وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة فرع العلوم العامة	دورة سنة ٢٠٠٦ العادية
عدد المسائل : ست	مسابقة في مادة الرياضيات المدة: اربع ساعات	الاسم: الرقم:

ملاحظـة : يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المُرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I – (1.5 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of each question and give, **with justification**, the answer corresponding to it.

Nº	Questions	Answers a	b	c	d
1	$z=-2e^{-i\frac{\pi}{6}}$. An argument of z is :	$\frac{-\pi}{6}$	$\frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{6}$
2	$\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^{12}=$	1	$2\sqrt{2}$	2^6	-1
3	$C_n^0+C_n^1+C_n^2+...+C_n^n=$	2^n	n!	n^2	2n
4	a is a natural integer. Consider the propositions: p : a is even. q : a $\geq$ 20. The proposition $\neg(p\wedge q)$ is :	a is odd and a < 20.	a is odd and $a\geq 20$	a is odd or a < 20	a is even or a < 20
5	If $F(x)=\int_1^x\sqrt{1+t^2}\,dt$, then $\lim\limits_{x\to 1}\frac{F(x)}{x-1}=$	1	0	$\sqrt{2}$	$+\infty$

II– (2.5points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider the plane (P) of equation $2x + y - 2z - 2 = 0$, and the points A (–1 ; 1 ; 3), B (1 ; 2 ; 1) and C(0 ;4 ;1).

1) Show that the line (AB) is perpendicular to plane (P) at B.

2) Let (T) be the circle, lying in the plane (P), of center B and of radius $\sqrt{5}$.
Show that the point C belongs to (T).

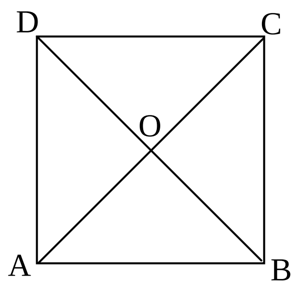
3) Write an equation of the plane (Q) that is determined by A, B and C.

4) Designate by (d) the line that is perpendicular to plane (Q) at C.
a- Give a system of parametric equations of (d).
b- Calculate the distance from A to (d).
c- Prove that the line (d) is tangent to the circle (C).

III– (3 points)

Consider, in an oriented plane, the direct square ABCD with center O such that $(\overrightarrow{AB},\overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.

Let r be the rotation with center O and angle $\frac{\pi}{2}$ and h be the dilation (homothecy) with center C and ratio 2.
Designate by S the transformation $r \circ h$.



1) Determine the nature of S and specify its ratio and its angle.

2) Designate by W the center of S.
a- Show that S(C) = D and that S(O) = B.
b- Construct the point W, specifying clearly the steps of this construction .

3) The plane is referred to an orthonormal system $(A\,;\,\overrightarrow{AB}\,,\,\overrightarrow{AD})$.
a- Write the complex form of S and deduce the affix of the center W.
b- Determine the image of the square ABCD under S .

IV– (2 points)

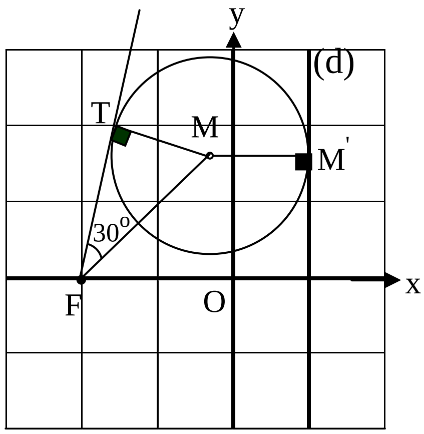
An urn contains **ten** balls: **five** white, **two** red and **three** green balls.

1) **Three** balls are drawn, simultaneously and randomly, from this urn.
Calculate the probability of each of the following events:
A : « **the three drawn balls have the same colour** »
B : « **at least one of the three drawn balls is red** »

2) **Two** balls are drawn randomly and successively from the given urn in the following manner :
If the first ball drawn is white, then it is replaced back in the urn after which a second ball is drawn.
But if the first ball is not white then it is kept outside the urn after which a second ball is drawn.
Designate by X the random variable that is equal to the number of times a white ball is drawn.
a- Show that $P(X=1) = \frac{19}{36}$.
b-Determine the probability distribution of X.

V– (3.5 points)

In the plane referred to an orthonormal system $(O; \vec{i}, \vec{j})$, consider the point F(–2 ; 0) and the line (d) of equation x = 1 .
Let (C) be a variable circle with center M such that:
- The line (d) is tangent to (C) at M'.
- (FT) is tangent to (C) at T.
- The angle TFM remains equal to 30º.



1) Prove that $\frac{MF}{MM'} = 2$ and deduce that M moves on a conic (H) whose nature, focus, directrix and eccentricity are to be specified.

2) Verify that the points O and A (4 ; 0) are the vertices of (H) and deduce the center and the second focus of (H).

3) a- Write an equation of (H) and determine its asymptotes.
b- Verify that the point B(6 ;6) belongs to (H) and write an equation of the line (Δ) that is tangent to (H) at the point B.
c- Draw (Δ) and (H).

4) Let (D) be the region that is bounded by the conic (H), the tangent (Δ) and the line of equation x = 4.
Calculate the volume generated by the rotation of (D) about the axis of abscissas.

VI– (7.5 points)

A- Consider the function g that is defined on IR by $g(x) = 3x + \sqrt{9x^2 + 1}$ and let (G) be its representative curve in an orthonormal system.

1) a- Calculate $\lim_{x \to +\infty} g(x)$ and show that the line of equation y = 6x is an asymptote of (G).

b- Show that the axis of abscissas is an asymptote to (G) at $-\infty$

2) Verify that g is strictly increasing on IR.

3) Draw the curve (G).

B- Consider the function f that is expressed by f(x) = ln (g(x)) and let (C) be its representative curve in a new orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Justify that the domain of f is IR.

b- Calculate f(x) + f(–x) and prove that O is a center of symmetry of (C).

2) a- Verify that $f'(x) = \dfrac{3}{\sqrt{9x^2 + 1}}$.

b- Find an equation of the line (d) that is tangent at O to (C).

c- Show that O is a point of inflection of (C).

3) a- Calculate $\lim_{x \to +\infty} f(x)$ and verify that $\lim_{x \to +\infty} \dfrac{f(x)}{x} = 0$.

Deduce $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to -\infty} \dfrac{f(x)}{x}$

b- Set up the table of variations of f.

4) a- Draw the line (d) and the curve (C).

b- The equation f(x) = x has three roots, one of them is a positive number α .

Show that 2.7 < α < 2.9 .

5) a- Prove that the function f has, on its domain, an inverse function h and plot (H), the representative curve of h, in the system $(O; \vec{i}, \vec{j})$.

b- Show that h(x) = $\dfrac{1}{6}(e^{x} - e^{-x})$.

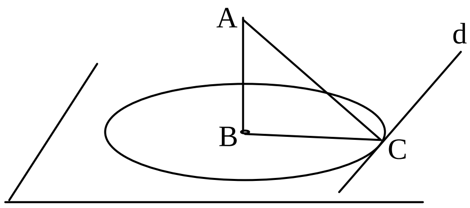
6) Suppose that α = 2.8 .

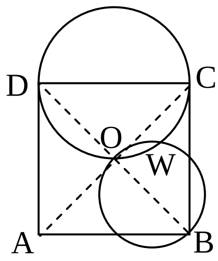
Calculate the area of the two regions that are bounded by (C) and (H).

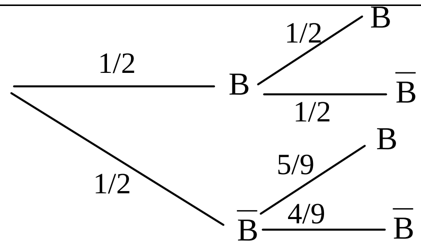
PREMIERE SESSION 2006

MATHEMATIQUES SG

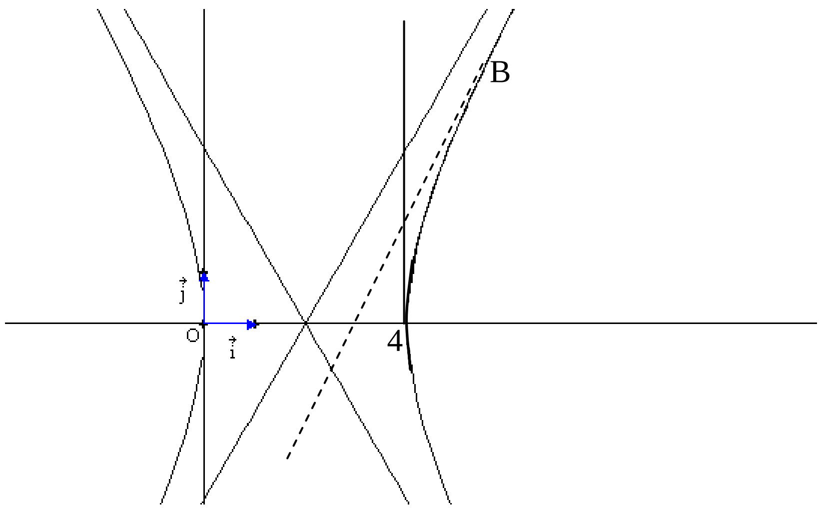
I	Eléments des réponses		Notes
1	$Z = -2\, e^{-i\frac{\pi}{6}} = 2e^{i(-\frac{\pi}{6}+\pi)} = 2e^{i\frac{5\pi}{6}}$	d	3
2	$\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^{12} = \left(e^{i\frac{\pi}{4}}\right)^{12} = e^{i(3\pi)} = -1$	d	
3	$C_n^0 + C_n^1 + \ldots + C_n^n = (1+1)^n = 2^n$	a	
4	$\urcorner (p \wedge q)$ est équivalente à $(\urcorner p) \vee (\urcorner q)$ $\urcorner (p \wedge q)$: a est impair ou a < 20	c	
5	$\lim\limits_{x\to 1} \frac{F(x)}{x-1} = \lim\limits_{x\to 1} \frac{F(x)-F(1)}{x-1} = F'(1) = f(1) = \sqrt{2}$ avec $f(x) = \sqrt{1+x^2}$. ➔OU : $\lim\limits_{x\to 1} \frac{F(x)}{x-1} = \frac{0}{0}$; D'après la règle de l'Hôspital ; $\lim\limits_{x\to 1} \frac{F(x)}{x-1} = F'(1) = f(1) = \sqrt{2}$	c	

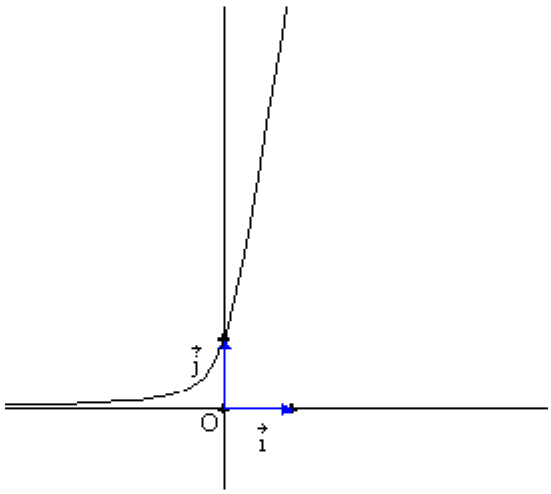
II	Eléments des réponses	Notes
1	$\vec{AB}(2\,;1\,;-2)$, $\vec{N}_P(2\,;1\,;-1)$ donc $\vec{AB} = \vec{N}_P$ avec B est un point de (P) car $2x_B + y_B - 2z_B - 2 = 0$	½
2	$C \in (P)$ car $2x_C + y_C - 2z_C - 2 = 0$; $\vec{BC}(-1\,;2\,;0)$; $BC = \sqrt{5} = R$ donc $C \in (T)$.  	1
3	$\vec{AM}.(\vec{AB} \wedge \vec{AC}) = 0$; $\begin{vmatrix} x+1 & y-1 & z-3 \\ 2 & 1 & -2 \\ 1 & 3 & -2 \end{vmatrix} = 0$; (Q) : $4x + 2y + 5z - 13 = 0$.	1
4.a	$\vec{CM} = t\,\vec{N}_Q$; (d) : $x = 4t$, $y = 2t + 4$, $z = 5t + 1$.	½
4b	$(d) \perp (ABC)$ donc $(d) \perp (AC)$ et $d(A\,;(d)) = AC = \sqrt{14}$.	1
4.c	$(d) \subset (P)$ car $8t + 2t + 4 - 10t - 2 - 2 = 0$ $(d) \perp (ABC)$ donc $(d) \perp (BC)$ (OU $\vec{V}_d . \vec{BC} = 0$) d'où (d) est tangente à (T).	1

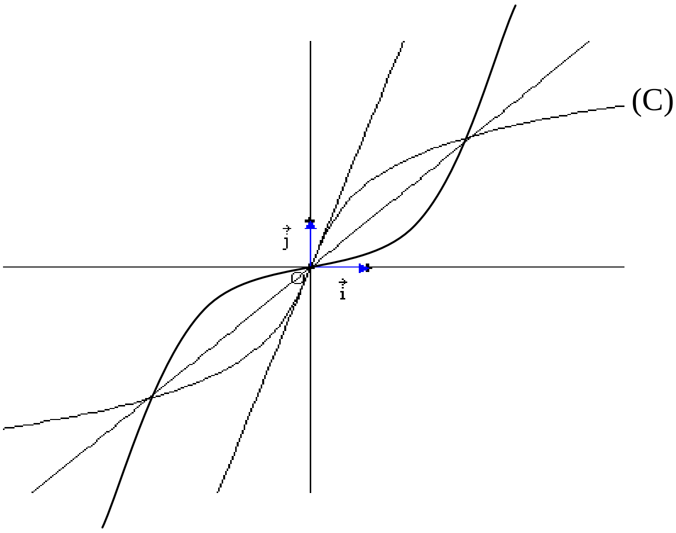
III	Eléments des réponses	Notes
1	S = r o h S est la composée d'une homothétie positive (de rapport 2) et d'une rotation (d'angle $\frac{\pi}{2}$) , c'est donc une similitude de rapport 2 et d'angle $\frac{\pi}{2}$. 	1
2.a	S(C) = r o h (C) = r(h(C)) = r(C) = D ; S(O) = r o h (O) = r(A) = B.	1
2.b	$(\overrightarrow{WC},\overrightarrow{WD}) = \frac{\pi}{2}$ et $(\overrightarrow{WO},\overrightarrow{WB}) = \frac{\pi}{2}$ d'où W est un point commun aux deux cercles de diamètres respectifs [DC] et [OB] ; ces deux cercles ont en commun deux points dont l'un d'eux est O, or S(O) $\neq$ O onc le centre W est le second point.	1 ½
3.a	z ' = az + b avec a = $2e^{i\frac{\pi}{2}} = 2i$; z ' = 2iz + b or S(C) = D donc $z_D = 2iz_C + b$; i = 2i(1+i) + b ; b = – i + 2 d'où z ' = 2iz +2 – i. $z_W = 2iz_W + 2 - i$; $z_W = \frac{2-i}{1-2i} = \frac{4}{5} + \frac{3}{5}i$.	1 ½
3.b	S(C) = D et S(O) = B . Le transformé du carré ABCD est le carré direct de centre B et dont l'un des sommets est D. ➔OU : S(A) = A' avec $z_{A'} = 2 - i$; S(B) = B' avec $z_{B'} = 2 + i$ S(C) = D et S(D) = D' avec $z_{D'} = -i$. le transformé de ABCD est A' B' D D'.	1

IV	Eléments des réponses	Notes
1	P(A) = P(BBB) + P(VVV) = $\frac{C_5^3 + C_3^3}{C_{10}^3} = \frac{11}{120}$. P(B) = P($R\overline{R}\overline{R}$)+ P(RR $\overline{R}$) = $\frac{C_2^1.C_8^2 + C_2^2.C_8^1}{C_{10}^3} = \frac{64}{120} = \frac{8}{15}$. ➔OU : P(B) = 1 – P($\overline{R}\ \overline{R}\,\overline{R}$) = 1 – $\frac{C_8^3}{C_{10}^3}$ =1 – $\frac{56}{120} = \frac{8}{15}$.	1 ½
2.a	 P(X = 1) = p($B \cap \overline{B}$) + p($\overline{B} \cap B$) = $\frac{1}{4} + \frac{5}{18} = \frac{19}{36}$	1
2.b	see table below	1 ½

x_i	0	1	2
p_i	$\frac{4}{18} = \frac{8}{36}$	$\frac{19}{36}$	$\frac{1}{4} = \frac{9}{36}$

VI	Eléments des réponses	Notes
1	$\frac{MF}{MM'} = \frac{MF}{MT}$; le triangle MTF est semi-équilatéral donc $\frac{MF}{MT} = \frac{1}{\sin \hat{F}} = \frac{1}{1/2} = 2$, d'où $\frac{MF}{MM'} = 2$ et M décrit l'hyperbole (H) de foyer F , de directrice (d) et d'excentricité 2.	1 ½
2.a	(OF) est l'axe focal ; $\frac{OF}{OK} = \frac{AF}{AK} = 2$ (K : intersection de (d) avec x'x)donc O et A sont les sommets de (H). Le centre I est le milieu de [OA] d'où I(2 ; 0) . I est le milieu de [FF'] d'où F'(6 ; 0).	1 ½
3.a	M(x ; y) , M'(1 ; y) , F(– 2 ; 0) $M \in (H)$ ssi $\frac{MF}{MM'} = 2$; $MF^2 = 4\,MM'^2$; $(x + 2)^2 + y^2 = 4(x – 1)^2$; $3x^2 – y^2 – 12x = 0$ ➜**OU** : 2a = OA = 4 ; a = 2 ; 2c = FF' = 8 ; c = 4 donc $b^2 = c^2 – a^2 = 12$ I(2 ; 0) est le centre de (H), donc (H) : $\frac{(x-2)^2}{4} - \frac{y^2}{12} = 1$ Asymptotes : $\frac{(x-2)^2}{4} - \frac{y^2}{12} = 0$; $y = \sqrt{3}\,(x – 2)$ ou $y = – \sqrt{3}\,(x – 2)$.	1
3.b	Pour x = 6 et y = 6 on a $\frac{(6-2)^2}{4} - \frac{6^2}{12} = 1$, d'où B est un point de (H). Equation de (Δ) : $\frac{(x-2)(x_B-2)}{4} - \frac{yy_B}{12} = 1$; y = 2x – 6. ➜**OU** : 6x – 2yy' – 12 = 0 ; $y' = \frac{3(x-2)}{y}$; $y'_B = 2$; $y – y_B = 2\,(x – x_B)$; y =2x – 6.	1
3c		1
4	$V = \pi\int_4^6 [4(x-3)^2 - (3x^2 - 12x)]dx = \pi\left[\frac{4}{3}(x-3)^3 - x^3 + 6x^2\right]_4^6 = \frac{8\pi}{3}u^3.$	1

VI	Eléments des réponses	Notes
A.1.a	$g(x) = 3x + \sqrt{9x^2+1}$ $\lim\limits_{x\to+\infty} g(x) = +\infty$; lorsque $x \to +\infty$, g(x) se comporte comme y = 3x + 3 |x| c.à.d y = 6x donc la droite d'équation y = 6x est une asymptote à (G). ➜**OU** : $\lim\limits_{x\to+\infty} (g(x)-6x) = \lim\limits_{x\to+\infty} (\sqrt{9x^2+1}-3x) = \lim\limits_{x\to+\infty} \frac{9x^2+1-9x^2}{\sqrt{9x^2+1}+3x} = 0.$	1
A.1.b	$\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} \frac{(3x+\sqrt{9x^2+1})(3x-\sqrt{9x^2+1})}{3x-\sqrt{9x^2+1}} = \lim\limits_{x\to-\infty} \frac{-1}{3x-\sqrt{9x^2+1}} = 0.$ D'où l'axe des abscisses est une asymptote à (G). ➜**OU** : lorsque $x \to -\infty$, g(x) se comporte comme y = 3x + 3 |x| c.à.d y = 0	1
A.2	$g'(x) = 3 + \frac{9x}{\sqrt{9x^2+1}}$ g '(x) = 0 pour $\frac{9x}{\sqrt{9x^2+1}} = -3$ c.à.d $81x^2 = 9(9x^2+1)$ avec $x \le 0$, soit 0 = 9 ce qui est impossible. g '(x) est continue sur IR et ne s'annulant pas, garde un signe constant celui de g '(0) = 3, donc g '(x) > 0 pour tout réel x. ➜OU : pour $x \ge 0$, g '(x) > 0 pour x < 0, supposons que g '(x) > 0 ; $\frac{9x}{\sqrt{9x^2+1}} > -3$; $\frac{81x^2}{9x^2+1} < 9$; 0 < 9 (vrai) d'où g '(x) > 0 pour tout réel x et g est strictement croissante.	1
A.3		1
B.1.a	g(x) > 0 pour tout x car (G) est au–dessus de l'axe des abscisses d'où f est définie sur IR.	½
B.1.b	$f(x) + f(-x) = \ln(3x + \sqrt{9x^2+1}) + \ln(-3x + \sqrt{9x^2+1})$ $= \ln(9x^2+1-9x^2) = \ln 1 = 0.$ f(– x) = – f(x) d'où f est impaire et O est un centre de symétrie de (C).	1
B.2.a	$f'(x) = \frac{g'(x)}{g(x)} = \frac{3\sqrt{9x^2+1}+9x}{\sqrt{9x^2+1}} \times \frac{1}{3x+\sqrt{9x^2+1}} = \frac{3}{\sqrt{9x^2+1}}.$	½

B.2.b	$y = x\, f'(0) = 3x$	½
B.2.c	$f''(x) = \dfrac{-27x}{(9x^2+1)\sqrt{9x^2+1}}$; f''(x) s'annule pour x = 0 en changeant de signe, O est un point d'inflexion de (C).	1
B.3.a	$\lim\limits_{x\to+\infty} f(x) = \ln(+\infty) = +\infty$; $\lim\limits_{x\to+\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to+\infty} \dfrac{f'(x)}{1} = \lim\limits_{x\to+\infty} \dfrac{3}{\sqrt{9x^2+1}} = 0$ $\lim\limits_{x\to-\infty} f(x) = -\infty$ et $\lim\limits_{x\to-\infty} \dfrac{f(x)}{x} = 0$ car O est un centre de symétrie de (C).	1
B.3.b	f '(x) > 0 x : $-\infty$ … $+\infty$ f '(x) : + f(x) : $-\infty$ ↗ $+\infty$	½
4.a		1 ½
4.b	f(2,7) = 2,78 > 2 , 7 et f(2,9) = 2,85 < 2,9 donc $2,7 < \alpha < 2,9$ car f est continue et strictement croissante sur R.	1
B.5.a	f est continue et strictement croissante sur R, elle admet une fonction réciproque h.	1
B.5.b	$6y = e^x - \dfrac{1}{e^x}$; $e^{2x} - 6ye^x - 1 = 0$; $(e^x - 3y)^2 = 9y^2 + 1$; $e^x = 3y + \sqrt{9y^2+1}$ (> 0) ou $e^x = 3y - \sqrt{9y^2+1} < 0$ (impossible) ; $x = \ln(3y + \sqrt{9y^2+1}) = f(y)$	1
6.	A = 4(aire comprise entre (H) et les droites d'équations y = x, x = 0 et x =α). $A = 4\int_0^{2,8}\left[x - \frac{1}{6}(e^x - e^{-x})\right]dx = 4\left[\frac{x^2}{2} - \frac{1}{6}\left(e^x + e^{-x}\right)\right]_0^{2,8}$ $= 4\left[\frac{(2,8)^2}{2} - \frac{1}{6}(e^{2,8} + e^{-2,8}) + \frac{1}{3}\right] \cong 6,009\ u^2$.	1 ½