وزارة التربية والتعليم العالي ا لمديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع العلوم العامة	دورة سنة 2005 العادية
عدد المسائل : ست	مسابقة في الرياضيات المدة : ٤ ساعات	الاسم : الرقم :

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– (2.5points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider:

- the points A (1 ; –2 ; 1) , B (2 ;–1 ; 3) , C(1 ;1 ;4) and H(0 ; 0 ; 2) ,
- the line (d) defined by : $\begin{cases} x = t \\ y = t \\ z = -t + 2 \end{cases}$ (t is a real parameter).

1) Write an equation of the plane (P) that is determined by the points A, B and C.
2) a- Prove that the line (d) is perpendicular to the plane (P) at the point H.
 b- Prove that H is equidistant from A , B and C.
 c- Find a system of parametric equations of a bisector of the angle AHB.
3) Let M be a movable point on line (d), and E(2 ; 2 ; 0) be a fixed point on (d).
 Find the values of t for which the volume of the tetrahedron MABC is equal to twice the volume of the tetrahedron EABC.

II– (2 points)

Consider the sequence (U_n) that is defined, on IN , by :

$U_0 = \int_0^1 \frac{1}{x+1}dx$, and for $n \geq 1$, $U_n = \int_0^1 \frac{x^n}{x+1}dx$

1) Calculate U_0 and U_1.
2) Prove that, for every $n \geq 1$, $U_{n+1} + U_n = \frac{1}{n+1}$, and deduce the value of U_2.
3) a- Prove that, for $0 \leq x \leq 1$, we get $0 \leq \frac{x^n}{x+1} \leq x^n$,

 and deduce that $0 \leq U_n \leq \frac{1}{n+1}$.

 b- Calculate $\lim_{n \to +\infty} U_n$.

III– (2points)

The management of a certain supermarket organized a lottery for their customers, on every Monday during the sale month.

To run this lottery, the management uses two urns **U** and **V**.

- The urn **U** contains **4** red balls and **3** white balls.
- The urn **V** contains **10** gift coupons of four different categories whose values are as shown in the table below :

	First category	Second category	Third category	Fourth category
Number of gift coupons	2	3	4	1
Value of the gift coupon in LL	100 000	50 000	10 000	0

A customer draws randomly a ball from the urn **U** :

- If the drawn ball is white, this customer does not win anything.
- If the drawn ball is red, this customer draws randomly a gift coupon from the urn **V**.

1) Consider the following events :

E : « the customer participating in this lottery wins 10 000 LL ».

N : « the customer participating in this lottery does not win anything ».

G : « the customer participating in this lottery achieves a non-zero gain ».

a- Verify that the probability of E is equal to $\frac{8}{35}$.

b- Calculate the probability of each of the events N and G.

2) Designate by X the random variable that is equal to the value won (positive or zero) by the customer who participated in this lottery.

a- Determine the probability distribution of X.

b- Calculate the mean (expected value) E(X).

IV – (3points)

In the plane referred to an orthonormal system $(O ; \vec{i} , \vec{j})$, consider the points

$F(3 ; 0)$, $F'(-3 ; 0)$ and $L (3 ; \frac{16}{5})$.

Designate by (E) the ellipse that passes through L and whose foci are F and F' .

1) a- Calculate LF + LF' .

b- Determine the coordinates of the vertices of (E).

c- Deduce that $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is an equation of (E), and draw (E).

2) Let (d) be the line of equation $x = \frac{25}{3}$.

a- What does the line (d) represent to the ellipse (E) ?

b- Find an equation of the straight line (T) that is tangent to (E) at the point L.

c- Prove that the lines (d) and (T) intersect each other at a point I belonging to the axis of abscissas.

3) Calculate the area of the region that is bounded by the ellipse (E) , the tangent (T), the axis of abscissas and the axis of ordinates.

V– (3.5 points)

The complex plane is referred to a direct orthononormal system $(O\,;\vec{u},\vec{v})$.
Let A be the point of affix 2, and B be the point of affix 2i .

Designate by E the image of A under the rotation R with center O and angle $\frac{\pi}{3}$,
and by F the image of B under the transformation *T* that is defined by the complex form : $z' = (\frac{-1}{2} - i\frac{\sqrt{3}}{2})\,z$.

1) a- Determine the nature and the characteristic elements of *T* .
   b- Prove that the four points A, B, E and F belong to the same circle with center O and whose radius is to be determined.

2) a- Prove that $\frac{z_E - z_A}{z_F - z_B}$ is real .
   b- Verify that $\frac{z_F - z_A}{z_E - z_B} = -i$.
   c- Deduce that AEBF is an isosceles trapezoid and that $(\overrightarrow{BE}\,,\overrightarrow{AF}) = -\frac{\pi}{2}\ (2\pi)$.

3) Consider : the dilation (homothecy) h that transforms A onto F and E onto B ,
   and the rotation r with angle $\frac{\pi}{2}$, that transforms B onto F .
   a- Determine W, the center of h.
   b- Prove that $h\circ r = r\circ h$.
   c- Let $S = h\circ r$.
   Determine the nature and the characteristic elements of S .

VI– (7 points)

Consider the function f that is defined, on $]\,0\,;+\infty[$, by :
$f(x) = (\ln x)^2 + 2\ln x - 3$.

Designate by (C) the representative curve of f in an orthonormal system $(O\,;\vec{i}\,,\vec{j})$.

1) a- Calculate $\lim_{x \to +\infty} f(x)$.
   b- Calculate $\lim_{x \to 0} f(x)$ and deduce an asymptote to (C).
2) Determine the abscissas of the points of intersection of (C) with the axis of abscissas.
3) a- Calculate $f'(x)$ and set up the table of variations of f .
   b- Verify that $f''(x) = \frac{-2\ln x}{x^2}$; Show that (C) has a point of inflection I, and write an equation of the tangent (d) to (C) at the point I.

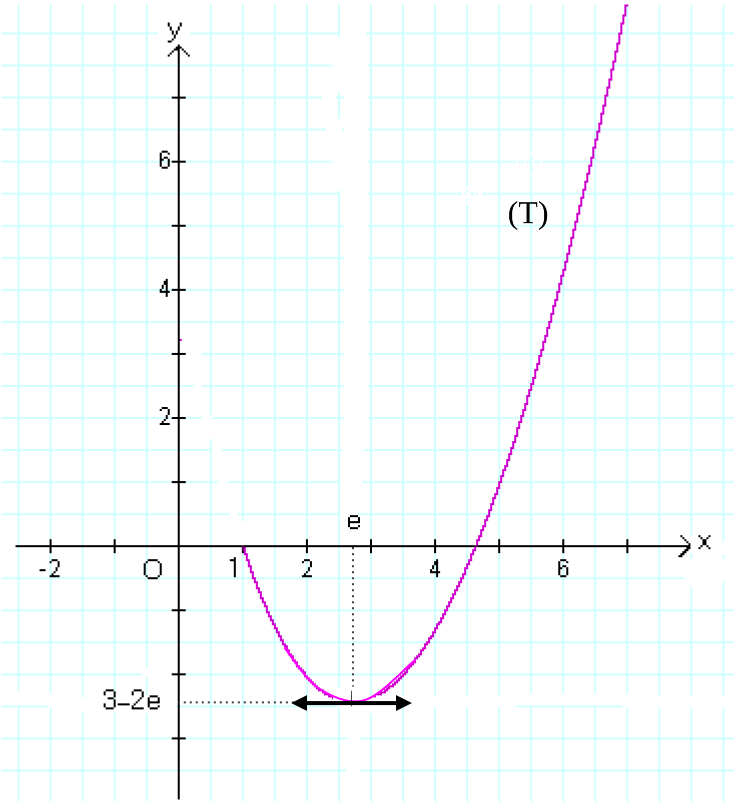
4) Draw the line (d) and the curve (C).

5) a- Prove that the function f has, on $[1 ; +\infty[$, an inverse function g and determine the domain of definition of g .

b- Verify that the point $A(5 ; e^2)$ belongs to (G) , the representative curve of g, and write an equation of the tangent to the curve (G) at A.

6) Determine graphically, according to the values of the real number m, the number of roots of the equation $(\ln x)^2 + 2\ln x = m$.

7) The curve (T) shown below is the representative curve ,on $[1 ; +\infty[$, of a function F, where F is a primitive (anti derivative) of the function f:

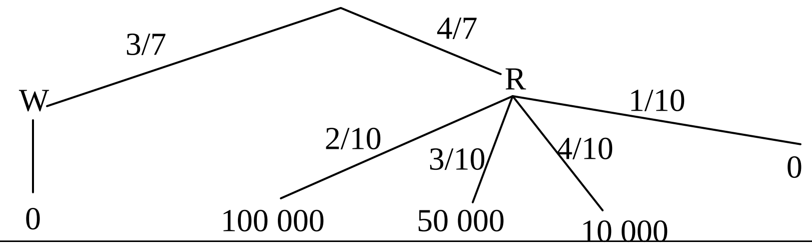


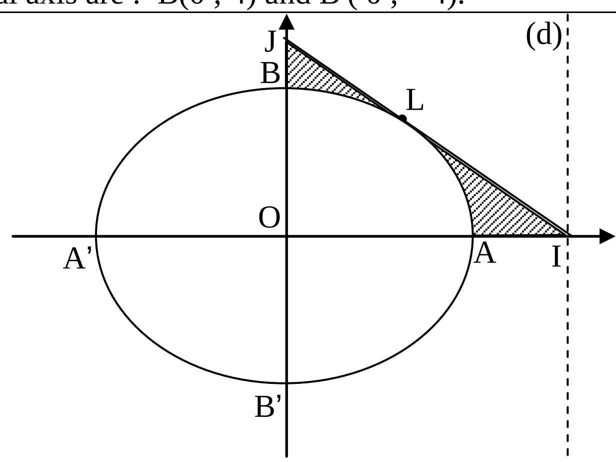
Calculate the area of the region that is bounded by the curve (C) , the axis of abscissas and the two lines of equations $x = 1$ and $x = e$.

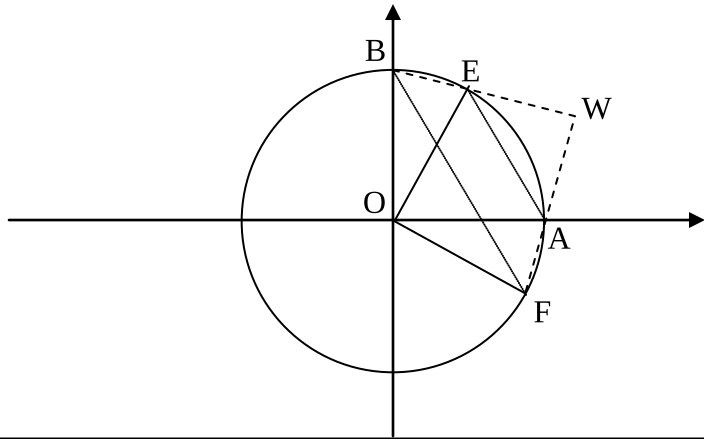
GS	MATH	1^{st} session 2005

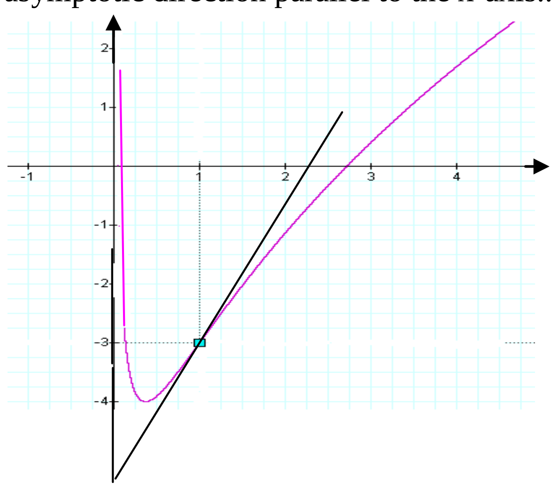
Q 1	Short Answers	M
1	The equation of plane (P) is determined by : $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & 1 & 2 \\ 0 & 3 & 3 \end{vmatrix} = 0$; $x + y - z + 2 = 0$	1
2a	$\vec{V}_d(1,1,-1)$ and $\vec{N}_P(1,1,-1)$, $\vec{V}_d = \vec{N}_P$, then (d) is perpendicular to (P). Intersection of (d) and (P) : $t + t + t - 2 + 2 = 0$; $t = 0$ (d) cuts (P) at point H(0 ; 0 ; 2).	1
2b	$\vec{HA}(1 ; -2 ; -1)$; $\vec{HB}(2 ; -1 ; 1)$; $\vec{HC}(1 ; 1 ; 2)$; $HA = HB = HC = \sqrt{6}$.	½
2c	HAB is isosceles of vertex H , $\vec{HA}+\vec{HB}:(3 ; -3 ; 0)$; $\vec{u}(1 ; -1 ; 0)$ is a direction vector of a bisector (Δ)of the angle $A\hat{H}B$. (Δ) : $x = m$; $y = -m$, $z = 2$. ►OR : W(3/2 ; – 3/2 ; 2) is the midpoint of [AB] ; $\vec{HW}(3/2 ; -3/2 ; 0)$ is direction vector of (Δ) ; $\vec{u}(1 ; -1 ; 0) = 2/3\ \vec{HW}$ also is direction vector of a bisector (Δ)	1
3	$V(MABC) = \frac{1}{3} MH \times area(ABC)$; $V(EABC) = \frac{1}{3} EH \times area(ABC)$ $V(MABC) = 2V(EABC)$; $MH = 2\,EH$; $MH^2 = 4EH^2$ so $3t^2 = 48$; $t^2 = 16$ Thus $t = -4$ or $t = 4$.	1½

Q 2	Short Answers	M
1	$U_0 = \int_0^1 \frac{dx}{x+1} = \ln(x+1)\Big\|_0^1 = \ln 2$ $U_1 = \int_0^1 \frac{x}{x+1}dx = \int_0^1 \frac{x+1-1}{x+1}dx = \int_0^1 (1-\frac{1}{x+1})dx = [x - \ln(x+1)]_0^1 = 1 - \ln 2$	1
2	$U_{n+1} + U_n = \int_0^1 \frac{x^{n+1}+x^n}{x+1}dx = \int_0^1 \frac{x^n(x+1)}{x+1}dx = \int_0^1 x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$. For $n = 1$; $U_2 + U_1 = \frac{1}{2}$; $U_2 = \ln 2 - \frac{1}{2}$.	1
3a	$0 \le x \le 1$; $1 \le x + 1 \le 2$; $\frac{1}{2} \le \frac{1}{x+1} \le 1$; $0 \le \frac{1}{x+1} \le 1$; $0 \le \frac{x^n}{x+1} \le x^n$ $0 \le \int_0^1 \frac{x^n}{x+1}dx \le \int_0^1 x^n dx$; $0 \le U_n \le \frac{1}{n+1}$.	1½
3b	Since $\lim_{n\to+\infty} \frac{1}{n+1} = 0$, then $\lim_{n\to+\infty} U_n = 0$.	½

<table>
<tr><th>Q 3</th><th>Short Answers</th><th>M</th></tr>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>1a</td><td>$P(E) = \frac{4}{7}\times\frac{4}{10}=\frac{8}{35}$</td><td>½</td></tr>
<tr><td>1b</td><td>$P(N) = \frac{3}{7}+\frac{4}{7}\times\frac{1}{10} = \frac{17}{35}$; $P(G) = 1 – P(N) = \frac{18}{35}$</td><td>1½</td></tr>
<tr><td>2a</td><td><table>
<tr><td>x_i</td><td>0</td><td>10 000</td><td>50 000</td><td>100 000</td></tr>
<tr><td>p_i</td><td>$\frac{17}{35}$</td><td>$\frac{8}{35}$</td><td>$\frac{4}{7}\times\frac{3}{10}=\frac{6}{35}$</td><td>$\frac{4}{7}\times\frac{2}{10}=\frac{4}{35}$</td></tr>
</table></td><td>1½</td></tr>
<tr><td>2b</td><td>$E(X) = \frac{10000}{35}(8 + 30 + 40) = \frac{156000}{7} \approx 22\ 285\ .\ 7$</td><td>½</td></tr>
</table>

<table>
<tr><th>Q 4</th><th>Short Answers</th><th>M</th></tr>
<tr><td>1a</td><td>$\vec{LF}(0;\frac{-16}{5})$; $\vec{LF'}(-6;\frac{-16}{5})$, then $LF + LF' = \frac{16}{5}+\sqrt{36+\frac{256}{25}}=\frac{50}{5} = 10.$</td><td>½</td></tr>
<tr><td>1b</td><td>$LF + LF' = 2a$ then a = 5, x’Ox is the focal axis of (E) and O is its center,
Thus the vertices on the focal axis are : A(5 ; 0) and A’(– 5 ; 0).
$b^2 = a^2 - c^2 = 25 – 9 = 16$ then b = 4 .
The vertices on the non-focal axis are : B(0 ; 4) and B’(0 ; – 4).</td><td>1</td></tr>
<tr><td>1c</td><td>The ellipse (E) has as an equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Thus we get $\frac{x^2}{25}+\frac{y^2}{16}=1.$

</td><td>1½</td></tr>
<tr><td>2a</td><td>$x = \frac{25}{3}=\frac{a^2}{c}$ then (d) is a directrix of (E).</td><td>½</td></tr>
<tr><td>2b</td><td>(T) : $\frac{x_L x}{25}+\frac{y_L y}{16}=1$; $\frac{3x}{25}+\frac{y}{5}=1$ thus 3x + 5y – 25 = 0.</td><td>½</td></tr>
<tr><td>2c</td><td>(T) cuts (d) at point (25/3 ; 0) which is a point on the axis of abscissas</td><td>½</td></tr>
<tr><td>3</td><td>Required area= A=area(OIJ) – $\frac{1}{4}$ $(\pi ab) = \frac{OI\times OJ}{2}-\frac{20\pi}{4} = (\frac{125}{6}-5\pi)u^2$</td><td>1½</td></tr>
</table>

Q 5	Short Answers	M
1a	Z’ = $(-\frac{1}{2}-i\frac{\sqrt{3}}{2})Z = e^{i\frac{-2\pi}{3}}Z$; T : rotation of center O and angle $\frac{-2\pi}{3}$.	1
1b	E = R(A) ; OE = OA = 2 and F = T(B) ; OF = OB = 2 A , B , E and F belong to the circle of center O and radius 2. 	1
2a	$Z_E = e^{i\frac{\pi}{3}}Z_A = (\frac{1}{2}+i\frac{\sqrt{3}}{2})Z_A = 1 + i\sqrt{3}$; $Z_F = (-\frac{1}{2}-i\frac{\sqrt{3}}{2})Z_B = \sqrt{3} - i$ $\frac{Z_E - Z_A}{Z_F - Z_B} = \frac{-1+i\sqrt{3}}{\sqrt{3}-3i} = \frac{-1+i\sqrt{3}}{-\sqrt{3}(-1+i\sqrt{3})} = \frac{-1}{\sqrt{3}}$ which is real.	1
2b	$\frac{Z_F - Z_A}{Z_E - Z_B} = \frac{-i+\sqrt{3}-2}{1+i\sqrt{3}-2i} = \frac{-i(1+i\sqrt{3}-2i)}{1+i\sqrt{3}-2i} = -i$.	½
2c	$\frac{Z_E - Z_A}{Z_F - Z_B}$ is real, then the vectors $\vec{AE}$ and $\vec{BF}$ have the same direction . On the other hand: $\left\|\frac{Z_F - Z_A}{Z_E - Z_B}\right\| = \|-i\| = 1$; EA = FB , and AEBF is an isosceles trapezoid. $\frac{Z_F - Z_A}{Z_E - Z_B} = e^{-i\frac{\pi}{2}}$ then $(\vec{BE};\vec{AF}) = -\frac{\pi}{2} (\text{mod.} 2\pi)$	1
3a	h (A) = F and h(E) = B, then W is the point of intersection of (AF) and (BE)	½
3b	hor and roh are two similitudes having the same ratio and the same angle. We still have to prove that they have the same center. The triangle WBF is right isosceles at W, hence W is the center of r, Consequently hor and roh both have the same center W.	1
3c	S is a similitude of center W, of ratio $\sqrt{3}$ and of angle $\frac{\pi}{2}$.	1

Q 6	Short Answers	M
1a	$\lim\limits_{x\to+\infty} f(x) = +\infty + \infty - 3 = +\infty$	½
1b	$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \ln x(\ln x + 2 - \frac{3}{\ln x}) = -\infty\,(-\infty + 2 - 0) = +\infty.$ The axis of ordinates is an asymptote to (C).	1
2	$(\ln x)^2 + 2\ln x - 3 = 0$; $\ln x = 1$ or $\ln x = -3$; $x = e$ or $x = e^{-3}$.	1
3a	$f'(x) = \frac{2\ln x}{x} + \frac{2}{x} = \frac{2}{x}(\ln x + 1)$; $f'(x) = 0$ for $\ln x = -1$ thus $x = \frac{1}{e}$. $\begin{array}{c\|ccccc} x & 0 & & 1/e & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & +\infty & \searrow & -4 & \nearrow & +\infty \end{array}$	1½
3b	$f''(x) = 2\left[-\frac{1}{x^2}(\ln x + 1) + \frac{1}{x}(\frac{1}{x})\right] = \frac{-2\ln x}{x^2}$; $f''(x)$ vanishes for $x = 1$ and changes signs, thus (C) has a point of inflection, namely $I(1\,;-3)$. Eq. of (d) : $y + 3 = f'(1)(x-1)$; $y = 2(x-1) - 3$; $y = 2x - 5$.	1½
4	$\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \frac{(\ln x)^2 + 2\ln x - 3}{x} = \lim\limits_{x\to+\infty} (\frac{2\ln x}{x} + \frac{2}{x}) = 0$, so (C) has, at $+\infty$, an asymptotic direction parallel to the x-axis.. 	2
5a	f is continuous and strictly increasing on $[\,1\,;+\infty\,[$ hence it has an inverse function g on $D_g = f([\,1\,;+\infty\,[) = [-3\,;+\infty\,[$.	1
5b	▪ $f(e^2) = 4 + 4 - 3 = 5$ then $g(5) = e^2$ and $A(5;e^2)$ is a point on (G). ▪ An equation of the tangent to (G) at the point A is : $y - e^2 = g'(5)(x-5)$ $g'(5) = \frac{1}{f'(e^2)} == \frac{e^2}{6}$; $y - e^2 = \frac{e^2}{6}(x-5)$; $y = \frac{e^2}{6}x + \frac{e^2}{6}$.	2
6	$(\ln x)^2 + 2\ln x = m$ is equivalent to $(\ln x)^2 + 2\ln x - 3 = m - 3$, ie $f(x) = m-3$ ▪ If $m - 3 < -4$ ie $m < -1$ then no roots ▪ If $m - 3 = -4$ ie $m = -1$ then there is one root ▪ If $m - 3 > -4$ ie $m > -1$ then there are two roots.	2
7	$A = -\int_1^e f(x)dx = -[F(e) - F(1)] = -(3 - 2e - 0) = (2e - 3)$ u^2.	1½