امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٦ العادية

مسابقة في مادة الفيزياء
المدة: ساعتان

الاسم:
الرقم:

This exam is formed of three exercises

<u>The use of a non-programmable calculator is recommended</u>

<u>First exercise</u> ***(6 ½ pts)*** ***<u>Verification of Newton's second law</u>***

A puck (S) of mass M = 100 g and of center of mass G, may slide along an inclined track that makes an angle α with the horizontal so that $\sin\alpha = 0.40$. Thus G moves along an axis *x'x* parallel to the track as shown in figure (1). Take $g = 10 \text{ m/s}^2$.

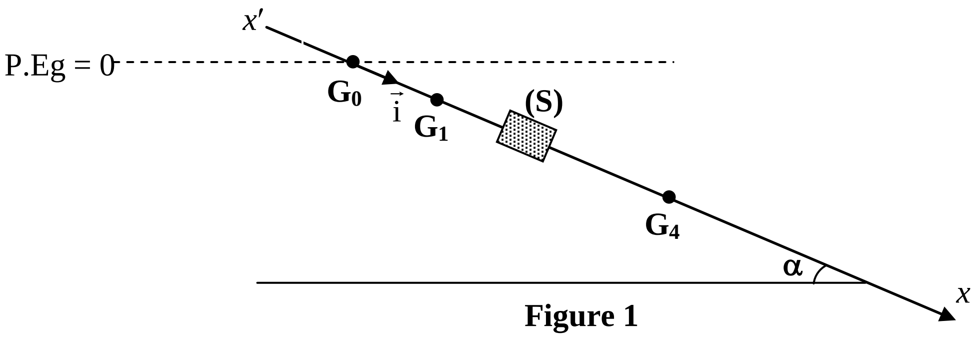


Figure 1

We release (S) without initial velocity at the instant $t_o = 0$ and at the end of each interval of time $\tau = 50$ ms, some positions G_o , G_1 , G_2 , …G_5 of G are recorded at the instants $t_o = 0$,t_1 , t_2 , ... t_5 respectively.

The values of the abscissa *x* of G ($x = \overline{G_oG}$) are given in the table below.

t	**0**	τ	**2**τ	**3**τ	**4**τ	**5**τ
x (cm)	**0**	**G_oG_1= 0.50**	**G_oG_2= 2.00**	**G_oG_3 = 4.50**	**G_oG_4 = 8.00**	**G_oG_5 =12.50**

1) Verify that the speed of the puck at the instants $t_2 = 2\tau$ and $t_4 = 4\tau$ are $V_2 = 0.40$ m/s and $V_4 = 0.80$ m/s respectively.

2) a) Calculate the mechanical energy of the system (puck-Earth) at the instants t_0, t_2 and t_4 knowing that the horizontal plane through G_0 is taken as a gravitational potential energy reference.

b) Why can we suppose that the puck moves without friction along the rail?

3) Determine the variation in the linear momentum $\overrightarrow{\Delta P} = \overrightarrow{P_4} - \overrightarrow{P_2}$ of (S) during $\Delta t = t_4 - t_2$.

4) a) Name the forces acting on (S) during its motion.

b) Show that the resultant $\Sigma\vec{F}$ of these forces may be written as $\Sigma\vec{F} = (Mg \sin\alpha)\ \vec{i}$.

5) Assuming that Δt is very small, $\dfrac{\Delta\vec{P}}{\Delta t}$ may be considered equal to $\dfrac{d\vec{P}}{dt}$. Show that Newton's second law is verified between the instants t_2 and t_4.

<u>Second exercise</u> (6 ½ pts) Measurement of the speed of a bullet

In order to measure the speed of a bullet, a convenient setup is used. The principle of functioning of this setup is based on the charging of a capacitor.

A- Study of the charging of a capacitor

We are going to study the charging of a capacitor using a series circuit formed of a resistor of resistance R, a switch K and a capacitor of capacitance C initially neutral across the terminals of a generator of constant emf E and of negligible internal resistance (figure 1).

The switch K is closed at the instant $t_0 = 0$. The capacitor starts to charge. At the instant t , the circuit carries a current i and the armature A of the capacitor carries the charge q.

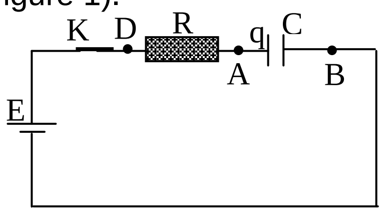


Figure 1

1) Applying the law of addition of voltages, determine the differential equation that describes the variation of the voltage $u_C = u_{AB}$ across the capacitor as a function of time.

2) **a)** Verify that $u_C = E(1 - e^{\frac{-t}{\tau}})$ is the solution of the differential equation where $\tau = RC$.

b) What does the time interval τ represent?

3) After what time would the steady state be practically attained?

B – Measurement of the speed of a bullet

The setup used to measure the speed V of a bullet is represented in figure 2.

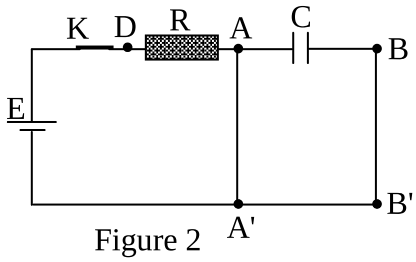


Figure 2

AA' and BB' are two thin parallel connecting wires lying in a vertical plane and are of negligible resistance . AA' and BB' are separated by a distance L.

Given: E = 100 V ; R = 1000 Ω ; C = 4 μF ; L = 1 m.

The capacitor being neutral, the switch K is closed.

1) **a)** The potential difference between A and A′ is zero. Why?

b) The charging of the capacitor did not start. Why?

2) K being closed , we shoot the bullet normally at AA' and BB' with a speed V (fig.3).

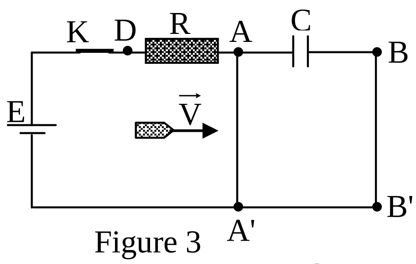


Figure 3

At the instant $t_0 = 0$, the bullet cuts the wire AA' and the capacitor starts to charge (fig.4).

The bullet continues its motion which is considered uniform rectilinear of the same speed V.

At the instant t_1, the bullet cuts the wire BB' and the phenomenon of charging stops. The voltage across the capacitor is then 45.7 V.

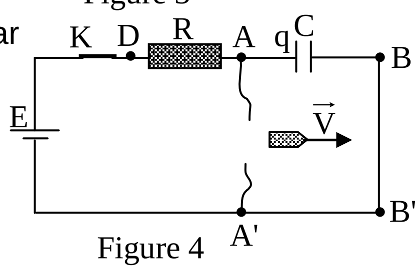


Figure 4

a) Taking into consideration the study in part A, determine the time interval t_1 taken by the bullet to cover the distance L.

b) Calculate V.

3) In order to measure precisely the value of V , the distance L between AA' and BB' must not exceed a maximum value L_{max}. Determine the value of L_{max} .

<u>Third exercise</u> (7 pts) Measurement of the age of Earth

One of the questions that preoccupied man long ago since he started to explore the universe was the age of Earth. As from 1905, Rutherford proposed a measurement of the age of minerals through radioactivity.

In 1956, Clair Paterson used the method (uranium - lead) to measure the age of a meteorite assuming that it originates from a planet that is formed approximately at the same time as that of Earth.

I – A radioactive family of uranium $^{238}_{92}\mathrm{U}$

Uranium 238, of radioactive period T= 4.5×10^9 y (year) is at the origin of a radioactive family leading finally to the stable lead isotope $^{206}_{82}\mathrm{Pb}$.

Each of these successive disintegrations is accompanied with the emission of an α particle or a β^- particle.

The diagram (Z, N) gives all the radioactive nuclei originating from the uranium $^{238}_{92}\mathrm{U}$ leading to the stable isotope $^{206}_{82}\mathrm{Pb}$. (**page 4**)

The tables (**page 4**) give the radioactive period of each nuclide.

1) In the first disintegration, a uranium nucleus $^{238}_{92}\mathrm{U}$ gives thorium nucleus $^{234}_{90}\mathrm{Th}$ and a particle denoted by $^{A_1}_{Z_1}\mathrm{X}$.

a) Write the equation of this disintegration and calculate the values of A_1 and Z_1.

b) Specify the type of radioactivity corresponding to this transformation.

2) In the second disintegration, the thorium nucleus $^{234}_{90}\mathrm{Th}$ undergoes a β^- decay. The daughter nucleus is the protactinium $^{A_2}_{Z_2}\mathrm{Pa}$. Calculate A_2 and Z_2.

3) a) Referring to the diagram, tell how many α particles and how many β^- particles are emitted when the uranium nucleus $^{238}_{92}\mathrm{U}$ is transformed into lead nucleus $^{206}_{82}\mathrm{Pb}$.

b) Write the overall nuclear equation of the decay of uranium 238 into lead 206.

4) Using the diagram (Z,N) of the figure and the tables, tell why after few billions of years we can neglect the presence of the intermediary nuclei among the products of the disintegration uranium-lead.

II- The age of Earth

We have studied a sample of a meteorite whose age is equal to that of Earth. At the instant t , the sample studied contains 1g of uranium 238 and 0.88g of lead 206.

We suppose that at the instant of its formation $t_0 = 0$, the meteorite does not contain any atom of lead.

Numerical data: molar mass of uranium: 238 g/mol; molar mass of lead: 206 g/mol;
Avogadro's number: $N_A = 6.02\times 10^{23}$ /mol.

1) Calculate, at the instant t :

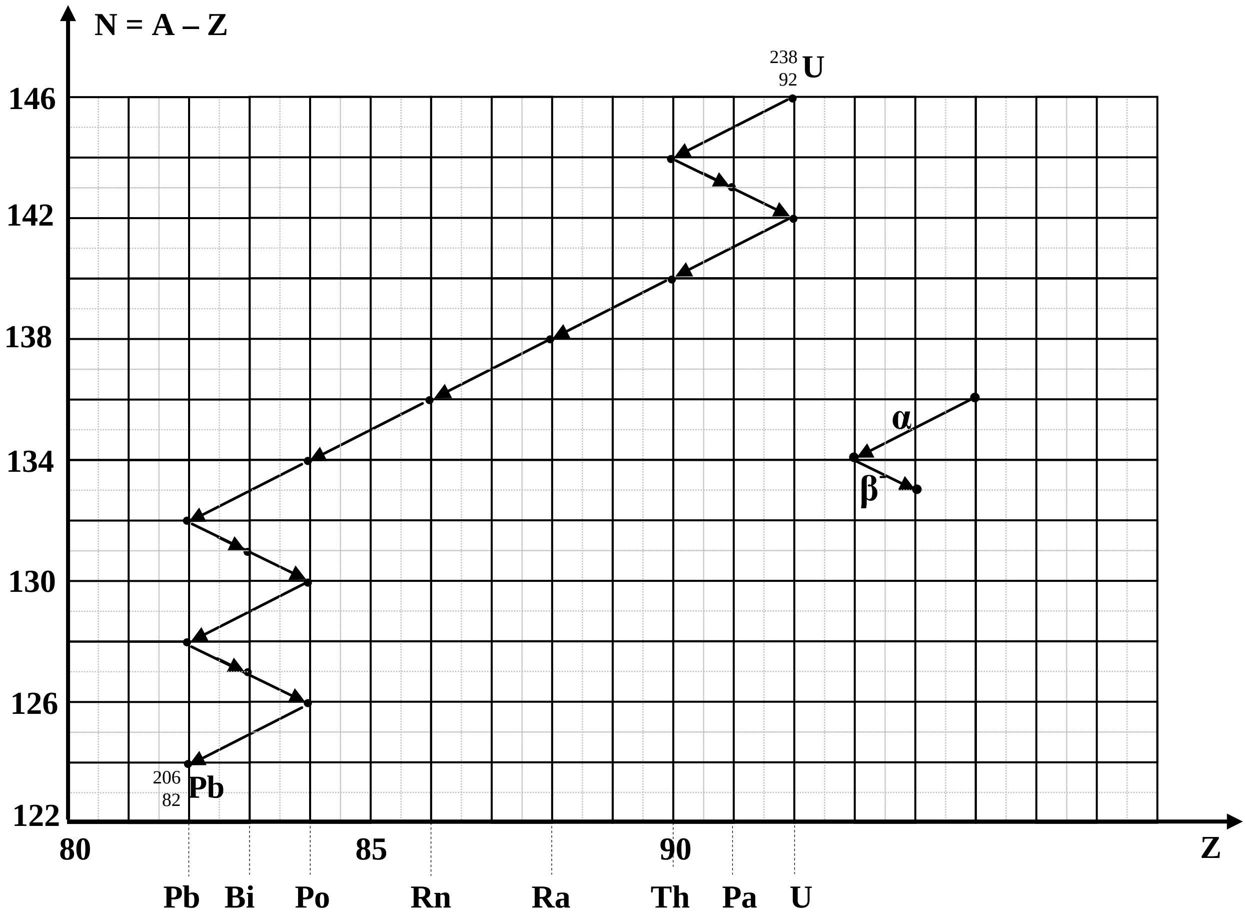
a) the number of uranium 238 nuclei , denoted by $N_U(t)$, present now in the sample;

b) the number of lead 206 nuclei , denoted by $N_{Pb}(t)$, present now in the sample.

2) Deduce the number of uranium 238 nuclei $N_U(0)$, present in the sample at the instant $t_0 = 0$.

3) Give the expression of $N_U(t)$ as a function of $N_U(0)$, t, and T.

4) Deduce the age of Earth at the instant t.



Nucleus	$^{238}_{92}U$	$^{234}_{90}Th$	$^{234}_{91}Pa$	$^{234}_{92}U$	$^{230}_{90}Th$	$^{226}_{88}Ra$	$^{222}_{86}Rn$
Radioactive period	4.5×10^{9} y	24 d	6.7 h	2.5×10^{5} y	7.5×10^{3} y	1.6×10^{3} y	3.8 d

Nucleus	$^{218}_{84}Po$	$^{214}_{82}Pb$	$^{214}_{83}Bi$	$^{214}_{84}Po$	$^{210}_{82}Pb$	$^{210}_{83}Bi$	$^{210}_{84}Po$
Radioactive period	3.1 min	27 min	20 min	1.6×10^{-4}s	22 y	5d	138 d

أسس التصحيح لشهادة علوم الحياة لمادة الفيزياء . دورة ٢٠٠٦ العادية

First exercise: (6 ½ pts)

1) $V_2 = \dfrac{G_1G_3}{2\tau} = \dfrac{G_0G_3 - G_0G_1}{2\tau} = \dfrac{(4.5-0.5)\times 10^{-2}}{0.1} = 0.4$ m/s. **($\frac{1}{2}$ pt)**

$V_4 = \dfrac{G_3G_5}{2\tau} = \dfrac{G_0G_5 - G_0G_3}{2\tau} = \dfrac{(12.5-4.5)\times 10^{-2}}{0.1} = 0.8$ m/s. **($\frac{1}{2}$ pt)**

2) a) M.E = K.E + $P.E_g$;

$M.E_0 = K.E_0 + PE_{g0} = 0 + 0 = 0$

$M.E_2 = K.E_2 + PE_{g2} = \frac{1}{2}\ M(V_2)^2 - Mgh_2$;

$h_2 = G_0G_2 \times \sin\alpha = 2 \times 0.4 = 0.8$ cm $= 0.008$ m $\Rightarrow M.E_2 = 0$ J.

$M.E_4 = K.E_4 + PE_{g4} = \frac{1}{2}\ M(V_4)^2 - Mgh_4$;

$h_4 = G_0G_4 \times \sin\alpha = 8 \times 0.4 = 3.2$ cm $= 0.032$ m $\Rightarrow M.E_4 = 0$ J. **(2 pts)**

b) $M.E_0 = M.E_2 = M.E_4 \Rightarrow$ the mechanical energy is conserved during motion $\Rightarrow$ No friction. **($\frac{1}{2}$ pt)**

3) $\Delta\vec{P} = \vec{P}_4 - \vec{P}_2 = M\,(V_4\vec{i} - V_2\vec{i}) = 0.04\vec{i}$ **($\frac{3}{4}$ pt)**

4) a) The forces acting on (S) :

The weight $\vec{W}$ of (S) and the normal reaction $\vec{N}$ of the path **($\frac{1}{4}$ pt)**

b) $\Sigma\vec{F} = \vec{W} + \vec{N} = \vec{W}_1 + \vec{W}_2 + \vec{N}$

where : $\vec{W}_1 = Mg\sin\alpha\,\vec{i}$, $\vec{W}_2 = -Mg\cos\alpha\,\vec{j}$,

$\vec{N} = N\,\vec{j}$; $\vec{W}_2 + \vec{N} = \vec{0}$ (no motion along y'y)

$\Rightarrow \Sigma\vec{F} = \vec{W}_1 = mg\sin\alpha\,\vec{i}$ **(1 pt)**

5) The 2nd Law of is given by : $\Sigma\vec{F} = \dfrac{d\vec{P}}{dt} = \dfrac{\Delta\vec{P}}{\Delta t}$.

We have: $\Sigma\vec{F} = mg\sin\alpha\,\vec{i} = 0.4\vec{i}$ and $\dfrac{\Delta\vec{P}}{\Delta t} = \dfrac{0.04\vec{i}}{0.1} = 0.4\,\vec{i}$

$\Rightarrow$ The 2nd Law of Newton is thus verified. **(1pt)**

Second exercise (6 ½ pts)

A- 1) $E = u_R + u_C \Rightarrow E = Ri + u_C = R\dfrac{dq}{dt} + u_C = RC\dfrac{du_C}{dt} + u_C$ **(1 pt)**

2) a) $\dfrac{du_C}{dt} = \dfrac{E}{RC}\,e^{\frac{-t}{\tau}} \Rightarrow E = RC \Rightarrow \dfrac{E}{RC}\,e^{\frac{-t}{\tau}} + E\,(1 - e^{\frac{-t}{\tau}}) = E$ **(1 pt)**

b) τ is the time taken for $u_c = 63\%$ E. **($\frac{1}{2}$ pt)**

3) The steady state is reached for t = 5RC. **($\frac{1}{2}$ pt)**

B - 1) a) because AA' is a connecting wire of negligible resistance. **($\frac{1}{4}$ pt)**

b) $u_C = u_{AA'} = 0$ **($\frac{1}{4}$ pt)**

2) a) We have $u_C = E\,(1 - e^{\frac{-t}{\tau}})$. We can write :

$1 - e^{\frac{-t}{\tau}} = \dfrac{u_C}{E} \Rightarrow e^{\frac{-t}{\tau}} = 1 - \dfrac{u_C}{E} \Rightarrow -\dfrac{t_1}{RC} = \ln\left(1 - \dfrac{u_C}{E}\right)$

$\Rightarrow t_1 = -RC \times \ln\left(1 - \dfrac{u_C}{E}\right) = -0.004 \times \ln\left(1 - \dfrac{45.7}{100}\right) = 2.44$ ms.

t_1 is the time of charging which is the same as the time taken by the bullet **(1$\frac{1}{2}$ pt)**

b) $V = \dfrac{L}{t_1} = \dfrac{1}{2.44\times 10^{-3}} = 410$ m /s. **($\frac{1}{2}$ pt)**

3) Because if this time exceeds 5RC, the steady state is attained and will no more vary. Thus $t \le 5RC \Rightarrow \dfrac{L}{V} \le 5RC$

$\Rightarrow L \le 5RCV \Rightarrow L \le 8.02$ m $\Rightarrow L_{max} = 8.02$ m. **(1 pt)**

Third exercice : (7 pts)

I-1) a) $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{A_1}_{Z_1}X$

$238 = 234 + A_1 \Rightarrow A_1 = 4$.

$92 = 90 + Z_1 \Rightarrow Z_1 = 2$ **(1pt)**

b) nucleus $^{A_1}_{Z_1}X$ is Helium $\Rightarrow$ The type of the radioactivity is α. **($\frac{1}{2}$ pt)**

2) $^{234}_{90}Th \rightarrow ^{0}_{-1}e + ^{A_2}_{Z_2}Pa$

$234 = 0 + A_2 \Rightarrow A_2 = 234$

$90 = -1 + Z_2 \Rightarrow Z_2 = 91$ **(1pt)**

3) **a)** There are 8 particles of α and 6 particles of β^-. **($\frac{3}{4}$ pt)**

b) $^{238}_{92}U \rightarrow ^{206}_{82}Pb + 8\ ^{4}_{2}He + 6\ ^{0}_{-1}e$ **($\frac{1}{2}$ pt)**

4) The radioactive period of each nucleus of the family is too small as compared to that of the nucleus of $^{238}_{92}U$. **($\frac{1}{4}$ pt)**

II - 1) a) $Nu(t) = \dfrac{N_A \times 1}{238} = 252941 \times 10^{16}$ nuclei **($\frac{1}{2}$ pt)**

b) $N_{Pb}(t) = \dfrac{N_A \times 0.88}{206} = 257165 \times 10^{16}$ nuclei **($\frac{1}{2}$ pt)**

2) $N_U(0) = N_U(t) + N_{Pb}(t) = 510106 \times 10^{16}$ nuclei **($\frac{3}{4}$ pt)**

3) $N_U(t) = N_U(0) \times e^{\frac{-0.693t}{T}}$ **($\frac{1}{2}$ pt)**

4) $t = \dfrac{T}{0.693} \times \ln \dfrac{N_U(0)}{N_U(t)} = 4.55 \times 10^9$ years.

The age of the Earth is 4.55×10^9 years **($\frac{3}{4}$ pt)**