وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٥ العادية

عدد المسائل : أربع

مسابقة في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (3 points)

In the complex plane referred to a direct orthonormal system $(O ; \vec{u} , \vec{v})$, consider the points E , M and M' of respective affixes i , z and z' , where $z' = iz + 1 + i$.

1) Find the algebraic form of z' when $z = \sqrt{2}e^{i\frac{\pi}{4}}$.
2) Determine the modulus and an argument of z if $z' = 1 + \sqrt{3} + 2i$.
3) Determine the value of z , for which the points M and M' are confounded .
4) a- Show that $z' - i = i (z - i)$.
   b- Deduce that when M moves on the circle (C) of center E and radius 3, then the point M' moves on the same circle.

II - (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i} , \vec{j} , \vec{k})$, consider :

- the plane (P) of equation $2x + y - 3z - 1 = 0$;
- the plane (Q) of equation $x + 4y + 2z + 1 = 0$;
- the line (d) defined by : $\begin{cases} x = 2t + 1 \\ y = -t - 1 \\ z = t \end{cases}$ (t is a real parameter).

1) Prove that the line (d) is included in the plane (P) .
2) Find an equation of the plane (S) that is determined by the point O and the line (d).
3) Consider the point $E\left(0 ; -\frac{1}{2} ; -\frac{1}{2}\right)$.
   Prove that E is the orthogonal projection of the point O on the line (d).
4) a- Show that the planes (P) and (Q) are perpendicular.
   b- Let (D) be the line of intersection of (P) and (Q) .
   Calculate the distance from E to (D).

III - (5 points)

A certain store sells only jackets, coats and shirts.
During a week, **120** customers were served in this store.
90 of those customers bought each one jacket, while the other **30** customers bought each one coat.
40% of those who bought jackets bought each also a shirt, while **20%** of those who bought coats bought each also a shirt.

A customer is chosen at random from those **120** customers and is interviewed.

1) Consider the following events :
    J : « the interviewed customer has bought a jacket ».
    C : « the interviewed customer has bought a coat ».
    S : « the interviewed customer has bought a shirt ».

a- Verify that the probability of the event $S \cap J$ is equal to $\frac{3}{10}$.

b- Calculate the following probabilities :
$P(S \cap C)$, $P(S)$, $P(C/S)$ and $P(C/\overline{S})$.

2) The prices of the clothes in this store are as shown in the following table :

Kind	Jacket	Coat	Shirt
Price in LL	150 000	200 000	60 000

Let X designate the random variable that is equal to the amount paid by a customer.

a- Give the four possible values of X.
b- Determine the probability distribution of X .
c- Calculate the mean (expected value) E(X) .
d- Estimate the amount of sales collected by the store during that week.

IV- **(8 points)**

Consider the function f that is defined, on $I =]\,1\,;+\infty\,[$, by $f(x) = x + 1 - \dfrac{3e^x}{e^x - e}$

and let (C) be its representative curve in an orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,)$.

1) a- Prove that the line of equation $x = 1$ is an asymptote to (C) .
    b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and show that the line (d) of equation $y = x - 2$ is an asymptote to (C).
    c- Determine the relative position of (C) and (d) .

2) Prove that $f'(x) > 0$ for all values of x in I, and set up the table of variations of f.

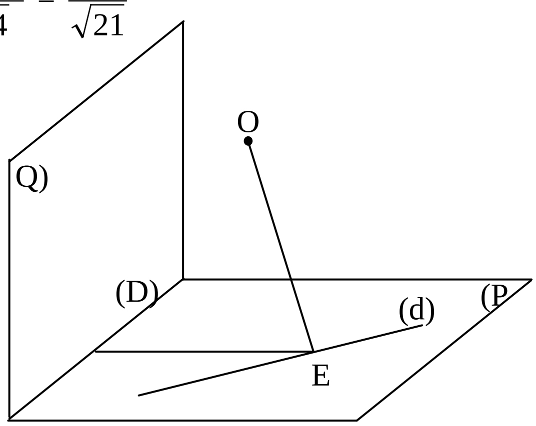
3) Prove that the equation f(x) = 0 has a unique root α and verify that $2.6 < \alpha < 2.7$.

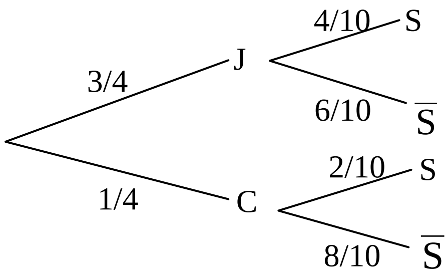
4) Draw the curve (C) .

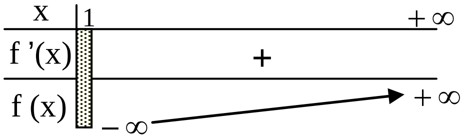
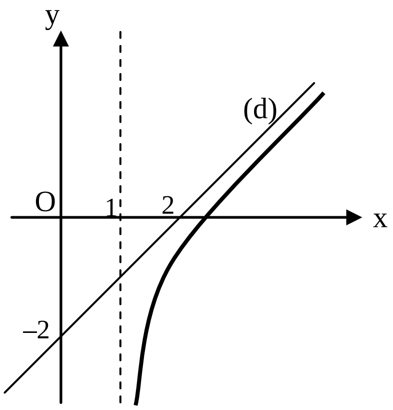
5) Designate by (D) the region that is bounded by (C) , the line (d) and the lines of equations x = 3 and x = 4.
    Calculate $\displaystyle\int_3^4 \frac{e^x}{e^x - e}\,dx$ and deduce the area of the region (D).

6) a- Prove that f, on the interval I, has an inverse function g.
    b- Prove that the equation f(x) = g(x) has no roots .

L.S.		*MATH* 1^{st} session 2005	
Question		Short Answers	M
I	1	$z' = i(\sqrt{2}\ e^{i\frac{\pi}{4}}) + 1 + i = i(1 + i) + (1 + i) = 2i$.	½
	2	$1+\sqrt{3} + 2i = iz + 1 + i$; $iz = \sqrt{3} + i$; $z = 1 - i\sqrt{3}$; $\lvert z\rvert = 2$ and $\arg(z) = -\frac{\pi}{3}$	½
	3	$z' = z$ for $z = iz + 1 + i$; $z(1 - i) = 1 + i$; $z = \frac{1+i}{1-i}$; $z = i$.	½
	4a	$z' - i = iz + 1 = i(z - i)$	½
	4b	$\lvert z' - i\rvert = \lvert i\rvert\ \lvert z - i\rvert = \lvert z - i\rvert$; EM' = EM. M moves on the circle (C) , EM = 3 , then EM' = 3, thus M' moves on the same circle.	1

II	1	Every point M(2t + 1; – t – 1 ; t) on (d) is a point in (P) because 2(2t + 1) – t – 1 – 3t – 1 = 0 ; 0t=0 , hence (d) is included in (P).	½
	2	I(1 , – 1 , 0) is a point on (d), the equation of (S) is given by : $\overrightarrow{OM}.(\overrightarrow{OI} \wedge \overrightarrow{V_d}) = 0$; $\begin{vmatrix} x & y & z \\ 1 & -1 & 0 \\ 2 & -1 & 1 \end{vmatrix} = 0$; $x + y - z = 0$.	1
	3	E is a point on (d) (for t = – ½). $\overrightarrow{OE}.\overrightarrow{V_d} = 0 + ½ - ½ = 0$. ►OR : Find the coordinates of the orthogonal projection of O on (d).	1
	4a	$\overrightarrow{n}_P(2\,;1;-3)$ and $\overrightarrow{n}_Q(1\,;4;\,2)$; $\overrightarrow{n}_P\ .\ \overrightarrow{n}_Q = 0$; (P) and (Q) are perpendicular.	½
	4b	(P) and (Q) are perpendicular, E is a point in (P) ; $d(E/(D)) = d(E/(Q)) = \frac{\lvert 0-2-1+1\rvert}{\sqrt{1+16+4}} = \frac{2}{\sqrt{21}}$ ►OR : Find a system of Parametric equations of (D) and calculate the distance from E to (D). 	1

<table>
<tr><th colspan="2">Question</th><th>Short Answers</th><th>M</th></tr>
<tr><td rowspan="7">III</td><td></td><td>
</td><td></td></tr>
<tr><td>1a</td><td>$P(S\cap J) = P(J) \times P(S/J) = \frac{3}{4}\times\frac{4}{10}=\frac{3}{10}.$</td><td>½</td></tr>
<tr><td>1b</td><td>$P(S\ \cap C) = P(C) \times P(S/C) = \frac{1}{4}\times\frac{2}{10}=\frac{1}{20}.$
$P(S) = P(S\ \cap J) + P(S\ \cap C) = \frac{6}{20}+\frac{1}{20} = \frac{7}{20}.$
$P(C/S) = \frac{P(C\cap S)}{P(S)} = \frac{1/20}{7/20} = \frac{1}{7}.$
$P(C/\overline{S}) = \frac{P(C\cap\overline{S})}{P(\overline{S})} = \frac{P(C)\times P(\overline{S}/C)}{1-P(S)} = \frac{(1/4)(8/10)}{1-(7/20)} = \frac{4}{13}.$</td><td>1½</td></tr>
<tr><td>2a</td><td>A customer has bought only one out of the following four choices :
only a jacket, only a coat, a jacket and a shirt, a coat and a shirt.
$X(\Omega) = \{ 150\ 000 ,\ 200\ 000 ,\ 210\ 000 ,\ 260\ 000\}$</td><td>½</td></tr>
<tr><td>2b</td><td><table>
<tr><td>$X = x_i$</td><td>150 000</td><td>200 000</td><td>210 000</td><td>260 000</td></tr>
<tr><td>P_i</td><td>$\frac{3}{4}\times\frac{6}{10}=\frac{18}{40}$</td><td>$\frac{1}{4}\times\frac{8}{10}=\frac{8}{40}$</td><td>$\frac{3}{4}\times\frac{4}{10}=\frac{12}{40}$</td><td>$\frac{1}{4}\times\frac{2}{10}=\frac{2}{40}$</td></tr>
</table></td><td>1 ½</td></tr>
<tr><td>2c</td><td>$E(X) = \frac{10000}{40}(15 \times 18 + 20 \times 8 + 21 \times 12 + 26 \times 2) = 183\ 500$</td><td>½</td></tr>
<tr><td>2d</td><td>The sales amount during that week is equal to the product of the mean amount by the number of the customers :
183 500 × 120 = 22 020 000 LL.</td><td>½</td></tr>
</table>

Question		Short Answers	M
IV	1a	$\lim\limits_{\substack{x\to1\\x>1}} e^x = e$; $\lim\limits_{\substack{x\to1\\x>1}}(e^x - e) = 0^+$; $\lim\limits_{\substack{x\to1\\x>1}} f(x) = -\infty$ The line of equation x = 1 is an asymptote to (C) .	½
	1b	$\lim\limits_{x\to+\infty}\frac{3e^x}{e^x-e}=3$, consequently $\lim\limits_{x\to+\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty}[f(x)-(x-2)] = \lim\limits_{x\to+\infty}[3-\frac{3e^x}{e^x-e}]=0$ The line (d) of equation y = x – 2 is an asymptote to(C).	1
	1c	f(x) – (x – 2) = 3 – $\frac{3e^x}{e^x-e} = \frac{-3e}{e^x-e}$ x > 1 , $e^x > e$, then f(x) – (x – 2) < 0 so (C) is below (d).	½
	2	f ’(x) = 1– $3\frac{e^x(e^x-e)-e^x(e^x)}{(e^x-e)^2}$ = 1+ $3\frac{e^{x+1}}{(e^x-e)^2} > 0$  	1
	3	On I, f is continuos and changes signs, thus the equation f(x) = 0 has at least one root α. But since f is strictly increasing on I ,then α is unique. f(2.6) = – 0.158 and f(2.7) = 0.0294 , thus 2.6 < α < 2.7	1
	4	 	1
	5	• $\int_3^4 \frac{e^x}{e^x-e}dx = [\ln(e^x-e)]_3^4 = \ln(e^4-e)-\ln(e^3-e) = \ln\frac{e^3-1}{e^2-1}$. • $\mathscr{A} = \int_3^4 (x-2-f(x))dx = \int_3^4 (-3+3\frac{e^x}{e^x-e})dx = [-3x]_3^4 + 3\ln\frac{e^3-1}{e^2-1}$ $= [-3+3\ln\frac{e^3-1}{e^2-1}]\ u^2 \quad \approx 0.28\ u^2$	1 ½
	6a	On I, f being continuous and strictly increasing, it has an inverse function g.	½
	6b	The equation f(x) = g(x) is equivalent to f(x) = x, so $1-\frac{3e^x}{e^x-e}=0$ gives $2\,e^x = -e$ which is impossible. ►OR : graphically , the curve (C) does not cut the first bisector y = x.	1