وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع العلوم العامة

دورة سنة٢٠٠٦ الاستثنائية

عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I – (1.5 points)

In the table below, only one of the proposed answers to each question, is correct.
Write down the number of each question and give, **with justification**, the answer corresponding to it.

Nº	Questions	Answers a	b	c	d
1	$z = -2e^{-i\frac{5\pi}{6}}$. An argument of z is :	$\frac{-\pi}{6}$	$\frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{6}$
2	The solution set of the inequality $\ln(x^2 - 2x+2) > 0$ is :	IR	$]0\,;+\infty[$	$IR - \{1\}$	$]1\,;+\infty[$
3	$\lim\limits_{x \to +\infty} x\ln\left(1+\frac{1}{x}\right) =$	1	-1	$+\infty$	$-\infty$
4	z and z ' are two complex numbers. If $z' = \frac{\bar{z} - i}{z + i}$, then $\lvert z' \rvert =$	$\lvert z \rvert$	2	$\lvert z \rvert \times \lvert \bar{z} \rvert$	1

II-(2.5 points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider the points A(1 ;-1 ;1), B(2 ; 0 ; 3), C(-1 ;1 ;1) and G(4 ;2 ;4) and designate by (P) the plane that is determined by A, B and C.

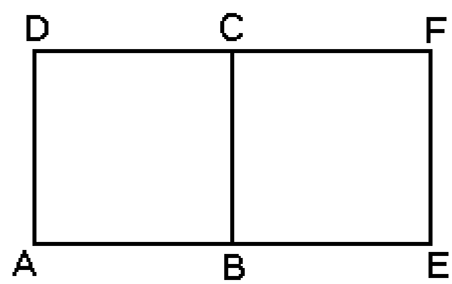
1) a- Calculate the area of triangle ABC.
   b- Calculate the volume of the tetrahedron GABC and deduce the distance from G to plane (P).

2) Prove that $x + y - z + 1 = 0$ is an equation of plane (P).

3) a- Show that the point F(2 ;0 ;6) is symmetric of G with respect to plane (P).
   b- Give a system of parametric equations of the line (d) that is the symmetric of the line (AF) with respect to plane (P).
   c- Prove that the line (AB) is a bisector of the angle $\widehat{FAG}$.

III-(3 points)

Consider, in an oriented plane, the two direct squares ABCD and BEFC.
Let S be the direct plane similitude that transforms A onto E and E onto F.



1) a- Determine the ratio k and an angle α of S.
   b- Construct geometrically the center W of S.
   c- Find the point G that is the image of F under S.

2) Let h be the transformation that is defined by $h = S \circ S$.
   a- Determine the nature and the elements of h.
   b- Specify h(A), and express $\overrightarrow{WA}$ in terms of $\overrightarrow{WF}$.

3) The complex plane is referred to an orthonormal system $(A;\overrightarrow{AB},\overrightarrow{AD})$.
   a- Determine the affixes of the points E, F and W.
   b- Find the complex form of S.
   c- Give the complex form of h and find the affix of h(E).

IV- (3 points)

Given two identical boxes B_1 and B_2 .

The box B_1 contains **four** red balls and **two** white balls, and the box B_2 contains **four** red balls**, three** white balls and **one** black ball.

A- The two boxes B_1 and B_2 are placed inside the same bag. **One** box is drawn randomly from this bag, after which three balls are then drawn, randomly and simultaneously, from this box.

1) Consider the following events:

E : « the drawn balls are three red balls from the box B_1».

F : « the three drawn balls are red ».

a- Show that the probability of E is equal to $\frac{1}{10}$.

b- Calculate the probability of 87F .

2) a- What is the probability of obtaining the black ball among the three drawn balls ?

b- What is the probability of drawing three balls having three different colours ?

B- All the balls in the boxes B_1 and B_2 are now emptied in an urn U.

Three balls are drawn, simultaneously and randomly, from the urn U.

Let X be the random variable that is equal to the number of white balls among the drawn balls.

1) Determine the probability distribution of X.
2) Calculate the mean (expected value) E(X).

V-(2.5 points)

Consider, in the plane referred to an orthonormal system $(O ; \vec{i}, \vec{j})$, the hyperbola (H) of equation $x^2 - 3y^2 = 3$.

1) a- Determine the coordinates of the vertices and of the foci of (H) and find its eccentricity.

b- Write the equations of the asymptotes and of the directrices of (H).

c- Draw the hyperbola (H).

2) Let (D) be the region bounded by the hyperbola (H) and the line of equation $x = 2$.

Calculate the volume generated by the rotation of (D) about the axis of abscissas.

3) Designate by K and L the points on (H) having the same abscissa 2.

Show that the tangents to (H) at K and L intersect on a directrix of (H).

VI- (7.5 points)

A – Consider the differential equation (E) : $y' + 2y = 6xe^{-2x}$.

Let $z = y - 3x^2e^{-2x}$.

1) Write a differential equation (E') satisfied by z, and solve (E').

2) Deduce the general solution of (E), and find a particular solution y of (E) that verifies y(0) = 0.

B- Let f be the function that is defined, on IR, by $f(x) = 3x^2e^{-2x}$, and let (C) be its representative curve in an orthonormal system $(O;\ \vec{i},\ \vec{j})$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote of (C).

b- Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to-\infty} \dfrac{f(x)}{x}$.

2) a- Calculate f '(x) and set up the table of variations of f.

b- Prove that the curve (C) has two points of inflection.

3) a- Draw the curve (C).

b- Determine, according to the values of the real number m, the number of roots of the equation : $me^{2x} - 3x^2 = 0$.

4) Let F be the function that is defined, on IR, by $F(x) = (ax^2 + bx + c)e^{-2x}$.

a- Determine a, b and c for which F is a primitive of f.

b- Calculate the area of the region that is bounded by the curve (C), the axis of abscissas and the lines of equations x = –1 and x = 0.

5) The tangent to (C) at the point $A(1;\ 3e^{-2})$ cuts again the curve (C) at a point E of abscissa t .

a-Verify that $-0.3 < t < -0.2$.

b-Let h be the function that is defined, on IR, by $h(x) = -e^{x-1}$.

Prove that h(t) = t.

6) Let g be the function that is defined by $g(x) = e^{f(x)}$.

a- Set up the table of variations of g .

b- Find the number of solutions of the equation g(x) = e.

c- Solve the inequality g(x) > 1.

GENERAL SCIENCES – MATHEMATICS ; 2nd SESSION – **2006**			
I			
1	$z = -2e^{-i\frac{5\pi}{6}} = 2e^{i(\frac{-5\pi}{6}+\pi)} = 2e^{i\frac{\pi}{6}}$.	b	3
2	$\ln(x^2 - 2x + 2) > 0$; $x^2 - 2x + 2 > 1$; $(x-1)^2 > 0$; $x \neq 1$.	c	
3	$\lim\limits_{x\to+\infty} x\ln(1+\frac{1}{x}) = \lim\limits_{t\to 0}\frac{\ln(1+t)}{t} = 1$, where $t = \frac{1}{x}$.	a	
4	$\lvert z' \rvert = \frac{\lvert \bar{z} - i \rvert}{\lvert z+i \rvert} = \frac{\lvert \overline{z+i} \rvert}{\lvert z+i \rvert} = 1$.	d	

II		
1.a	$S = \frac{\lVert \vec{AB} \wedge \vec{AC} \rVert}{2}$; $\vec{AB} \wedge \vec{AC} = -4\vec{i} - 4\vec{j} + 4\vec{k}$; $S = 2\sqrt{3}\ u^2$.	1/2
1.b	$V = \frac{\lvert \vec{AG}.(\vec{AB} \wedge \vec{AC}) \rvert}{6} = \frac{\lvert -12 \rvert}{6} = 2\ u^3$; $V = \frac{d \times S}{3}$,thus $d = \frac{3V}{S} = \frac{6}{2\sqrt{3}} = \sqrt{3}\ u$.	1
2	$\vec{AM}.(\vec{AB} \wedge \vec{AC}) = 0$; $-4(x-1) - 4(y+1) + 4(z-1) = 0$; $x + y - z + 1 = 0$. ⇨OR : The coordinates of A , B and C verify the equation of (P) .	1/2
3.a	$\vec{FG}(2 ; 2 ; -2)$; $\vec{N_P}(1 ; 1 ; -1)$; $\vec{FG} = 2\vec{N_P}$, so $(FG) \perp (P)$. I : midpoint of [FG] ; I(3 ; 1 ; 5) ; 3 + 1 – 5 + 1 = 0 thus I belongs to (P). ⇨OR : prove that (P) is the mediator plane of [FG] .	1
3.b	(d) is the line (AG) : x = 3m + 1 ; y = 3m – 1 and z = 3m + 1.	1
3.c	(AI) is a bisector of $F\hat{A}G$ since AF = AG and I is the midpoint of [FG], moreover, $\vec{AI}(2;2;4)$ and $\vec{AB}(1;1;2)$ hence $\vec{AI} = 2\vec{AB}$,thus B belongs to (AI) .	1

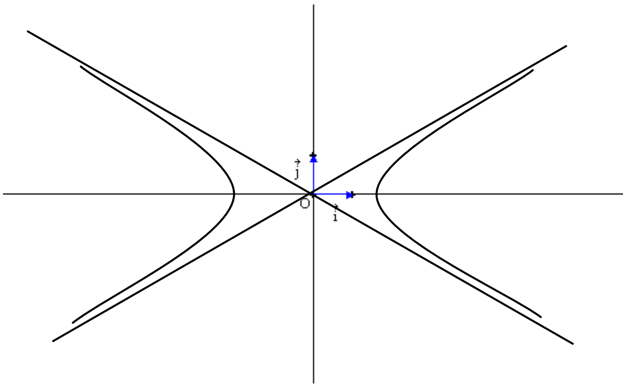
III		
1.a	S(A) = E ; S(E) = F. $k = \frac{EF}{AE} = \frac{1}{2}$; $(\vec{AE}, \vec{EF}) = \frac{\pi}{2}(2\pi)$, $\alpha = \frac{\pi}{2}$.	1/2
1.b	Since $\alpha = (\vec{WA}, \vec{WE}) = \frac{\pi}{2}$ then W belongs to circle of diameter [AE] ; Also since $(\vec{WE}, \vec{WF}) = \frac{\pi}{2}$ then W belongs to circle of diameter [EF] ; Thus W is the point of intersection of the 2 circles , other than E (S(E) = F ≠E).	1/2
1.c	S(E) = F and S(F) = G so $(\vec{EF}, \vec{FG}) = \frac{\pi}{2}$ and G belongs to the semi st. line [FD) ; and since $\frac{FG}{EF} = \frac{1}{2}$ then G is the midpoint of [FC].	1/2
2.a	h is a direct plane similitude of center W , of ratio $\frac{1}{4}$ and of angle π , thus it becomes the negative homothecy of center W and of ratio $-\frac{1}{4}$.	1

2.b	S(A) = E and S(E) = F then h(A) = S(S(A)) = S(E) = F, so $\overrightarrow{WF} = -\frac{1}{4}\overrightarrow{WA}$.	1/2
3.a	$z_E = 2$; $z_F = 2 + i$; $\overrightarrow{WF} = -\frac{1}{4}\overrightarrow{WA}$; $z_F - z_W = -\frac{1}{4}(z_A - z_W)$; $z_W = \frac{8}{5} + \frac{4}{5}i$.	1
3.b	$z' - z_W = \frac{1}{2}e^{i\frac{\pi}{2}}(z - z_W)$; $z' - \frac{8}{5} - \frac{4}{5}i = \frac{1}{2}i\,(z - \frac{8}{5} - \frac{4}{5}i)$; $z' = \frac{1}{2}i\,z + 2$.	1
3.c	$z' - z_W = -\frac{1}{4}(z - z_W)$; $z' = = -\frac{1}{4}z + 2 + i$. For z = 2 ; $z' = -\frac{1}{2} + 2 + i = \frac{3}{2} + i$.	1

IV		
A1.a	$P(E) = \frac{1}{2}\times\frac{C_4^3}{C_6^3} = \frac{1}{2}\times\frac{4}{20} = \frac{1}{10}$.	1/2
A1.b	P(F) = P(3R from B_1) + P(3 R from B_2) = $\frac{1}{10} + \frac{1}{2}\times\frac{C_4^3}{C_8^3} = \frac{1}{10} + \frac{1}{2}\times\frac{4}{56} = \frac{19}{140}$.	1
A2.a	Obtaining the black ball among the chosen 3 balls means that B_2 is chosen ,so 1 black and 2 non black are chosen from B_2 ; $p_1 = \frac{1}{2}\times\frac{C_1^1\times C_7^2}{C_8^3} = \frac{1}{2}\times\frac{21}{56} = \frac{3}{16}$.	1
A2.b	Obtaining 3 balls having different colors means that B_2 is chosen from which one ball of each color is chosen; $p_2 = \frac{1}{2}\times\frac{C_1^1\times C_4^1\times C_3^1}{C_8^3} = \frac{12}{2\times 56} = \frac{3}{28}$.	1
B	The possible values of X are 0 ; 1 ; 2 and 3.	2
	E(X) = $\frac{390}{364}$ = 1.07.	

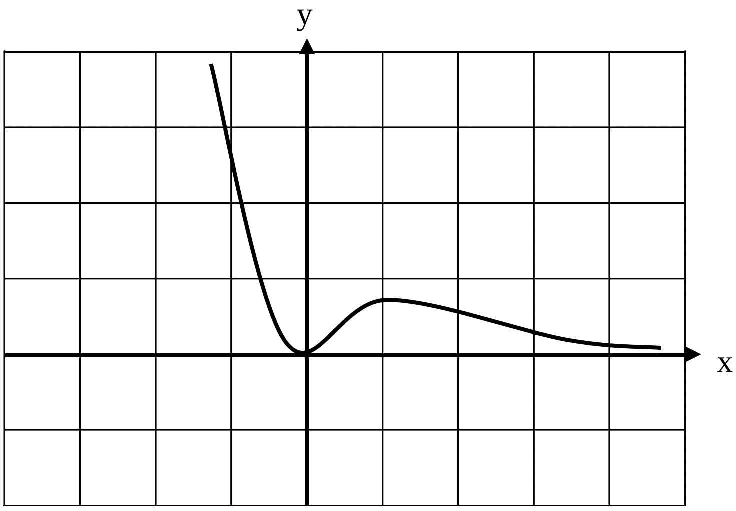
x_i	0	1	2	3
p_i	$\frac{C_9^3}{C_{14}^3} = \frac{84}{364}$	$\frac{C_5^1\times C_9^2}{C_{14}^3} = \frac{180}{364}$	$\frac{C_5^2\times C_9^1}{C_{14}^3} = \frac{90}{364}$	$\frac{C_5^3}{C_{14}^3} = \frac{10}{364}$

V		
1.a	$x^2 - 3y^2 = 3$. $\frac{x^2}{3} - y^2 = 1$. $a^2 = 3$ so A($\sqrt{3}$; 0) and A($-\sqrt{3}$; 0) are the vertices. $c^2 = a^2 + b^2 = 4$ then the foci are F(2 ; 0) and F'(– 2 ; 0) , and $e = \frac{c}{a} = \frac{2\sqrt{3}}{3}$.	1 1/2
1.b	The asymptotes of (H) have equations : $y = \frac{1}{\sqrt{3}}x$ and $y = \frac{-1}{\sqrt{3}}x$. Directrices : $x = \frac{a^2}{c} = \frac{3}{2}$; $x = -\frac{a^2}{c} = -\frac{3}{2}$.	1

		1/2
2	$V = \pi \int_{\sqrt{3}}^{2} y^2dx = \pi \int_{\sqrt{3}}^{2} (\frac{x^2}{3} - 1)dx = \pi \left[\frac{x^3}{3} - x \right]_{\sqrt{3}}^{2} = \frac{6\sqrt{3} - 10}{9}\ u^3$.	1
3	For $x = 2$, $y^2 = \frac{1}{3}$ and $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$, so $K(2 ; \frac{1}{\sqrt{3}})$ and $L(2 ; -\frac{1}{\sqrt{3}})$. Equation of tangent at K : $y = f'(x_K)(x - x_K) + f(x_K)$; $2x - 6yy' = 0$; $y' = \frac{x}{3y}$. $y = \frac{2}{\sqrt{3}}(x-2) + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}x - \sqrt{3}$. By symmetry , the equation of the tangent at L is $y = -\frac{2}{\sqrt{3}}x + \sqrt{3}$. The two asymptotes intersect at a point of abscissa $x = \frac{3}{2}$	1

VI		
A1	$y = z + 3x^2e^{-2x}$; $y' = z' + 3(2xe^{-2x} - 2x^2e^{-2x})$; $z' + 2z = 0$ (E') ; $z = Ce^{-2x}$.	1 1/2
A2	$y = Ce^{-2x} + 3x^2e^{-2x}$; $y(0) = 0$ gives $C = 0$ and $y = 3x^2e^{-2x}$.	1
B1a	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{3x^2}{e^{2x}} = 3 \lim_{x\to+\infty} \left(\frac{x}{e^x}\right)^2 = 0$; the axis of abscissas is an asymptote to (C) at ($+\infty$).	1
B1b	$\lim_{x\to-\infty} f(x) = +\infty$; $\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} 3xe^{-2x} = -\infty$. (C) has a vertical asymptotic direction (parallel to axis of ordinates).	1
B2a	$f'(x) = 6x(1-x)e^{-2x}$.	1

x	$-\infty$		0		1		$+\infty$
f '(x)		–	0	+	0	–	
f(x)	$+\infty$	↘	0	↗	$3e^{-2}$	↘	0

B2b	f ”(x) = $6e^{-2x}(2x^2 - 4x + 1)$. f ‘’ (x) vanishes twice, changing signs, at the points of abscissas $\frac{2-\sqrt{2}}{2}$ and $\frac{2+\sqrt{2}}{2}$, thus (C) has 2 points of inflection.	1
B3a		1 1/2
B3b	$me^{2x} = 3x^2$; $m = 3x^2e^{-2x}$. For m < 0 no roots . For m = 0 a double root. For $0 < m < 3e^{-2}$; three roots For $m = 3e^{-2}$ a simple root and a double root. For $m > 3e^{-2}$ one root.	1
B4a	F’(x) = f(x) gives : $(2ax + b)e^{-2x} - 2e^{-2x}(ax^2 + bx + c) = 3x^2e^{-2x}$; –2a = 3 ; 2a – 2b = 0 and b – 2c = 0 a = –3/2 ; b = – 3/2 and c = – 3/4 ; $F(x) = -\frac{3}{2}(x^2 + x + \frac{1}{2})e^{-2x}$.	1
B4b	$A = \left[-\frac{3}{2}(x^2 + x + \frac{1}{2})e^{-2x}\right]_{-1}^{0} = \frac{3}{4}(e^2 - 1)$ u^2.	1
B5a	$3e^{-2}$= 0.406 ; f(– 0.3) = 0.4919 > 0.406 and f(– 0.2) = 0.179 < 0.406 thus –0.3 < t < – 0.2. ⇨OR : f(– 0.3) – $3e^{-2}$=0.0859 > 0 and f(– 0.2) – $3e^{-2}$ = – 0.227 < 0.	1
B5b	$3t^2e^{-2t} = 3e^{-2}$; $t^2\frac{e^{-2t}}{e^{-2}} = 1$; $e^{-2t+2} = \frac{1}{t^2}$ $\left[e^{-(t-1)}\right]^2 = \frac{1}{t^2}$; $e^{-(t-1)} = -\frac{1}{t}$ (car t < 0) thus h(t) = t .	1
	x: – ∞ … 0 … 1 … + ∞ g’(x): – 0 + 0 – g(x): + ∞ ↘ 1 ↗ $e^{3e^{-2}}$ ↘ 1	1
B6b	g(x) = e ; $e^{f(x)}$ = e ; f(x) = 1 ; but (C) cuts the line of equation y = 1 in a single point thus the equation g(x) = e has a unique solution .	1/2
B6c	g(x) > 1 ; f(x) > 0 then x ≠ 0. ⇨OR : Use the table of variations of g.	1/2