وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة الإكمالية الإستثنائية للعام ٢٠٠٧
عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة : يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, M and M' of affixes i , z and z' respectively, where $z' = \dfrac{iz}{z-i}$ $(z \neq i)$.

1- Determine the points M such that $z' = z$.

2- If $z = \dfrac{\sqrt{2}}{2}e^{i\frac{3\pi}{4}}$, find an argument of z'.

3- Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

a) Calculate x' and y' in terms of x and y.

b) Determine the set of points M for which z' is real.

4- a) Show that $z' - i = \dfrac{-1}{z-i}$.

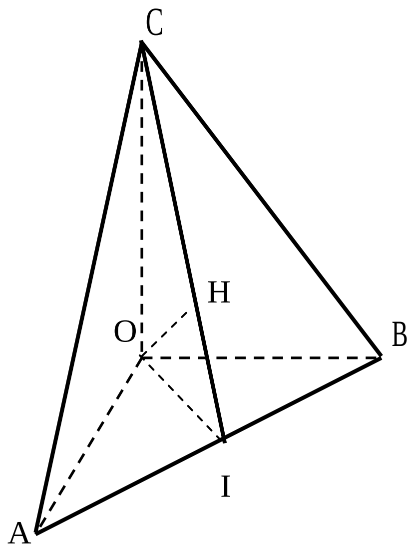
b) Show that when M moves on the circle (ω) of center A and radius 1 then M' moves on the same circle.

II- (4 points)

The adjacent figure is considered in a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$ where:

$\overrightarrow{OA} = \vec{i}$, $\overrightarrow{OB} = \vec{j}$ and $\overrightarrow{OC} = 2\vec{k}$.

Let I be the midpoint of [AB].



1- Justify that an equation of plane (ABC) is $2x + 2y + z - 2 = 0$.

2- Consider the point $H\left(\frac{4}{9};\frac{4}{9};\frac{2}{9}\right)$.

a) Show that C, H and I are collinear.

b) Prove that (OH) is perpendicular to the plane (ABC).

c) Prove that the two planes (OIC) and (ABC) are perpendicular.

3- a) Write a system of parametric equations of the straight line (Δ) passing through C and parallel to (OB).

b) Let F be a variable point on (Δ).
Prove that the tetrahedron FOAB has a constant volume to be calculated.

III- (4 points)

In a public library, every visitor has to either choose a book or use a computer.
70% of these visitors use the computer.

Out of those who use the computer, 45% do a research.
Out of the visitors who choose a book, 80% do a research.

A) We meet, at random, one of the visitors in this library.
Consider the following events:
C: « the visitor uses the computer».
B: « the visitor chooses a book».
R: « the visitor does a research».

1- Verify that the probability $P(C \cap R)$ is equal to 0.315 .
2- Calculate $P(B \cap R)$ then $P(R)$.
3- The visitor did a research, calculate the probability that he used the computer.

B) On a Monday morning, 30 persons visited this library. We choose, simultaneously and at random, three of these visitors. Designate by X the random variable that is equal to the number of visitors who used the computer among the three chosen visitors.
1- Determine the values of X.
2- Determine the probability distribution of X.

IV- (8 points)

Let f be the function that is defined, on $[0;+\infty[$, by $f(x) = (x+1)e^{-x}$ and designate by (C) its representative curve in an orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j})$. (unit: 2cm)

1- Calculate $\lim_{x \to +\infty} f(x)$ and deduce an asymptote of (C).
2- a) Calculate $f'(x)$ and set up the table of variations of f.
b) Calculate $f'(0)$ and interpret the result graphically.
3- a) Prove that the curve (C) has a point of inflection $W(1, \frac{2}{e})$.
b) Write an equation of the line (d) that is tangent to (C) at the point W.
4- Draw (d) and (C).
5- a) Calculate the real numbers a and b so that the function F defined on $[0;+\infty[$ by $F(x) = (ax+b)e^{-x}$ is an antiderivative of f.
b) Calculate, in cm^2, the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.
6- Let g be the inverse function of f and designate by (G) the representative curve of g.
a) Draw (G) in the preceding system.
b) Find an equation of the tangent to the curve (G) at the point of abscissa $\frac{2}{e}$.

معيار التصحيح

مسابقة في مادة الرياضيات

I- (4 points)

Part of the Q	Answer	Mark
1	$z = \frac{iz}{z-i}$ then z(z-2i) = 0 ; z = 0 or z = 2i , so M(0;0) or M(0;2).	0.5
2	$z=\frac{\sqrt{2}}{2}e^{i\frac{3\pi}{4}} = \frac{-1}{2}+\frac{i}{2}; z' = \frac{\frac{-1}{2}-\frac{i}{2}}{\frac{-1}{2}-\frac{i}{2}} = 1;\ \arg z' = 0(2\pi)$.	0.5
3.a	$z' = \frac{-x}{x^2+(y-1)^2} + i\frac{x^2+y^2-y}{x^2+(y-1)^2}; x' = \frac{-x}{x^2+(y-1)^2}$; $y' = \frac{x^2+y^2-y}{x^2+(y-1)^2}$.	0.5
3.b	z' is real then $x^2+y^2-y=0$; then $x^2+(y-\frac{1}{2})^2=\frac{1}{4}$ and $z\neq i$, so the set is a circle of center (0 , 1/2) and radius 1/2 excluding point A(0;1).	1
4.a	$z' = \frac{iz}{z-i}$ then $z'-i = \frac{iz}{z-i} - i$ thus $z'-i = \frac{-1}{z-i}$.	0.5
4.b	Since AM = 1 then $\|z-i\|=1$ then $\|z'-i\| = \left\|\frac{-1}{z-i}\right\| = \frac{\|-1\|}{1} = 1$, so AM' = 1 then M' moves on the same circle (ω).	1

II- (4 points)

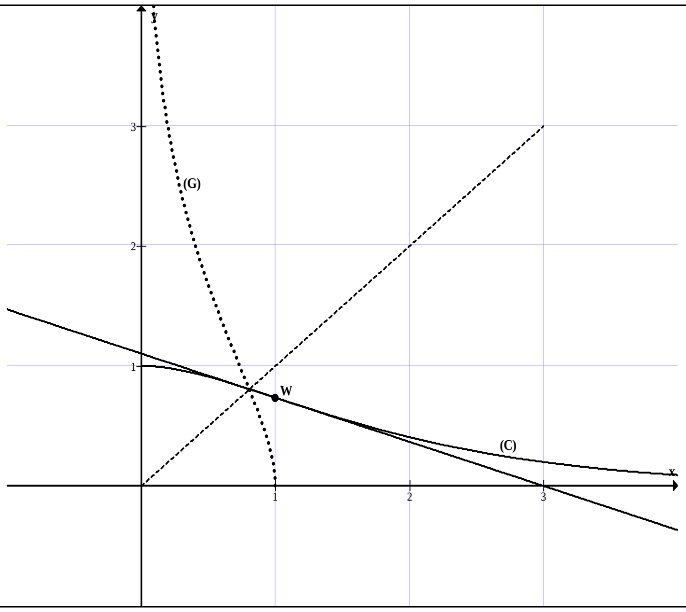
Part of the Q	Answer	Mark
1	The coordinates of A, B and C verify the given equation since: $2x_A+2y_A+z_A-2=2+0+0-2=0$. Also, $2x_B+2y_B+z_B-2=0+2+0-2=0$; and $2x_C+2y_C+z_C-2=0$.	0.5
2.a	$\vec{CH}\left(\frac{4}{9};\frac{4}{9};-\frac{16}{9}\right)$; $\vec{CI}\left(\frac{1}{2};\frac{1}{2};-2\right)$; so, $\vec{CH}=\frac{8}{9}\vec{CI}$, hence C , H and I are collinear.	0.5
2.b	$\vec{n}(2\ ;\ 2;\ 1)$ is a normal vector to plane (ABC), but $\vec{OH}\left(\frac{4}{9};\frac{4}{9};\frac{2}{9}\right)=\frac{2}{9}\vec{n}$, so $\vec{OH}$ is perpendicular to plane (ABC).	0.5

2.c	(OH) is perpendicular to plane (ABC) and (OH) $\subset$(OCI) so the plane (OCI) is perpendicular to plane (ABC). OR : $\vec{n'} = \vec{OI} \wedge \vec{OC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{vmatrix} = \vec{i} - \vec{j}$; $\vec{n'}$ is normal to plane(OIC) and $\vec{n} \cdot \vec{n'} = 0$	1
3.a	$\vec{OB}(0;1;0)$ is a direction vector of (Δ), and C is a point of (Δ). consequently, (Δ): $\begin{cases} x = 0 \\ y = t \\ z = 2 \end{cases}$	0.5
3.b	(Δ) // (OAB) , so the distance from F to (OAB) is constant hence the volume is constant. area of triangle OAB $= \frac{OA \times OB}{2} = 0.5u^2$ and d(F ;OAB) = OC = 2 Consequently V = $\frac{0.5 \times 2}{3} = \frac{1}{3}u^3$. ►**OR** : calculate $\vec{OF} \cdot \left(\vec{OA} \wedge \vec{OB}\right) = 2$ (independent of t), and V= $\frac{\left\|\vec{OF} \cdot \left(\vec{OA} \wedge \vec{OB}\right)\right\|}{6} = \frac{1}{3}u^3$	1

III- (4 points)

Part of the Q	Answer	Mark
A. 1	$P(C \cap R) = P(C).P(R/C) = (0.7)(0.45) = 0.315$	0.5
A. 2	$P(B \cap R) = P(B).P(R/B) = (0.3)(0.8) = 0.24$ $P(R) = P(C \cap R) + P(B \cap R) = 0.315 + 0.24 = 0.555$	1
A. 3	$P(C/R) = \frac{P(C \cap R)}{P(R)} = 0.567$	0.5
B.1	The values of X are : 0 ; 1 ; 2 ; and 3	0.5
B.2	**If the total number is 30, then there are 21 who use the computer.** $P(X = 0) = \frac{C_9^3}{C_{30}^3} = \frac{3}{145}$; $P(X = 1) = \frac{C_{21}^1 C_9^2}{C_{30}^3} = \frac{27}{145}$ $P(X = 2) = \frac{C_{21}^2 C_9^1}{C_{30}^3} = \frac{27}{58}$; $P(X = 3) = \frac{C_{21}^3}{C_{30}^3} = \frac{19}{58}$	1.5

IV- (8 points)

Part of the Q	Answer	Mark
1	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\frac{x+1}{e^{x}} = \lim\limits_{x\to+\infty}\frac{1}{e^{x}} = 0$, the axis of abscissas is an asymptote to (C) .	0.5
2.a	$f'(x) = -x\,e^{-x}$. x : 0 ; $+\infty$ f'(x) : 0 ; □ f(x) : 1 ↘ 0	1
2.b	f '(0) = 0. Then the tangent at A(0 ; 1) is parallel to x-axis.	1
3.a	$f''(x) = (x-1)e^{-x}$; $f''(x)$ vanishes for x = 1 and changes sign; consequently (C) has a point of inflection $W(1, \frac{2}{e})$.	1
3.b	$y-\frac{2}{e} = -\frac{1}{e}(x-1)$ or $y = -\frac{1}{e}x+\frac{3}{e}$	0.5
4		1
5.a	F'(x) = f(x) ; a – b – ax = x + 1 so a = –1 and b = – 2 .	1
5.b	$A = \int_{0}^{1} f(x)dx = \left[(-x-2)e^{-x}\right]_{0}^{1} = (2-\frac{3}{e})u^{2} = 0.896\ u^{2} = 0.896 \times 4\ cm^{2} = 3.58cm^{2}$.	1
6.a	Graph	0.5
6.b	By symmetry of (d) w.r.t first bisector: $x = -\frac{1}{e}y+\frac{3}{e}$ or y = -ex + 3. ► **OR :** $g'(\frac{2}{e}) = \frac{1}{f'(1)} = -e$; an equation of tangent at point $(\frac{2}{e}\ ; 1)$ to (G) is : $y-1 = -e(x-\frac{2}{e})$; $y = -ex+3$.	0.5