P(S ∩ C) = (18/30)× (7/10) = 21 / 50 P(A ∩ C) = (12/30)× (1/2) = 1/5 P(C) = P(A ∩ C) + P(S ∩ C) = 1/5+ 21/50 = 31/50	1 ½
2	$P(A/C) = \frac{P(A \cap C)}{P(C)}$ = 1/5 ÷ 31/50 = 10/31		1
3.a	x_i | –2 | n | 5 p_i | 19/50 | 1/5 | 21/50		2
3.b	E(X) = – 38 / 50 + n/5 + 105/50 = 1.34 + 0.2n		1 ½
3.c	E(X) = 2.54 ; 1.34 + 0.2n = 2.54 ; n = 6.		1

Q2	Short Answers	M
1	$U_1 = (20\,000\,000 \times 0.06)/12 + 20\,000\,000 - 500\,000 = 19\,600\,000$	½
2	$U_{n+1} = (U_n \times 0.06) / 12 + U_n - 500\,000 = 1.005\,U_n - 500\,000$	1
3.a	$\frac{V_{n+1}}{V_n} = \frac{U_{n+1} - 100\,000\,000}{U_n - 100\,000\,000} = \frac{1.005U_n - 100500\,000}{U_n - 100\,000\,000}$ $= \frac{1.005(U_n - 100\,000\,000)}{U_n - 100\,000\,000} = 1.005$ $V_1 = U_1 - 100\,000\,000 = 19\,600\,000 - 100\,000\,000 = -80\,400\,000$	2
3.b	$V_n = V_1 \times q^{n-1} = -80\,400\,000 \times (1.005)^{n-1}$ $U_n = -80\,400\,000 \times (1.005)^{n-1} + 100\,000\,000$	1 ½
3.c	$U_n = 0$; $-80\,400\,000 \times (1,005)^{n-1} + 100\,000\,000 = 0$ $(1.005)^{n-1} = 1000 / 804$; $(n-1)\ln(1.005) = \ln(1000/804)$; $n - 1 = 43.74$ n = 44.74 ie 45 months are needed. ▼ or : $C = R.\frac{1-(1+i)^{-n}}{i}$; $20\,000\,000 = 500\,000 \times \frac{1-(1+\frac{0.06}{12})^{-n}}{\frac{0.06}{12}}$; n =44.74	1
3.d	$U_{44} = -80\,400\,000 \times (1.005)^{43} + 100\,000\,000 = 368\,491.879$ The paid amount is 368 491. 879(1.005) = 370 334. 338 4 LL ▼ or : $U_{45} = -80\,400\,000 \times (1.005)^{44} + 100\,000\,000 = -129\,665.661$ The paid amount is 500 000 – 129 665 . 661 = 370 334. 339	1

Q4	**Short Answers**	M
A1.a	$\lim\limits_{x\to+\infty} f(x) = +\infty + 0 = +\infty$	½
A1.b	$\lim\limits_{x\to+\infty} (f(x) - x - \frac{1}{2}) = \lim\limits_{x\to+\infty} e^{1-x} = 0$, then the line of equation $y = x + ½$ is an asymptote of (C) .	1
A2.	$f'(x) = 1 - e^{1-x}$ $f'(x) > 0$ for $1 > e^{1-x}$ $0 > 1 - x$; $x > 1$. x | 0 | 1 | $+\infty$ $f'(x)$ | − | 0 | + $f(x)$ | ½ | 5/2 | $+\infty$	2 ½
A.3		2
A.4	$A = \int_0^1 (x + \frac{1}{2} + e^{1-x} - x - \frac{1}{2})dx = \int_0^1 e^{1-x}dx = -\left[e^{1-x}\right]_0^1 = -(1 - e) = (e - 1)\ u^2$	2
B.1	Fixed costs = C (0) = ½ + e = 3.218 281 ie 3 218 281 LL	1
B.2	500 batteries correspond to x = 5 ; $f(5) = 5 + ½ + e^{1-5} = 5.518\,315$ ie 5 518 315 LL	1 ½
B.3.a	The selling price of a unit is 20 000 × 100 = 2 000 000, ie 2 million LL. Number of sold units for a production of x hundred batteries is 0.9x $R(x) = 2 \times 0.9x = 1.8x$	2
B.3.b	See figure.	½
B.3.c	The two curves intersect at a unique point then the equation R(x) = C (x) has a unique solution α. C (1.43) = 2.580 ; R(1.43) = 2.574 ; R(1.43) < C (1.43) C (1.44) = 2.584 ; R(1.44) = 2.592 ; R(1.44) > C (1.44) Then $1.43 < \alpha < 1.44$.	2
B.3.d	α is the number, in hundreds of batteries, to be produced by the factory in order to achieve zero profit (Break even value)	1 ½
B.3.e	The factory starts to make a profit at a production of 144 batteries.	1