وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع الاجتماع والاقتصاد

دورة سنة ٢٠٠٦ ألعادية

عدد المسائل : اربع

مسابقة في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I – **(4points)**

The development in the number of subscribers, **in hundreds**, of a network chain during the last 6 years is as shown in the following table:

Year	2000	2001	2002	2003	2004	2005
Rank of the year: x_i	1	2	3	4	5	6
Number of **hundreds** of subscribers: y_i	5	8	12	15	20	24

1) Draw, in a rectangular system, the scatter plot of the points associated to the distribution $(x_i ; y_i)$.
2) Calculate the coordinates of the center of gravity G and plot this point in the preceding system.
3) Determine an equation of $D_{y/x}$, the line of regression of y in terms of x, and draw this line in the same system.
4) Suppose that the above pattern remains valid till the year 2015.
   a- Estimate the number of subscribers of this chain in 2007.
   b- During which year would the number of subscribers of this chain exceed 4000 for the first time?

II- **(4points)**

A jeweler has, in his safe, **30** identical boxes each containing either a **necklace** or a **watch** or a **bracelet**, made of either **gold** or **platinum**. These articles are distributed as shown in the following table:

	Necklace	**Watch**	**Bracelet**
Platinum	5	2	6
Gold	3	6	8

A- A box is chosen at random from this safe.
1) What is the probability of obtaining a necklace?
2) What is the probability of obtaining a gold necklace?
3) What is the probability of obtaining a necklace knowing that it is made of gold?

B- A customer wants to buy 3 gifts. Suppose that he selects simultaneously and randomly 3 boxes from this safe.
1) Prove that the probability that this customer obtains **two** gold articles and **one** platinum article is $\frac{442}{1015}$.
2) Each platinum article is sold for 2 million LL, and each gold article is sold for 1.2 million LL.

Let X be the random variable that is equal to the sum paid by the customer to buy any 3 articles chosen at random.
a- Determine the four possible values of X.
b- Determine the probability distribution corresponding to this random variable .
c- Calculate the expected value E(X). What does the number obtained represent?.

III- (4points)

Rami deposited a capital of 50 000 000 LL in a bank $\mathbf{B_1}$ on October 1, 2005, at 8 % annual interest, compounded yearly.

1) What is the amount of money that would be in his account on October 1, 2006?

2) Let $U_0 = 50\,000\,000$. Designate by U_n the amount in his account on the first of October of the year $(2005 + n)$.

a- Find a relation between U_{n+1} and U_n, and deduce that the sequence (U_n) is a geometric sequence whose common ratio is to be determined.

b- Express U_n in terms of n.

c- Calculate U_8.

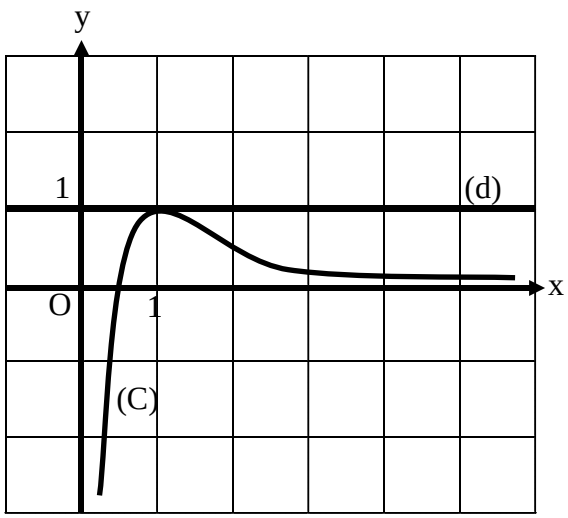
3) Another bank $\mathbf{B_2}$ advertises for the plan " **special investment**: double your capital in 8 years".

a- Is the plan **special investment** more profitable for Rami than investing his money in bank $\mathbf{B_1}$ for a period of 8 years? Justify your answer.

b- Determine the annual interest rate of the plan **special investment** knowing that this plan also earns compound interest that is compounded yearly.

IV- (8points)

Shown in the adjacent orthonormal system, the representative curve (C) of a function f that is defined on $]0 ; +\infty[$.



Indication : the line (d) of equation $y = 1$ is tangent to the curve (C) at the point $(1;1)$

A-

1) Determine f(1) and f '(1) and set up the table of variations of f.

2) The function f is expressed by $f(x) = \dfrac{a + b(\ln x)}{x}$, prove that $a = b = 1$.

3) Determine the abscissa of the point of intersection of (C) with the axis of abscissas, and solve the inequality $f(x) > 0$.

4) Calculate the area of the region bounded by the curve (C), the axis of abscissas and the line of equation $x = 1$.

5) F is a primitive (antiderivative) of f on $]0 ; +\infty[$; determine, according to the values of x, the sense of variations of F.

B-

In a certain company, the function f defined on $[0.1 ; 5]$ expresses the profit achieved upon selling x hundreds of the items produced. This profit is expressed in millions LL.

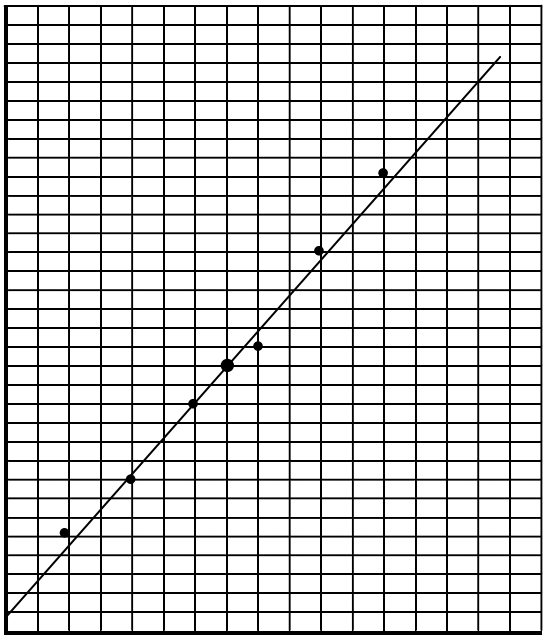
1) a- Does this company achieve a positive profit upon selling 30 items? Justify.

b- What is the minimal number of items that the company should sell in order to achieve a positive profit?

2) a- How many items should be sold in order to achieve the maximum profit?

b- What is the amount of this maximum profit?

MATHEMATICS SE FIRST SESSION 2006

<table>
<tr><td>Q</td><td colspan="4">ANSWERS</td><td>M</td></tr>
<tr><td>☏I</td><td colspan="4"></td><td></td></tr>
<tr><td rowspan="4">1</td><td rowspan="4">  </td><td rowspan="4">N</td><td>2</td><td>$\bar{x} = 3.5$, $\bar{y} = 14$
G(3.5 ; 14)</td><td>1 ½</td></tr>
<tr><td rowspan="3">1</td><td>3</td><td>y = 3.828x + 0,6</td><td>1 ½</td></tr>
<tr><td>4.a</td><td>In the year 2007 , x = 8
y = 3.828×8 + 0.6 = 31.224
Let 3122 member chips.</td><td>1 ½</td></tr>
<tr><td>4.b</td><td>3.828x + 0.6 > 40
3.828x > 39. 4
x > 10.29
Let x = 11
in the year 2010 the number of member chips will be greater than 4000 for the first time.</td><td>1 ½</td></tr>
<tr><td>☏II</td><td colspan="4"></td><td></td></tr>
<tr><td>A.1</td><td colspan="4">P(N) = 8/30 = 4/15</td><td>½</td></tr>
<tr><td>A.2</td><td colspan="4">$P(N \cap G) = 3/30 = 1/10$</td><td>½</td></tr>
<tr><td>A3</td><td colspan="4">P(N/G) = 3/17.</td><td>1</td></tr>
<tr><td>B1</td><td colspan="4">$P(2G \text{ and } 1P) = \frac{C_{17}^2 \times C_{13}^1}{C_{30}^3} = \frac{136 \times 13}{4060} = \frac{442}{1015}$</td><td>1</td></tr>
<tr><td>B.2.a</td><td colspan="4">The 4 possible values of X are :
3.6 for 3 articles in gold ; 4.4 for 2 articles in gold and platinum
5,2 for an article in gold and 2 in platinum ; 6 for 3 articles in platinum</td><td>½</td></tr>
<tr><td>B.2.b</td><td colspan="4"><table><tr><td>x_i</td><td>3,6</td><td>4,4</td><td>5,2</td><td>6</td></tr><tr><td>P_i</td><td>$\frac{C_{17}^3}{C_{30}^3} = \frac{680}{4060}$</td><td>$\frac{1768}{4060}$</td><td>$\frac{C_{17}^1 \times C_{13}^2}{C_{30}^3} = \frac{1326}{4060}$</td><td>$\frac{C_{13}^3}{C_{30}^3} = \frac{286}{4060}$</td></tr></table></td><td>2</td></tr>
<tr><td>B.2.c</td><td colspan="4">$E(X) = \frac{1}{4060}[3.6 \times 680 + 4.4 \times 1768 + 5.2 \times 1326 + 286 \times 6] = 4.64$
the average price of 3 articles is 4 640 000 LL</td><td>1 ½</td></tr>
</table>

☏III		
1	Rami will have in his account in october 1, 2006: $50\,000\,000(1 + 0.08) = 54\,000\,000$ LL	1
2.a	$U_{n+1} = U_n(1+0.08) = 1.08U_n$ (U_n) is a geometric sequence of common ratio 1.08.	1 ½
2.b	$U_n = U_0(1.08)^n = 50\,000\,000(1.08)^n$	1
2.c	$U_8 = 50\,000\,000(1.08)^8 \approx 92\,546\,510$LL.	½
3.a	The special investement is more profitable for Rami , since : $92\,546\,510 < 50\,000\,000 \times 2$.	1
3.b	$2C = C(1 + i)^8$; $2 = (1 + i)^8$; $8\ln(1 +i) = \ln 2$; $\ln(1 + i) = (\ln 2)/8$; $1 + i = e^{(\ln 2)/8}$ $i = e^{(\ln 2)/8} - 1 = 0.09$; then the annual interst rate is 9 %.	2
☏IV		
A1	$f(1) = 1$ and $f'(1) = 0$ x | 0 1 $+\infty$ f(x) $-\infty$ ↗ 1 ↘ 0	2
A.2	$f(1) = 1$ gives $a = 1$ $f'(x) = \frac{b - a - b\ln x}{x^2}$; $f'(1) = 0$; $b - a = 0$ so $b = a$; $b = 1$.	1 ½
A.3	(C) cuts the axis of abscissas at a point of absciss x so that $f(x) = 0$ we get $1 + \ln x = 0$; $x = 1/e$. $f(x) > 0$ for $x > 1/e$.	2
A.4	$A = \int_{\frac{1}{e}}^{1} \frac{1+\ln x}{x} dx$ (u^2) . Let $u(x) = 1 + \ln x$; $u'(x) = 1/x$ we get $\int_{\frac{1}{e}}^{1} \frac{1+\ln x}{x} dx = \int_{\frac{1}{e}}^{1} u(x).u'(x)dx = \frac{1}{2}\left[u^2(x)\right]_{1/e}^{1} = \frac{1}{2}\left[(1+\ln x)^2\right]_{1/e}^{1} = \frac{1}{2}(1-0) = \frac{1}{2}$ $A = \frac{1}{2}\ u^2$	2
A.5	$F'(x) = f(x)$ x | 0 1/e $+\infty$ F'(x) — 0 + F(x) ↘ ↗	1 ½
B.1.a	for selling 30 items, $x = 0.3$; using the graph $0.3 < 1/e$ and $f(x) < 0$, then the profit is not positive . ▶**OR** : $f(0.3) = -0.679$.	1 ½
B.1.b	The breaking event (zero profit) is $1/e = 0.367$ and $f(x) > 0$ for $x > 1/e$, so 37 items is the minimal number of items that the company should sell in order to achieve a positive profit..	1 ½
B.2.a	f has a maximum for $x = 1$ .We have to sell 100 items in order to achieve the maximum profit.	1
B.2.b	The maximum profit is 1 000 000 LL since $f(1) = 1$.	1