دورة سنة 2005 العادية	امتحانات شهادة الثانوية العامة فرع الاجتماع والاقتصاد	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم : الرقم :	مسابقة في : الرياضيات المدة : ساعتان	عددالمسائل : اربع

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

The table below shows the amount of advertising expenses x (in millions LL) of a certain car factory, and the corresponding number y of tens of cars sold.

x_i	10	12	14	14.5	15
y_i	20	25	30	35	40

1) Calculate the means $\overline{X}$ and $\overline{Y}$ of the variables x and y.
2) Represent graphically the scatter plot of the points $(x_i ; y_i)$ as well as the center of gravity G ($\overline{X}$; $\overline{Y}$), in a rectangular system.
3) Calculate the linear correlation coefficient r and give an interpretation of the value thus obtained.
4) Determine an equation of $D_{y/x}$, the line of regression of y in terms of x , and draw this line in the preceding system.
5) Suppose that the above pattern remains valid when this factory spends 18 000 000 LL on advertising.
   a- Estimate in this case the number p of cars to be sold (give answer to the nearest unit).
   b- The average cost of production of a car is 15 000 000 LL.
   Each car is sold for 20 000 000 LL.
   Estimate the profit achieved by this factory upon selling these p cars.

II - (4 points)

An urn contains **9** balls: 3 white, 4 red and 2 black.

A- Three balls are drawn randomly and **successively** from this urn, **without replacement**.
1) What is the probability that the three drawn balls are all white ?
2) What is the probability that the third drawn ball is the only white ball among the three drawn balls.

B- In all what follows, three balls are drawn randomly and **simultaneously** from the given urn.
1) Let C be the event : “***the three drawn balls have the same colour***”.
   Show that the probability of C is equal to $\frac{5}{84}$
2) Designate by X the random variable that is equal to the number of black balls obtained .
   a- Determine the probability distribution of X
   b- Calculate E(X), the expected value of X.

III - (4 points)

A statistical study of the population of a certain village revealed the following information :

- The population of this village was 6000 at the beginning of the year 2000.
- The annual increase in the population of this village is 2 %.
- 200 persons leave this village permanently every year (moving to the town, immigrating to other countries, ...)

Designate by U_n the number of inhabitants in this village in the year (2000 + n).

1) Let $U_0 = 6000$, verify that $U_1 = 5920$.
2) Show that $U_{n+1} = 1.02U_n - 200$.
3) Consider the sequence (V_n) that is defined by $V_n = U_n - 10\,000$; $(n \geq 0)$.
   a- Prove that (V_n) is a geometric sequence of common ratio 1.02.
   b- Calculate V_n in terms of n and deduce U_n in terms of n.
   c- During which year would the number of inhabitants in this village become less than 3000 for the first time?

IV- (8 points)

A - Let f be the function that is defined, on $[0\,;+\infty[$, by $f(x) = 3(x+1)e^{-x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

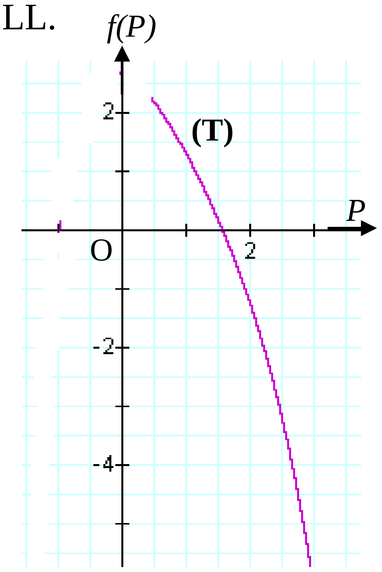
1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and determine an asymptote of (C).
2) Show that $f'(x) = -3xe^{-x}$ and set up the table of variations of f.
3) Draw the curve (C).
4) Let F be the function that is defined on $[0\,;+\infty[$ by $F(x) = 3(-x-2)e^{-x}$.
   a- Show that F is an antiderivative (a primitive) of f.
   b- Calculate the area of the region bounded by the curve (C) , the axis of abscissas and the lines of equations $x = 0$ and $x = 1$.

B - A factory produces a certain chemical liquid. The demand is modeled by :

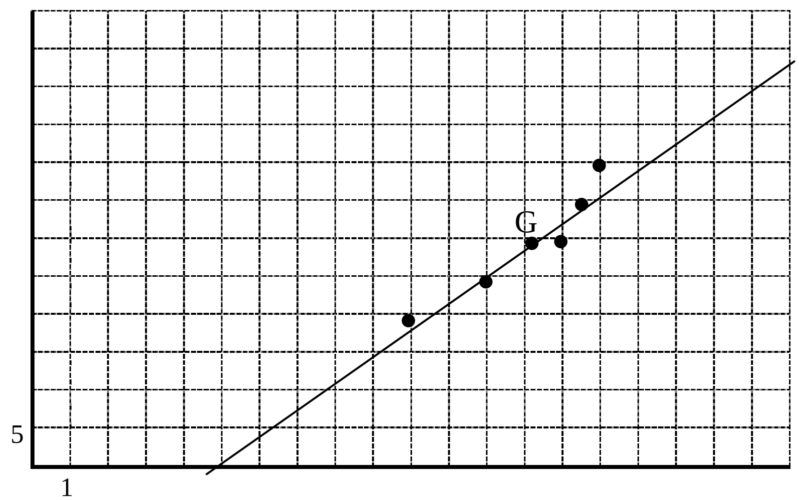
$f(p) = 3(p+1)e^{-p}$; where p is the unit price expressed in thousands LL and $f(p)$ is expressed in thousands of liters, for $0.5 \leq p \leq 4$.

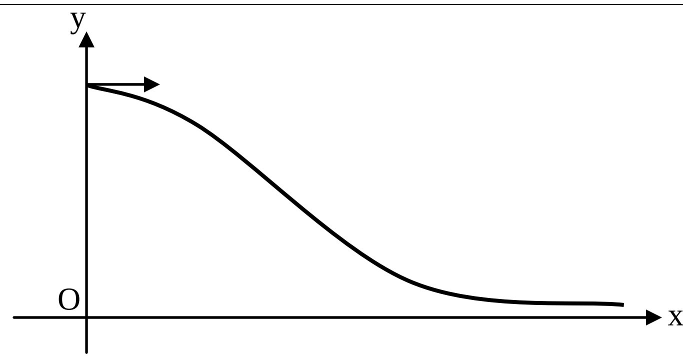
1) Calculate the demand corresponding to a unit price of 1000 LL.
2) The supply is modeled by $g(p) = \dfrac{e^{p}}{3}$.

   The adjacing curve (T) is the representative curve of the function h defined by $h(p) = f(p) - g(p)$, on $[0.5\,;\,4]$.

   a- Verify that the equation $h(p) = 0$ has a unique root α and prove that $1.57 < \alpha < 1.58$.
   b-Suppose that $\alpha = 1.575$.
   Give an economical interpretation of this value of α.
3) a-Calculate $E(p)$, the elasticity of the demand with respect to the price p .
   b- Determine the set of values of p for which the demand is elastic, and find the corresponding prices.

<table>
<tr><td colspan="4">SE   *MATH*   1st session 2005</td></tr>
<tr><th colspan="2">Q</th><th>Answers</th><th>M</th></tr>
<tr><td rowspan="6">I</td><td>1</td><td>$\overline{X} = 13.1$; $\overline{Y} = 30$</td><td>1</td></tr>
<tr><td>2</td><td></td><td>1</td></tr>
<tr><td>3</td><td>r = 0.953
There is a strong positive relationship.</td><td>1</td></tr>
<tr><td>4</td><td>$D_{Y/X}$: y = 3.633x – 17.60</td><td>1 ½</td></tr>
<tr><td>5a</td><td>x = 18 ; y = 3.633 × 18 – 17.60 = 47.794 , thus : 478 cars.</td><td>1</td></tr>
<tr><td>5b</td><td>Profit: (20 000 000 – 15 000 0000) × 478 – 18 000 000 = 2 372 000 000 LL.</td><td>1 ½</td></tr>
<tr><td rowspan="5">II</td><td>A1</td><td>P(www) = $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}=\frac{1}{84}$.</td><td>1</td></tr>
<tr><td>A2</td><td>The first drawn ball is not white as well as the second one ; $p(\overline{w};\overline{w};w)=\frac{6}{9}\times\frac{5}{8}\times\frac{3}{7}=\frac{5}{28}$.</td><td>1 ½</td></tr>
<tr><td>B1</td><td>We can draw 3 white balls or 3 black balls ; P(C) = $\frac{C_3^3+C_4^3}{C_9^3} = \frac{5}{84}$</td><td>1 ½</td></tr>
<tr><td>B2 a</td><td>X(Ω) = { 0 , 1 , 2}
<table><tr><td>x_i</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P_i</td><td>$\frac{C_7^3}{C_9^3}=\frac{5}{12}$</td><td>$\frac{C_2^1\times C_7^2}{C_9^3}=\frac{1}{2}$</td><td>$\frac{C_2^2\times C_7^1}{C_9^3}=\frac{1}{12}$</td></tr></table></td><td>2 ½</td></tr>
<tr><td>B2 b</td><td>E(X) = $0+\frac{1}{2}+\frac{2}{12}=\frac{4}{6}=\frac{2}{3}$.</td><td>½</td></tr>
<tr><td rowspan="6">III</td><td>1</td><td>U_1 = 6 000 + 6000 × 0.02 – 200 = 5920</td><td>1</td></tr>
<tr><td>2</td><td>$U_{n+1} = U_n + 0.02\times U_n - 200 = 1.02\times U_n - 200$</td><td>1</td></tr>
<tr><td>3a</td><td>$\frac{V_{n+1}}{V_n}=\frac{U_{n+1}-10000}{U_n-10000}=\frac{1.02U_n-200-10000}{U_n-10000}=\frac{1.02U_n-10200}{U_n-10000}=\frac{1.02(U_n-10000)}{U_n-10000}=1.02$</td><td>1 ½</td></tr>
<tr><td>3b</td><td>$V_n = V_o(1.02)^n$; $V_o = U_o - 10\,000 = -4\,000$; $V_n = -4000(1.02)^n$
$U_n = V_n + 10\,000 = -4000(1.02)^n + 10\,000$</td><td>1 ½</td></tr>
<tr><td>3c</td><td>$-4000(1.02)^n + 10\,000 < 3000$; $(1.02)^n > \frac{7}{4}$; $n\ln(1.02) > \ln\frac{7}{4}$; n > 28.25 ; n ≥ 29
In 2029 the population will become for the first time less than 3 000.</td><td>2</td></tr>
</table>

IV	A1	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{3(x+1)}{e^x} = \lim\limits_{x\to+\infty} \frac{3}{e^x} = 0$; the axis of abscissas is an asymptote of (C).	1
	A2	$f'(x) = 3[e^{-x} - (x+1)e^{-x}] = 3\,e^{-x}(1 - x - 1) = -3x\,e^{-x}$. x: 0 … $+\infty$ f '(x): 0 … – f(x): 3 ↘ 0	2
	A3		2
	A4 a	$F'(x) = 3[e^{-x} - (-x-2)\,e^{-x}] = 3\,e^{-x}(-1 + x + 2) = 3(x+1)\,e^{-x} = f(x)$	1
	A4 b	$A = \int_0^1 f(x)dx = [F(x)]_0^1 = F(1) - F(0) = 3(2 - \frac{3}{e})\ u^2$.	1 ½
	B1	For a price of 1000 LL ; p = 1 ; $f(1) = \frac{6}{e} = 2.207$ thus: 2207 liters.	1
	B2 a	The curve of the function h cuts the axis of abscissas at a unique point of abscissa α.The equation h(p) = 0 has a unique solution $p = \alpha$, h(1. 57) = 0.0018 > 0 and h(1. 58) = – 0.024 < 0 Hence 1. 52 < α < 1. 58.	1 ½
	B2 b	For a unit price that is equal to 1575 LL, the market is in equilibrium.	1
	B3 a	$E(p) = \frac{-pf'(p)}{f(p)} = \frac{p^2}{p+1}$	1 ½
	B3 b	The demand is elastic iff E(p) > 1 ; $\frac{p^2}{p+1} > 1$; $p^2 - p - 1 > 0$ $p < \frac{1-\sqrt{5}}{2}$ or $p > \frac{1+\sqrt{5}}{2}$ (with $0.5 \le p \le 4$) $\frac{1+\sqrt{5}}{2} < p \le 4$ or $1.618 < p \le 4$ The unit price belongs to the interval] 1618 ; 4000].	1 ½