عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I-(4 points)

The annual profit (in millions LL) of a service agency, starting from the year 2001, is as shown in the following table:

Year	2001	2002	2003	2004	2005
Rank of the year: x_i	1	2	3	4	5
Profit in millions LL : y_i	200	220	250	270	280

1) a- Determine the coordinates of the center of gravity G.
   b- Construct the scatter plot of the points associated to the distribution $(x_i ; y_i)$ and plot the point G, in a rectangular system.
2) Write an equation of $D_{y/x}$, the line of regression of y in terms of x, and draw this line in the preceding system.
3) Suppose that the above pattern remains valid till the year 2015.
   a- How much is the profit that this agency is expected to achieve in the year 2008 ?
   b- After which year would the profit of this agency exceed 400 millions LL for the first time ?

II- (4 points)

In a shop there are 1000 leather wallets, of which some are defective.
These wallets were manufactured by three factories α, β and γ according to the following table :

	Factory α	Factory β	Factory γ
Number of wallets	200	350	450
Percentage of defective wallets	5%	4%	2%

A wallet is chosen at random from these 1000 wallets, and consider the following events :

A : « The chosen wallet was produced by the factory α ».
B : « The chosen wallet was produced by the factory β ».
C : « The chosen wallet was produced by the factory γ ».
D : « The chosen wallet is defective ».

1) a- Prove that the probability $P(D \cap A)$ is equal to $\frac{1}{100}$.
   b- Calculate the following probabilities: $P(D \cap B)$, $P(D \cap C)$ and $P(D)$.
2) Knowing that the chosen wallet is not defective, what is the probability that it was manufactured by the factory α ?
3) A wallet is sold for 50 000LL if manufactured by the factory α, for 60 000LL if manufactured by the factory β and for 80 000LL if manufactured by the factory γ.
   The price of any defective wallet is reduced by 30 %.
   Designate by X the random variable that is equal to the final price of the wallet that was randomly chosen.
   Find the six values of X and determine the probability distribution of X.

III- (4 points)

Fadi deposits a capital of 100 million LL in a bank, at 10 % annual interest rate, compounded yearly. At the end of every year, Fadi withdraws 5 million LL from his account.

Let $U_0 = 100$ and designate by U_n the amount, in millions LL, that is in Fadi's account at the end of the nth year after withdrawing the 5 million LL.

1) a- Verify that $U_1 = 105$ and calculate U_2 .
   b- Show that the sequence (U_n) is not geometric.
   c- Justify the relation $U_{n+1} = 1.1U_n - 5$.

2) Let $V_n = U_n - 50$, for every natural integer n.
   a- Show that the sequence (V_n) is a geometric sequence of common ratio 1.1 .
   b- Calculate V_n in terms of n, and find the value of U_8 .

IV-**(8 points)**

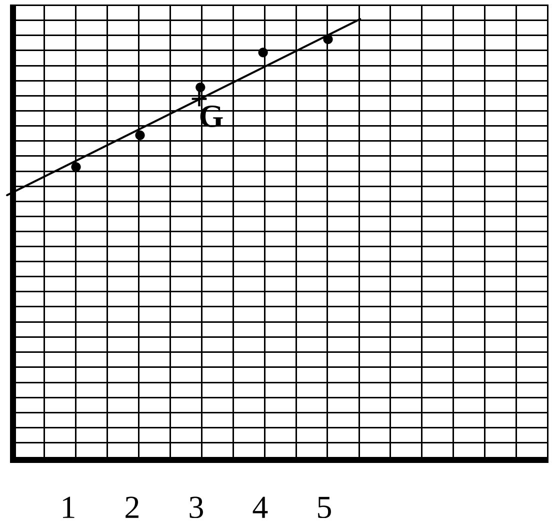
A- Let f be the function that is defined on $[0 ; +\infty[$ by $f(x)=(x^2+2x)e^{-x}$, and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Verify that the axis of abscissas is an asymptote of (C).
   b- Calculate $f(\sqrt{2})$ and give your answer to the nearest 10^{-3} .
2) a- Show that $f'(x)=(2-x^2)e^{-x}$ and set up the table of variations of f.
   b- Write an equation of the line (d) that is tangent to (C) at O.
3) Draw the line (d) and the curve (C).
4) Let F be the function that is defined on $[0 ; +\infty[$ by $F(x) = (-x^2 - 4x - 4)e^{-x}$.
   a- Show that F is an antiderivative (primitive) of f.
   b- Calculate the area of the region bounded by the curve (C) , the axis of abscissas and the lines of equations $x = 0$ and $x = 1$.

B- A factory produces a certain liquid detergent. The demand, in thousands of liters, is modeled by :
$d(p) = (p+2)e^{-p}$, where p is the unit price (price of one liter) in thousands LL. $(1 \le p \le 4)$.
1) Calculate the demand corresponding to a unit price of 2 000 LL.
2) Prove that the revenue function is expressed by $f(p)=(p^2+2p)e^{-p}$.
3) Calculate the unit price for which the revenue would be maximum. Determine this maximum.
4) a- Determine E(p), the elasticity of the demand with respect to the price.
   b- Calculate $E(\sqrt{2})$, and give an economical interpretation of the value thus obtained.

SE MATHEMATICS

2^{nd} SESSION - 2006

Q I	SHORT ANSWERS	M
1.a	$\overline{X} = 3$; $\overline{Y} = 244$	1
1.b		1 ½
2	Using a calculator: $y = 21x + 181$	1 ½
3.a	In 2008 : $x = 8$, $y = 21 \times 8 + 181 = 349$ The profit achieved in 2008 is estimated by 349 000 000 LL	1 ½
3.b	$y > 400$; $21x + 181 > 400$; $21x > 219$; $x > 10.42$. In 2011 the profit exceeds 400 million LL for the first time.	1 ½

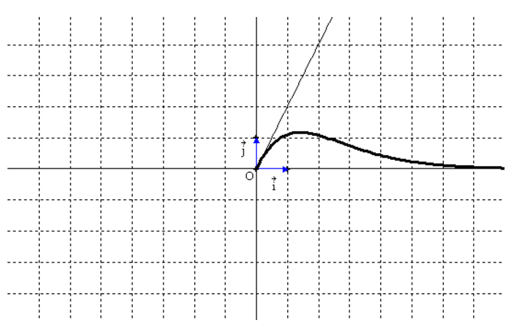
QII	SHORT ANSWERS	M

	Factory α	Factory β	Factory γ	Total
Number of defective wallets	10	14	9	33
Number of non-defective wallets	190	336	441	967
Total	200	350	450	1 000

QII	SHORT ANSWERS	M
1.a	$P(D \cap A) = \frac{10}{1000} = \frac{1}{100}$	1
1.b	$P(D \cap B) = \frac{14}{1000} = \frac{7}{500}$; $P(D \cap C) = \frac{9}{1000}$ $P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C) = \frac{10+14+9}{1000} = \frac{33}{1000}$ ▶ OR : $P(D) = \frac{33}{1000}$ by reading the table.	2
2	$P(A/\overline{D}) = \frac{190}{967}$	1
3	The values of X are : 35 000 ; 42 000 ; 50 000 ; 56 000 ; 60 000 and 80 0000	3

x_i	35 000	42 000	50 000	56 000	60 000	80 000	Total
p_i	0.01	0.014	0.19	0.009	0.336	0.441	1

QIII	SHORT ANSWERS	M
1.a	$U_1 = U_0\ (1 + 0.1) - 5 = 100 \times 1.1 - 5 = 105.$ $U_2 = U_1 \times 1.1 - 5 = 105\times 1.1 - 5 = 110.5.$	1
1.b	$\frac{U_1}{U_o} = \frac{105}{100} = 1.05$ and $\frac{U_2}{U_1} = \frac{110.5}{105} = 1.052$, thus $\frac{U_1}{U_o} \neq \frac{U_2}{U_1}$,and (U_n) is not geometric	1 ½
1.c	$U_{n+1} = U_n + 0.1U_n - 5 = 1.1U_n - 5.$	1
2.a	$\frac{V_{n+1}}{Vn} = \frac{U_{n+1} - 50}{U_n - 50} = \frac{1.1U_n - 55}{U_n - 50} = \frac{1.1(U_n - 50)}{U_n - 50} = 1.1$ (V_n) is a geometric sequence of common ratio q = 1.1.	1 ½
2.b	$V_o = U_o - 50 = 50$; $V_n = V_o \times q^n = 50(1.1)^n$; $U_8 = V_8 + 50 = 50(1.1)^8 + 50 = 157.179.$	2

QIV	SHORT ANSWERS	M
A1a	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x^2 + 2x}{e^x} = \lim_{x\to+\infty} \frac{2x+2}{e^x} = \lim_{x\to+\infty} \frac{2}{e^x} = 0$.	1
A1b	$f(\sqrt{2}) = (2 + 2\sqrt{2})\, e^{-\sqrt{2}} = 1.174$	1
A2a	$f'(x) = (2x + 2)\, e^{-x} - e^{-x}\ (x^2 + 2x) = e^{-x}\ (2 - x^2)$ x: 0 ; $\sqrt{2}$; $+\infty$ f '(x): + ; 0 ; − f '(x): 0 ↗ 1.174 ↘ 0	1 ½
A2b	(d) : y = f '(0)x = 2x	1
A3	 1 ½	
A4a	$F'(x) = (-2x - 4)\, e^{-x} - e^{-x}\,(-x^2 - 4x - 4)$ $= e^{-x}\,(x^2 + 2x) = f(x)$	1
A4b	$A = \int_0^1 f(x)dx = [F(x)]_0^1 = F(1) - F(0)$ $= -9e^{-1} + 4 = 0.689\ u^2$	1
B1	For a price of 2000 LL ; p = 2 ; $d(2) = 4e^{-2} = 0.541$, that is 541 liters.	1
B2	Revenue = demand × price, thus $f(p) = p\,(p+2)\,e^{-p} = (p^2+2p)e^{-p}$.	1
B3	The revenue is maximum for $p = \sqrt{2}$, i.e. for a price of 1414 LL ; this maximum is 1174 thousands LL.	1 ½
B4a	$E(p) = -p \times \frac{d'(p)}{d(p)} = -p \times \frac{(-p-1)\,e^{-p}}{(p+2)\,e^{-p}} = \frac{p(p+1)}{p+2}$	1
B4b	$E(\sqrt{2}) = \frac{\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}+2} = +1$; when the revenue is maximum, the demand has unit elasticity. (For an increase of 1% on the unit price of 1414LL, the demand decreases by 1%).	1 ½