وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	الدورة الإكمالية الإستثنائية للعام ٢٠٠٧

عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

The table below shows the percentage of damaged harvest in a certain village, in the even years 1982, 1984 . . . till 1994.

Year	1982	1984	1986	1988	1990	1992	1994
Rank of the year x_i	1	2	3	4	5	6	7
Percentage y_i	3.5	3.8	4.6	6.5	6.9	7.8	9

1- Calculate the means $\overline{X}$ and $\overline{Y}$ of the variables x and y.

2- Represent graphically the scatter plot of the points (x_i , y_i) as well as the center of gravity $G(\overline{X} , \overline{Y})$ in a rectangular system.

3- Calculate the correlation coefficient r and give an interpretation of the value thus obtained.

4- Determine an equation of $D_{y/x}$, the line of regression of y in terms of x, and draw this line in the preceding system.

5- Suppose that the above pattern remains valid till the year 2010.
Estimate the percentage of the damaged harvest in the year 2002.

6- In fact, the percentage of the damaged harvest was in reality 13 in the year 2002.
What is the percentage of error in the preceding estimation?

II- (4 points)

The manager of a sports club declares that, every year, 75% of the members of the club renew their membership and 800 new members join the club.
In 2005, the club had 1600 members.
Denote by u_n the number of members in the club in the year (2005 + n).

1- Verify that $u_1 = 2000$ and calculate u_2.

2- Show that $u_{n+1} = \frac{3}{4}u_n + 800$, for all natural integers n.

3- Consider the sequence (v_n) defined by: $v_n = 3200 - u_n$, for all natural integers n.
a) Show that (v_n) is a geometric sequence. Specify its common ratio and its first term.
b) Express v_n ,then u_n , in terms of n.

4- Assuming that the development of the number of members continues according to the given pattern, is it possible for the number of members to double?

III- (4 points)

An urn contains: four white balls each carrying the number 5 and three black balls each carrying the number 2. A game starts by drawing randomly one ball from this urn. If the drawn ball is white then the game ends, but if it is black then a second ball is drawn from the urn without replacing the previously drawn ball, and so on. The game continues on like this till a white ball appears and the game ends.

1- Calculate the probability that the game ends right after the second draw.

Let X be the random variable that is equal to the sum of numbers carried by the drawn balls.

2- Justify that the values of X are 5, 7, 9 and 11.

3- Prove that $P(X= 9) = \frac{4}{35}$.

4- Determine the probability distribution for X.

5- Calculate the expected value E(X).

IV- (8 points)

Consider the function f defined on $[0 ;+\infty[$ by $f(x) = 6(x - 2)e^{-0.5x} - 1$.
Let (C) be the representative curve of f in an orthonormal system.

A) 1- Calculate $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote of (C).

2- Verify that $f'(x) = 6e^{-0.5x}(2 - 0.5x)$ and set up the table of variations of f.

3- Draw (C).

4- Prove that the equation f(x) = 0 has two solutions α and β such that $2.6 < \alpha < 2.7$ and $6.6 < \beta < 6.7$.

B) In all what follows let $\alpha = 2.65$.

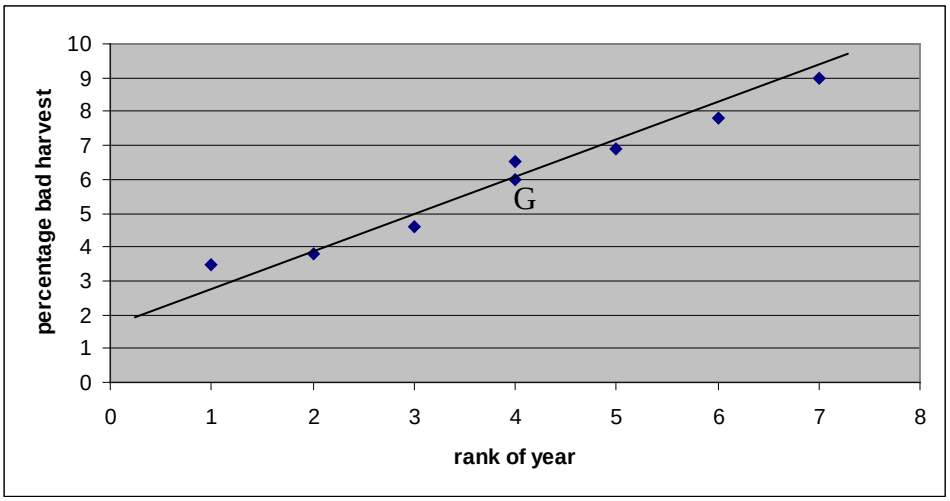
A factory produces a chemical product.
The total cost function is given by $g(x) = \frac{5}{1+12e^{-0.5x}}$, where x is expressed in tons and g(x) is expressed in millions LL $(0.5 \le x \le 6)$.

1- Precise the value of the fixed costs.

2- Determine the average cost function and the marginal cost function.

3- We admit that the average cost is minimum when it is equal to the marginal cost. Determine the level of production for which the average cost is minimum, and specify the value of this minimum.

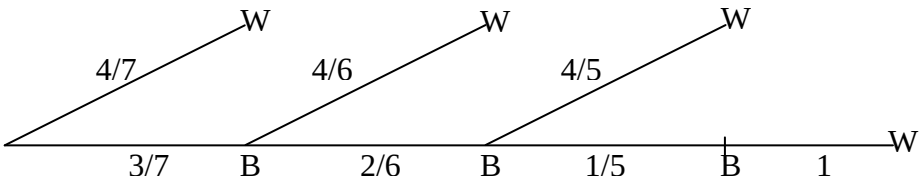
I - (7 points)

Part of the Q	Answer	Mark
1	$\overline{X} = 4$ and $\overline{Y} = 6.014$ (Using a calculator).	0.5
2	Scatter plot 	1
3	r = 0.98 . There is a strong positive correlation between x and y	1
4	a = 0.957 and b = 2.185 (using calculator) $D_{y/x}$: y = 0.957 x + 2.185	1.5
5	In the year 2002, x = 11 hence y = 0 .957 (11) + 2.185 = 12.712 Hence 12.712% of the harvest was damaged.	2
6	The estimation error is $\frac{13-12.712}{13} = 0.022$ so 2.2%	1

II - (7 points)

Part of the Q	Answer	Mark
1	$u_1 = 1600 \times \frac{75}{100} + 800 = 2000$ since $u_0 = 1600$. $u_2 = 2000 \times \frac{75}{100} + 800 = 2300$.	1.5
2	$u_{n+1} = u_n \times \frac{75}{100} + 800 = \frac{3}{4}u_n + 800$	0.5
3-a	$v_{n+1} = 3200 - u_{n+1} = 3200 - \left(\frac{3}{4}u_n + 800\right) = 2400 - \frac{3}{4}u_n = \frac{3}{4}(3200 - u_n) = \frac{3}{4}v_n$. So (v_n) is a geometric sequence of common ratio $\frac{3}{4}$, of first term $v_0 = 3200 - 1600 = 1600$.	2
3-b	$v_n = 1600 \times \left(\frac{3}{4}\right)^n$ and $u_n = 3200 - 1600 \times \left(\frac{3}{4}\right)^n$.	1
4	$u_n = 2 \times 1600 = 3200 - 1600 \times \left(\frac{3}{4}\right)^n$. Thus, $0 = -1600 \times \left(\frac{3}{4}\right)^n$ which is impossible since $\left(\frac{3}{4}\right)^n > 0$ for any natural integer n.	2

III - (7 points)

<table>
<tr><th>Part of the Q</th><th>Answer</th><th>Mark</th></tr>
<tr><td>1</td><td> 

Game ends after 2nd draw only if a black ball then a white ball are drawn, so $p = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$.</td><td>1.5</td></tr>
<tr><td>2</td><td>If game ends at first draw then X=5; if it ends at 2nd draw then X=2+5=7 ; at the 3rd then X=2+2+5=9 and at the 4th then X=2+2+2+5=11.</td><td>1.5</td></tr>
<tr><td>3</td><td>P(X=9)=P(BBW)=$\frac{3}{7} \times \frac{2}{6} \times \frac{4}{5} = \frac{4}{35}$.</td><td>1</td></tr>
<tr><td>4</td><td><table>
<tr><td>X= x_i</td><td>5</td><td>7</td><td>9</td><td>11</td></tr>
<tr><td>p_i</td><td>$\frac{4}{7}$</td><td>$\frac{2}{7}$</td><td>$\frac{4}{35}$</td><td>$\frac{1}{35}$</td></tr>
</table></td><td>2</td></tr>
<tr><td>5</td><td>E(X)=$\frac{100 + 70 + 36 + 11}{35} = 6.2$.</td><td>1</td></tr>
</table>

IV -(14 points)

<table>
<tr><th>Part of the Q</th><th>Answer</th><th>Mark</th></tr>
<tr><td>A1</td><td>$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} 6xe^{-0.5x} - \lim\limits_{x\to+\infty} 12e^{-0.5x} - 1 = 0 - 0 - 1 = -1.$
So (C) admits the line of equation y = –1 as an asymptote.</td><td>1.5</td></tr>
<tr><td>A2</td><td>$f'(x) = 6[e^{-0.5x} - 0.5(x-2)e^{-0.5x}] = 6e^{-0.5x}(1 - 0.5x + 1) = 6e^{-0.5x}(2 - 0.5x).$
<table>
<tr><td>x</td><td>0 4 $+\infty$</td></tr>
<tr><td>$f'(x)$</td><td>+ 0 --</td></tr>
<tr><td>$f(x)$</td><td>-13 ↗ $12e^{-2} - 1$ ↘ -1</td></tr>
</table></td><td>2.5</td></tr>
<tr><td>A3</td><td>Curve</td><td>2</td></tr>
<tr><td>A4</td><td>The curve cuts the axis of abscissas in two points, hence the equation $f(x)=0$ has two solutions α and β.
$f(2.6) = -0.018 < 0$ and $f(2.7) = 0.088 > 0$ thus $2.6 < \alpha < 2.7$.
$f(6.6) = 0.0179 > 0$ and $f(6.7) = -0.01 < 0$ thus $6.6 < \beta < 6.7$.</td><td>2.5</td></tr>
<tr><td>B1</td><td>$f(0) = \frac{5}{13} = 0.385$. The fixed costs amount to 385 000LL.</td><td>1</td></tr>
<tr><td>B2</td><td>The average cost is $\overline{C}(x) = \frac{5}{x(1+12e^{-0.5x})}$.
The marginal cost is $M_C(x) = g'(x) = \frac{30e^{-0.5x}}{(1+12e^{-0.5x})^2}$.</td><td>2</td></tr>
<tr><td>B3</td><td>$\overline{C}(x) = M_C(x)$ gives $\begin{cases} \frac{5}{x(1+12e^{-0.5x})} = \frac{6e^{-0.5x}}{(1+12e^{-0.5x})^2} \times 5 \\ (1+12e^{-0.5x}) = 6xe^{-0.5x} \quad (0.5 \le x \le 6) \\ 6(x-2)e^{-0.5x} - 1 = 0 \end{cases}$
The minimum average cost is obtained for α tons, that is 2.65 tons.
This minimum is equal to 0.450 million LL.</td><td>2.5</td></tr>
</table>