وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٦ الأستثنائية

عدد المسائل: اربع

مسابقة في مادةتايضايرلا
المدة: ساعتان

الاسم:
الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I-(3.5 points)

In the complex plane, referred to a direct orthonormal system $(O\,;\,\vec{u}\,,\,\vec{v})$, consider the points A, B and M of respective affixes 2, 4 and z (where $z \neq 2$).

Let M' be the point of affix z' such that $z' = \dfrac{z-4}{z-2}$.

1) a- Give a geometric interpretation of $|z'|$, $|z-4|$ and $|z-2|$.

b- Determine the set of points M when M' moves on the circle with center O and radius 1.

2) Let $z = x + iy$ and $z' = x' + iy'$.

a- Express x' and y' in terms of x and y.

b- When z' is real, find the line on which the point M moves.

II-**(4 points)**

An urn U contains **four** balls numbered **1**, **three** balls numbered **2** and **one** ball numbered **5**.
Another urn V contains **three** balls numbered **1** and **five** balls numbered **2**.

A- Two balls are drawn, simultaneously and randomly, from the urn U.
Calculate the probability of each of the following events:
E : « the two drawn balls carry the same number »
F : « the product of the two numbers, that are marked on the two drawn balls, is 10 ».

B- We draw randomly one ball from the urn U and one ball from the urn V.
Let X be the random variable that is equal to the sum of the two numbers that are marked on the two drawn balls.

1- Give the five possible values of X.

2- Verify that the probability of having $X = 3$ is equal to $\dfrac{29}{64}$.

3- Determine the probability distribution of X.

III-**(4 points)**

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider:

the plane (P) of equation $x + y + z - 4 = 0$,

the points A (3 ;1 ; 0) , B(1 ;2 ;1) , C(1 ; 1 ;2) and E(2 ; 0 ;-1).

1) Prove that the triangle ABC is right angled at B.

2) a- Verify that (P) is the plane that is determined by A , B and C.
   b- Show that the line (AE) is perpendicular to plane (P) .

3) Designate by (Q) the plane passing through A and perpendicular to (BE).
   Write an equation of (Q).

4) The planes (P) and (Q) intersect along a line (D).
   a- Prove that the lines (D) and (BC) are parallel.
   b- Let L be any point on (BC) and H be its orthogonal projection on (Q) .
   Show that LH remains constant as L moves on the line (BC).

IV-**(8.5 points)**

A- Given the differential equation (E) : $y' - y - e^{x} + 1 = 0$.

Let $z = y - xe^{x} - 1$.

1) Find a differential equation (E') that is satisfied by z, and determine its general solution.
2) Deduce the general solution of (E), and find a particular solution y of (E) that verifies y(0) = 0.

B- Let f be the function that is defined on IR by $f(x) = (x - 1)e^{x} + 1$, and designate by (C) its representative curve in an orthonormal system $(O;\,\vec{i}\,,\,\vec{j})$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$. Give f(2) in the decimal form.
   b- Calculate $\lim\limits_{x\to-\infty} f(x)$ and deduce an asymptote (d) to (C).
   c- Verify that the curve (C) cuts its asymptote (d) at the point E(1 ; 1).

2) a- Calculate f '(x) and set up the table of variations of f.

   b- Prove that the curve (C) has a point of inflection.

3) Draw the line (d) and the curve (C).

4) a- Prove that the function f has, on [0 ; + ∞ [, an inverse function g.
   b- Draw the curve (G) that represents g, in the system $(O;\,\vec{i}\,,\,\vec{j})$.
   c- Calculate the area of the region bounded by the two curves (C) and (G).

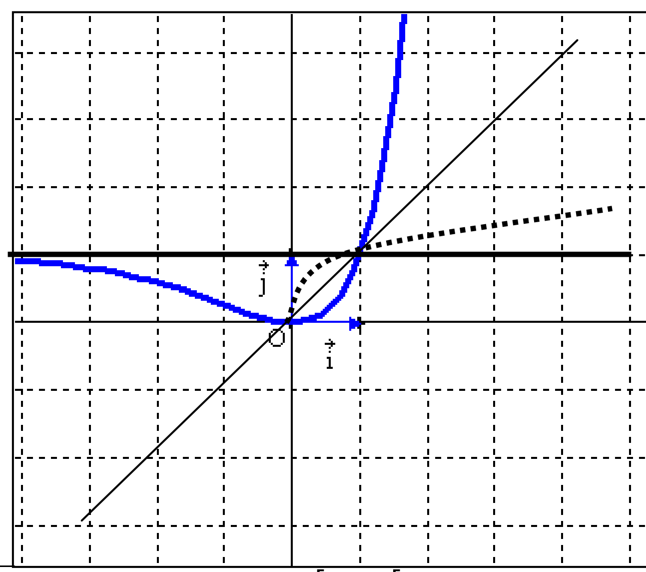
LS MATHEMATICS

SCHEME OF CORRECTION		2nd SESSION 2006
Q1		M
1.a	$\lvert z'\rvert = OM'$; $\lvert z-4\rvert = BM$ and $\lvert z-2\rvert = AM$	1
1.b	$\lvert z'\rvert = \frac{\lvert z-4\rvert}{\lvert z-2\rvert}$ gives $OM' = \frac{BM}{AM}$. $OM' = 1$ is equivalent to $BM = AM$ and thus the set of points M is the perpendicular bisector of [AB],	1
2.a	$x' + iy' = \frac{x+iy-4}{x+iy-2} = \frac{(x-4+iy)(x-2-iy)}{(x-2)^2+y^2}$;$x' = \frac{x^2+y^2-6x+8}{(x-2)^2+y^2}$,$y' = \frac{2y}{(x-2)^2+y^2}$	1
2.b	z' is real iff $y' = 0$, to get $y = 0$, so M moves on the axis of abscissas.	½

Q2		M
A	$P(E) = P(1\,;1) + P(2\,;2) = \frac{C_4^2}{C_8^2} + \frac{C_3^2}{C_8^2} = \frac{9}{28}$. $P(F) = P(2\,;5) = \frac{C_3^1 \times C_1^1}{C_8^2} = \frac{3}{28}$	1½
B-1	The five values of X are: 2 ; 3 ; 4 ; 6 ; 7 .	½
B-2	$P(X=3) = P(1\,;2) + P(2\,;1) = \frac{4}{8}\times\frac{5}{8}+\frac{3}{8}\times\frac{3}{8} = \frac{29}{64}$	½
B-3	see table below	1½

x_i	2	3	4	6	7
P_i	$\frac{4}{8}\times\frac{3}{8}=\frac{12}{64}$	$\frac{29}{64}$	$\frac{3}{8}\times\frac{5}{8}=\frac{15}{64}$	$\frac{1}{8}\times\frac{3}{8}=\frac{3}{64}$	$\frac{1}{8}\times\frac{5}{8}=\frac{5}{64}$

Q3		M
1	$\vec{AB}(-2,1,1)$, $\vec{BC}(0,-1,1)$; $\vec{AB}.\vec{BC} = 0$, thus ABC is right angled at B.	½
2.a	The coordinates of A , B and C verify the equation of (P). $\Rightarrow$ OR : $\vec{AM}.(\vec{AB}\wedge\vec{AC}) = 0$	½
2.b	$\vec{AE}(-1,-1,-1)$, $\vec{N_P}(1,1,1)$ so $\vec{AE} = -\vec{N_P}$ and (AE) is perpendicular to (P).	½
3	. $\vec{BE}(1,-2,-2) = \vec{N_Q}$ thus (Q) : $x - 2y - 2z + d = 0$, A belongs to (Q) gives $d = -1$ (Q) : $x - 2y - 2z - 1 = 0$.	½
4.a	$\vec{BC}(0,-1,1)$, $\vec{V_D} = \vec{N_P}\wedge\vec{N_Q} = 3\vec{j} - 3\vec{k}$ then $\vec{V_D} = -3\vec{BC}$ and B does not belong to (Q) , hence (D) // (BC) . $\Rightarrow$ OR : (BC) is perpendicular to plane (ABE) , then (BC) is perpendicular to (EB), and (EB) is perpendicular to (Q) , hence (BC) // (Q) [since (BC) $\not\subset$ (Q)] (P) contains (BC) then (P) cuts (Q) along (D) //(BC).	1
4.b	(D) //(BC) , then(BC) //(Q) .All the points on (BC) have the same distance from (Q) ; $\Rightarrow$ OR : Equations of (BC) : $x = 1$, $y = -m + 2$, $z = m + 1$ $d(M \to (Q)) = \frac{\lvert 1+2m-4-2m-2-1\rvert}{\sqrt{1+4+4}} = 2$ = constant	1

<table>
<tr><th>Q4</th><th></th><th>M</th></tr>
<tr><td>A1</td><td>$y' - y - e^x + 1 = 0$; $y = z + xe^x + 1$; $y' = z' + e^x + xe^x$; $z' + e^x + xe^x - z - xe^x - 1 - e^x + 1 = 0$
$z' - z = 0$; to get $z = Ce^x$.</td><td>1</td></tr>
<tr><td>A2</td><td>$y = Ce^x + xe^x + 1$; $y(0) = C + 1 = 0$; $C = -1$, then $y = -e^x + xe^x + 1$.</td><td>1</td></tr>
<tr><td>B1.a</td><td>$\lim_{x \to +\infty} f(x) = +\infty$; $f(2) = e^2 + 1 = 8.389$.</td><td>½</td></tr>
<tr><td>B1.b</td><td>$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (xe^x - e^x + 1) = 1$, so the line (d) of equation $y = 1$ is an asymptote to (C) .</td><td>1</td></tr>
<tr><td>B1.c</td><td>$y = 1$ and $1 = (1 - 1)e^x + 1$; so (C) cuts (d) at E (1 ;1) .</td><td>½</td></tr>
<tr><td>B2.a</td><td>$f'(x) = e^x + (x - 1)e^x = xe^x$.
<table>
<tr><td>x</td><td>$-\infty$</td><td>0</td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>f(x)</td><td>1 ↘</td><td>0</td><td>↗ $+\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td>B2.b</td><td>$f''(x) = (x + 1)e^x$; $f''(x)$ vanishes for $x = -1$ and changing signs; consequently (C) has a point of inflection .</td><td>½</td></tr>
<tr><td>B3</td><td>
</td><td>1</td></tr>
<tr><td>B4.a</td><td>f is continuous and strictly increasing on $[0;+\infty[$,so it has an inverse function g .</td><td>½</td></tr>
<tr><td>B4.b</td><td>(G) is the symmetric of (C) w.r.t. the first bisector of equation $y = x$.</td><td>½</td></tr>
<tr><td>B4.c</td><td>$\mathcal{A} = 2\int_0^1 [x - f(x)]dx = 2\int_0^1 (x - 1 + e^x - xe^x)dx = 2\left[\frac{x^2}{2} - x + e^x\right]_0^1 - 2\int_0^1 xe^x dx$
so $\int_0^1 xe^x dx = [xe^x]_0^1 - \int_0^1 e^x dx = e - (e - 1) = 1$
Thus $\mathcal{A} = 2(\frac{1}{2} - 1 + e - 1) - 2 = 2e - 5$ u^2.</td><td>1</td></tr>
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