المادة: الرياضيات الشهادة: الثانوية العامة الفرع: العلوم العامة نموذج رقم -٤- المدّة : أربع ساعات	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (2pts)

Consider the two sequences (U_n) $n \in IN$ and (V_n) , defined as :

$U_0 = 2$, $U_{n+1} = \sqrt{U_n}$ and $V_n = \ln(U_n)$ for all $n \in N$.

1) a- Use mathematical induction to show that $U_n > 1$ for all $n \in N$.

b- deduce that for all n in N , V_n is defined and $V_n > 0$.

2) a- Prove that (V_n) is a geometric sequence whose common ratio and first term should be determined

b- Express V_n in terms of n , then deduce an expression of U_n in terms of n .

c- Prove that the sequence (U_n) is decreasing . Deduce that (U_n) is convergent , then find its limit .

3) Let $S = V_0 + V_1 + \dots + V_n$. and $P = U_0 \times U_1 \times \dots \times U_n$.

Calculate S in terms of n , then deduce P in terms of n .

II- (3pts).

In the image and sound section in a grand store ,sets of a certain brand of TV and DVD are on sale .

. The probability that a client buys the TV is $\frac{3}{5}$.

. The probability that a client buys the DVD given that he bought the TV is $\frac{7}{10}$.

. The probability that a client buys the DVD is $\frac{23}{50}$.

Denote by T the event : the client buys the TV and by L the event :the client buys the DVD .

1) Determine the probabilities of the following events.

(The results should be expressed as fractions)

a) The client buys both items.

b) The client buys the DVD only .

c) The client buys at least one of the items.

d) The client does not buy any items.

2) Knowing the client does not buy the DVD , show that the probability to buy the TV is $\frac{9}{21}$.

3) Before the sale period , the TV costs 500 000 LL and the DVD costs 200 000 LL.

During the sale week , the store discounted 15 % on the cost if a client buys only one item and 25 % if a client buys both items .

Denote by S the effective sum paid by a certain client .

a) Determine the four possible value for S .

b) Determine the probability distribution for S .

c) Calculate the expected value for S .

4) Knowing that the client didn't buy a DVD , calculate the probability that the he didn't pay 425000LL . Explain .

III- (2 pts)

O, A, F and F' are fixed . OF'= 1, OF = 5 and OA = 6. Let (C) be a variable circle tangent to (OA), (FD) and (F'S). ***(See the figure below)***

Part A.

1) a- Calculate FD and F'S.

b- Prove that MF + MF' = 6.

c- Deduce that M moves on a ellipse (E) with foci and major axis to be determined .

1) a- Determine the center I of (E). Show that O and A are two vertices of (E).

b- Construct B and B', the vertices of (E) on the non-focal axis .Calculate e.

c- H is a point on [FA) so that AH = $\frac{3}{2}$ and (Δ) is the perpendicular at H to (OA).

Prove that (Δ) is a directrix to (E)

2) L is the point so that IFLB is a rectangle . Prove that $I\hat{L}H$ is a right angle .

Part B

The plane is referred to the system $(I;\vec{i},\vec{j})$ with $\vec{i}=\frac{1}{2}\overrightarrow{IF}$.

1) a-Write an equation of (E).

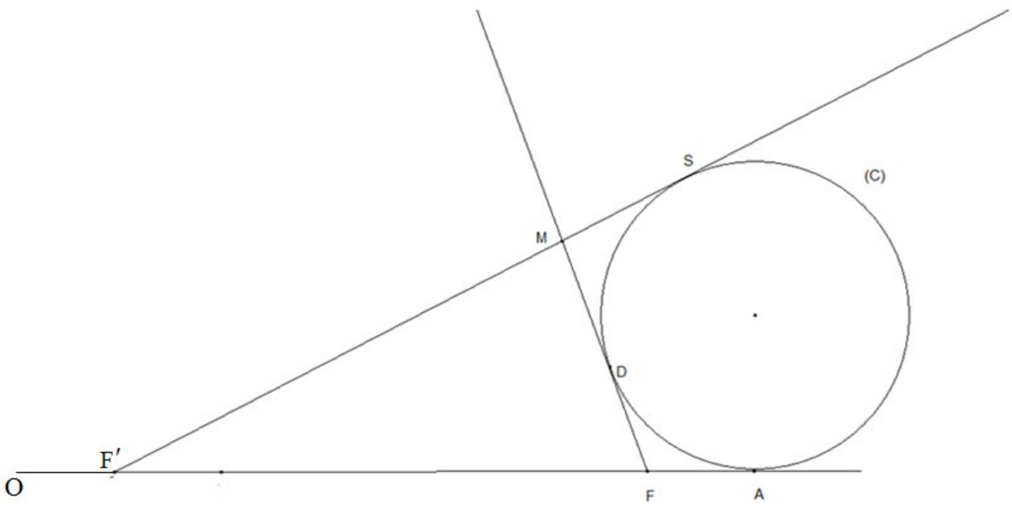
b-Find an equation of (Δ') , the 2nd directrix to (E).

2) The perpendicular at F' to (OA) meets (E) at G and G'. (Δ') intersects the x-axis at K.

a-Prove that (KG) and (KG') are tangent to (E).

b-Prove that $\frac{GF}{GF'}=\frac{KF}{KF'}$.

c-Calculate the area bounded be (E) , (KG) , (KG') and (IB).



IV- (3 pts)

In the space referred to an orthonormal system (O ; $\vec{i}$, $\vec{j}$, $\vec{k}$) , consider the points

A(1,-2,1) and B(2,-1,3). (P) is a plane containing (AB) and parallel to $\vec{v}$(0,1,1).

1) Prove that x + y – z + 2 = 0 is an equation of (P).

2) Consider the point E(2,2,0), and denote by (d) the line through E and perpendicular to (P) .

a- Write a system of parametric equations of (d) .

b- Find the coordinates of H orthogonal projection of E on (P) .

In what follows , suppose that H(0,0,2).

3) **a-** Prove that HA = HB .

b- Write a system of parametric equations of the bissector of AHB.

4) **a-** Calculate the angle that (AE) makes with (P) .

b- Write an equation of the plane (Q) containing (AE) and perpendicular to (P).

5) Consider in the plane (P) the circle (C) with center H and radius HA .

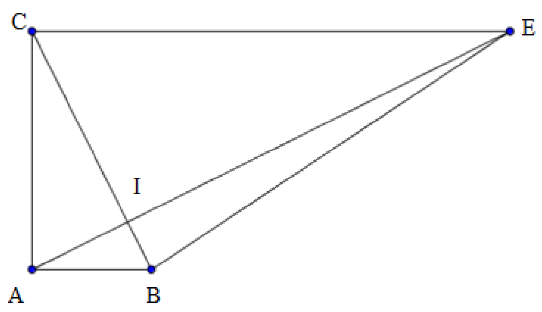
a-Prove that F($\sqrt{3}$, - $\sqrt{3}$, 2) is a point on (C).Then show that (HF) perpendicular to (AB) .

b- Write a system of parametric equations of the tangent at F to (C) .

6) N is a variable point on (d) . Find the coordinates of N so that the volume of the tetrahedron NABF is twice that EABF.

V- (3 points)

In the next figure , ABEC is a right trapezoid so that AB =1 , AC = 2 , and CE = 4 .
S is the similitude that maps A onto C and C onto E . (BC) intersects (AE) at I.



1) Calculate the scalar product $(\overrightarrow{BA}+\overrightarrow{AC}).(\overrightarrow{AC}+\overrightarrow{CE})$, deduce that (AE) is perpendicular to (BC)

2) Show that 2 is the scale factor of S and $-\frac{\pi}{2}$ is an angle of it .

3) **a-** Determine S(AE) and S(BC).

b- Deduce that I is the center of S .

c- Determine S(B).

4) G is the midpoint of [AB] and H is that of [EC] .

a- Prove that H = SoS(G) .

b- Express $\overrightarrow{IH}$ in terms of $\overrightarrow{IG}$.

5) F is the orthogonal projection of B on (EC) . h is the dilation with center F and scale factor $-\dfrac{1}{3}$

a-Determine an angle of hoS and so its scale factor .

b-Prove that C is the center of hoS.

6) a- The plane is referred to the direct orthonormal system $(A;\vec{u},\vec{v})$ with $\vec{u}=\overrightarrow{AB}$ and $\vec{v}=\dfrac{1}{2}\overrightarrow{AC}$.

b- Find the complex form for S . Deduce z_I.

7) M is a variable point that moves o the curve (C)with equation : $y=\dfrac{2}{1+e^x}$, and M'= S(M).

M' moves on the curve (C') = S((C)).

a- Prove that the midpoint H of [CE] is on (C').

b- Write an equation of the tangent (T) to (C') at H .

c- Show that $y=2[1-\ln(\dfrac{4-x}{x})]$ is an equation of (C').

VI- (7pts)

Part A.

f is a function defined over $]0;+\infty[$, as $f(x)=x^2-2+\ln x$;(C) is the graph of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Find $\lim f(x)$ as $x\to 0$ and as $x\to+\infty$ and $\lim\dfrac{f(x)}{x}$ as x $\to+\infty$.
2) **a-** Set up the table of variations of f .
   b- Prove that the equation $f(x)=0$ has a unique solution α so that $1.31<\alpha<1.32$.
   c- Determine , according to x ,the sign of $f(x)$.
3) Discuss , according to x , the concavity of (C).
4) **a-** Calculate $f(1)$, $f(2)$, then plot (C).
   b- Solve graphically $f(x)>-x$.

Part B.

g is the function defined over $]0,+\infty[$ as $g(x)=x^2+(2-\ln x)^2$; (C') is the graph of g in a new system of axes .

1) Find $\lim g(x)$ as $x\to 0$ and as $x\to+\infty$ and $\lim\dfrac{g(x)}{x}$ as $x\to+\infty$.

2) Show that $g'(x)=\dfrac{2f(x)}{x}$, then set up the table of variations of g .

Verify that $g(\alpha)=\alpha^2(1+\alpha^2)$.

3) Calculate g(1) , g(e) then plot (C').

4) a- Verify that $x(\ln x-1)$ is an antiderivative of $\ln x$.

b- Let $z = x(2-\ln x)^2$, calculate z' , then find $\int g(x)dx$.

5) a-For $x \leq \alpha$, prove that g has an inverse function h .

Find $D_h, R_h,$ then plot (C_h) the graph of h in the same system as (C').

b- Calculate the area of the region bounded by (C_h) ,in terms of α , the two lines y = α and $x = 5$.

c- Find the point of (C_h) where the tangent is parallel to the line with equation $y = -\frac{1}{2}x$.

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

Scale of Marks /80

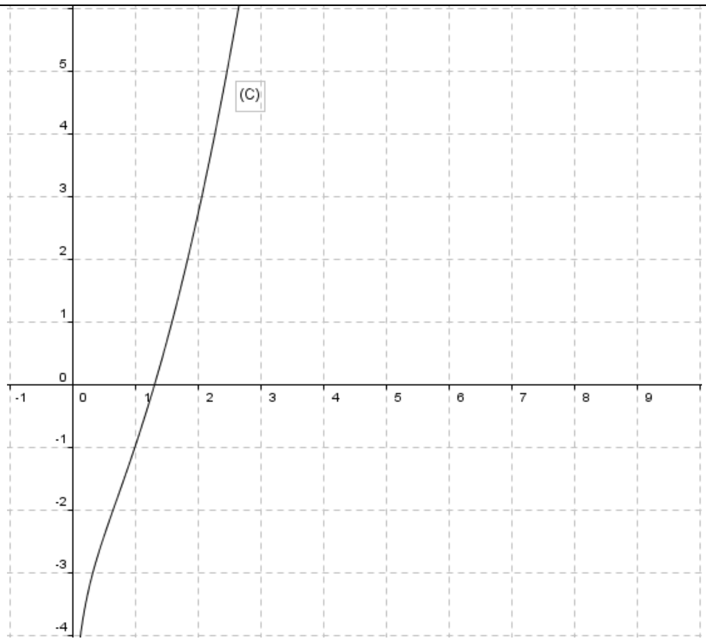
Question / Mark		Solution
		Question I
1.a	1	$U_0 = 2 \geq 1$, suppose that $U_K > 1$. $\sqrt{U_K} > 1$, hence $U_{K+1} > 1$.
1.b	1	Since $U_n > 1$, then ln $(U_n) > 0$ and V_n is defined .
2.a	1	$V_{n+1} = \ln(U_{n+1}) = \ln(\sqrt{U_n}) = \frac{1}{2}\ln(U_n) = \frac{1}{2}V_n$ (V_n) is a geometric sequence so that $V_0 = \ln 2$ and r = $\frac{1}{2}$
2.b	1	$V_n = V_0 \times r^n = \ln 2 \times (\frac{1}{2})^n$ $\ln(U_n) = V_n$; $U_n = e^{V_n} = e^{\ln 2 \times (\frac{1}{2})^n}$
2.c	2	$\frac{U_{n+1}}{U_n} = e^{\ln 2\times(\frac{1}{2})^{n+1} - \ln 2\times(\frac{1}{2})^n} = e^{(\frac{1}{2})^n(\frac{1}{2}\ln 2 - \ln 2)} = e^{(\frac{1}{2})^n \ln 2(\frac{1}{2}-1)} = e^{-(\frac{1}{2})^{n+1}\ln 2} < 1$. U_n is decreasing and having 1 as lower bound : (U_n) convergent . If n $\to +\infty$, then $(\frac{1}{2})^n \to 0$ and $U_n \to 1$
3.	2	S = $V_0 + V_1 + V_2 + ... + V_n = \frac{V_0(r^{n+1}-1)}{r-1} = \frac{\ln 2[(\frac{1}{2})^{n+1} - 1]}{\frac{1}{2}-1} = -2\ln 2[(\frac{1}{2})^{n+1} - 1]$ $S = \ln U_0 + \ln U_1 + \ln U_2 + ... + \ln U_n$ $= \ln(U_0 \times U_1 \times ... \times U_n) = \ln p$ Then P = e^S
		Question II
1.a	1.5	$P(T \cap L) = P(T) \times P(L/T) = \frac{3}{5} \times \frac{7}{10} = \frac{21}{50}$.

1.b	1.5	$P(L)=P(L\cap T)+P(L\cap\overline{T})=\frac{21}{50}+P(L\cap\overline{T})$ $Then\ P\left(L\cap\overline{T}\right)=\frac{23}{50}-\frac{21}{50}=\frac{1}{25}$
1.c	1.5	$P(T\cup L)=P(T)+P(L)-P\left(T\cap L\right)=\frac{3}{5}+\frac{23}{50}-\frac{21}{50}=\frac{16}{25}.$
1.d	1	$P(\overline{T}\cap\overline{L})=1-P\left(T\cup L\right)=\frac{9}{25}.$
2	1	$P\left(T/\overline{L}\right)=\frac{P\left(T\cap\overline{L}\right)}{P(\overline{L})}=\frac{P(T)\times P\left(\overline{L}/T\right)}{P\left(\overline{L}\right)}=\frac{\left(\frac{3}{5}\right)\times\left(\frac{3}{10}\right)}{\left(\frac{21}{50}\right)}=\frac{9}{21}$
3.a	1	425000 for TV only , 170000 dor DVD only , 525000 for both , 0 for nothing.
3.b	2	<table><tr><td>D_i</td><td>0</td><td>170000</td><td>425000</td><td>5250000</td></tr><tr><td>P_i</td><td>$\frac{18}{50}$</td><td>$\frac{2}{50}$</td><td>$\frac{9}{50}$</td><td>$\frac{21}{50}$</td></tr></table> $P(425)=P(T\cap\overline{L})=\frac{3}{5}\times\frac{3}{10};P(170)=P(L\cap\overline{T})=\frac{2}{5}\times\frac{1}{10}$
3.c	1	E(D) = $\sum D_iP_i=\frac{15190}{50}\approx 304000LL$.
4	1.5	$\overline{L}=(\overline{L}\cap T)or(\overline{L}\cap\overline{T})$; Since he didn't pay 425000 , then he didn't buy any item. $P\overline{T}/\overline{L})=\frac{P(\overline{T}\cap\overline{L})}{P(\overline{L})}=\frac{18/50}{27/50}=\frac{2}{3}$
		Question III
		Part A
1.a	0.75	FD = OA =1 and F'S =F'A = 5.
1.b	0.75	b-MF + MF' = MD + DF + MF'= 1+ MS +MF' = 1+F'S = 1+5 =6 = OA.
1.c	0.75	c-MF + MF' = 6 ; M moves on the ellipse with foci F and F' and 2a = 6 The focal axis is (FF')
2.a	0.75	a- The center I is the midpoint of [FF']. IO =IA = 3 = a ; Since O and A are on the focal axis , then they are two vertices of (E).
2.b	1	b-B and B' are on the perpendicular bisector of [FF'] so that IB = IB'= $\sqrt{9-4}=\sqrt{5}$. e = $\frac{c}{a}=\frac{IF}{IA}=\frac{2}{3}$.
2.c	1	AH = $\frac{3}{2}$, then IH = 3+$\frac{3}{2}=\frac{9}{2}=\frac{a^2}{c}$. (Δ) is a directrix to (E).

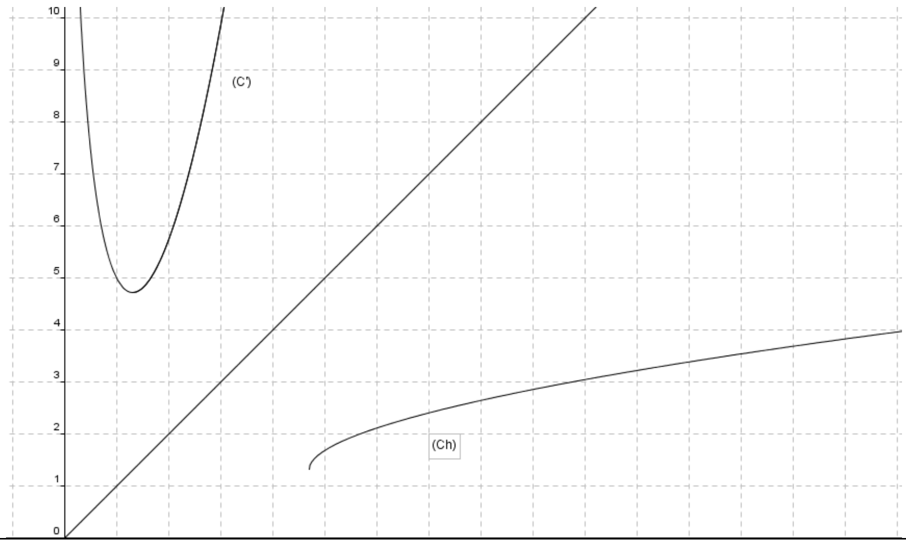
3		IL ^{2} = 9 ; IH ^{2} = $\frac{81}{4}$ and LH ^{2} = $5+\frac{25}{4}=\frac{45}{4}$. IH ^{2}= IL^2+LH^2 then the triangle ILH is right at L .
Part B		
1.a	0.5	$\frac{x^2}{9}+\frac{y^2}{5}=1$.
1.b	0.5	(Δ') : x = - $\frac{9}{2}$
2.a	0.5	G(-2, $\frac{5}{3}$) and G'(2,$\frac{5}{3}$) ; K(-$\frac{9}{2}$,0) . Derive wrt x : $\frac{2x}{9}+\frac{2yy'}{5}=0$; y' $_G$= $\frac{2}{3}=slope(KG)$. (KG) is tangent to (E) and by symmetry , (KG') is also tangent to (E).
2.b	0.5	$\frac{GF}{GF'}=\frac{KF}{KF'}$ (verification).
2.c	1	(KG) intersects (IB) at J(0,3). Half (area) = area (triangle KIJ) - $\frac{1}{4}$ area (E). $=\frac{1}{2}\times\frac{9}{2}\times3-\frac{1}{4}(\pi\times3\times\sqrt{5})=\frac{27-3\pi\sqrt{5}}{4}$. Total area = $\frac{27-3\pi\sqrt{5}}{2}u^2$
Question IV		
1	1	$\overrightarrow{AM}.(\overrightarrow{AB}\wedge\vec{v})$ = 0 ; x + y – z + 2 = 0 (P)
2.a	1	a)(d) : x = k + 2 ; y = k + 2 ; z = - k
2.b	1	b)E = $(d)\cap(P)$: k = -2 and H(0,0,2)
3 .a	0.5	HA = HB = $\sqrt{6}$
3.b	1	The bisector is (HG) with 2G ($\frac{3}{2}$,$\frac{-3}{2}$,2) midpoint of [AB]. x= m , y = -m , z = 2 .
4.a	1	The angle is HAE , ; cos HAE = $\frac{AH}{AE}$ or sin or tan…
4.b	1.5	M(x,y,x) $\in(Q)$. Then $\overrightarrow{AM}.(\overrightarrow{AE}\wedge\overrightarrow{n_P})=0$ Therefore x +z – 2 = 0
5.a **5.b**	2 1.5	$F(\sqrt{3},-\sqrt{3},2)\in(P)\ and\ HF=HA=\sqrt{6}$ $\overrightarrow{HF}.\overrightarrow{AB}=0$ b)the tangent at F is the line through F and parallel to (AB).x = t , y = t , z = 2

6	1.5	the base is ABF , then d(N,P)= d(E,P) $\frac{\lvert k+2+k+2+k+2\rvert}{\sqrt{3}}\ then EH = 2\sqrt{3}$. $\lvert 3x+6\rvert = 6$ hence k= 0 or k = -4
		Question V
1	1	1) $(\overrightarrow{BA}+\overrightarrow{AC}).(\overrightarrow{AC}+\overrightarrow{CE}) = 0$ then (BC) is perpendicular to (AE).
2	1	2) $k=\frac{CE}{AC}=\frac{4}{2}=2$ and $\alpha=(\overrightarrow{AC},\overrightarrow{CE})=\frac{-\pi}{2}+2k\pi$
3.a	1	3) a- S(AE) = line through C and perpendicular to (AE) .Then S(AE)=(BC). Similary S(BC) = (AE)..
3.b	1	b- S(I) = S(AE) ∩S(BC)= (BC)∩(AE)=I . I is the center of S .
3.c	1	c- Since CA= 2AB and $(\overrightarrow{AB},\overrightarrow{CA})=\frac{\pi}{2} and S(A)=C$ then S(B) =A.
4.a	0.5	4) a- S(G) = G' midpoint of [CA] and S(G')=H midpoint of [AC].
4.b	0.5	b- SoS= dilation (I;-4)then $\overrightarrow{IH}=-4\overrightarrow{IG}$
5.a	0.5	5)a-h(F; $\frac{-1}{3}$) $oS(I,2,\frac{-\pi}{2})=S'(?,\frac{2}{3},\frac{\pi}{2})$.
5.b	0.5	b- $\overrightarrow{FC}=\frac{-1}{3}\overrightarrow{FE}$ then C = h(E) but S(C)=E then hoS(C)=C and C is the center of hoS
6	1	z' = -2iz+b , z_C= -2iz_A+b . b = 2i. z' = -2iz +2i , z_I(1+2i) = 2i then $z_I=\frac{4}{5}+\frac{2i}{5}$
7.a	1	G' (0,1) is on (C) and H = S(G') is on (C').
7.b	1.5	$f'(x)=\frac{-2e^x}{(1+e^x)^2}$ and $f'(0)=-\frac{1}{2}=$ slope of the tangent therefore the slope of the tangent at H to (C') is 2 equation of (T) is : $y=2x-2$
7.c	1.5	$x'+iy'=-2i(x+iy)+2i \quad x'=2y$ and $y'=2-2x$ replace in(C) : $\frac{x'}{2}=\frac{2}{1+e^{\frac{2-y'}{2}}} \quad e^{\left(\frac{2-y'}{2}\right)}=\frac{4}{x'}-1$ $\frac{2-y'}{2}=\ln\left(\frac{4-x}{x}\right)$ *eq* of (C'): $y=2\left[1-\ln\left(\frac{4-x}{x}\right)\right]$.

<table>
<tr><th colspan="3">Question VI</th></tr>
<tr><th colspan="3">Part A</th></tr>
<tr><td>1</td><td>3</td><td>$\lim_{x\to 0} f(x) = -\infty$ (y’y) is an asymptote to (C).
$\lim_{x\to +\infty} f(x) = +\infty$ and $\lim_{x\to +\infty} f(x) = +\infty$.(C) has a parabolic branch parallel to (y’y).</td></tr>
<tr><td>2.a</td><td>1</td><td>f ‘(x)= 2x+ $\frac{1}{x}$ > 0
<table><tr><td>x</td><td>0</td><td>$+\infty$</td></tr><tr><td>f ‘(x)</td><td colspan="2">+</td></tr><tr><td>f(x)</td><td colspan="2">↗</td></tr></table></td></tr>
<tr><td>2.b</td><td>1</td><td>f is continuous and strictly increasing from $-\infty$ to $+\infty$
then f(x) = 0 has only one root α.
f(1.31) < 0 , f(α) = 0 and f(1.32) > 0
f(1.31) < f(α) < f(1.32), but f is increasing
therefore 1.31< α < 1.32</td></tr>
<tr><td>2.c</td><td>1</td><td>f(x) < 0 for x < α and f(x) > 0 for x > α.</td></tr>
<tr><td>3</td><td>1</td><td>f ’’ (x)= 2 - $\frac{1}{x^2}$
<table><tr><td>x</td><td>0</td><td>$\frac{\sqrt{2}}{2}$</td><td>$+\infty$</td></tr><tr><td>f ‘‘(x)</td><td>-</td><td>o</td><td>+</td></tr><tr><td>concavity</td><td>down</td><td></td><td>up</td></tr></table>$(\frac{\sqrt{2}}{2}, -\frac{3}{2} - \frac{1}{2}\ln 2)$ inflection point .</td></tr>
</table>

<table>
<tr><td>4.a</td><td>1</td><td>Graph .

</td></tr>
<tr><td>4.b</td><td>1</td><td>4.b-f(x) > -x , consider the part of (C) above (y= -x)
x > 1</td></tr>
<tr><td colspan="3">Part B</td></tr>
<tr><td>1</td><td>3</td><td>$\lim_{x\to 0} g(x) = +\infty$ (y'y) is an asymptote to (C') ;
$\lim_{x\to +\infty} g(x) = +\infty$

$\lim_{x\to +\infty} \frac{g(x)}{x} = +\infty$ (C') has a parabolic branch parallel to (y'y).</td></tr>
<tr><td>2</td><td>2</td><td>g'(x) = 2x + 2 (2-lnx)($\frac{-1}{x}$) = $\frac{2f(x)}{x}$.
<table>
<tr><td>x</td><td>0</td><td></td><td>α</td><td></td><td>$+\infty$</td></tr>
<tr><td>g'(x)</td><td></td><td>-</td><td>o</td><td>+</td><td></td></tr>
<tr><td>g(x)</td><td></td><td>↘</td><td>g(α)</td><td>↗</td><td></td></tr>
</table>
f(α) = 0 ; $\alpha^2 = 2 - \ln\alpha$.
$g(\alpha) = \alpha^2 + (2-\ln\alpha)^2 = \alpha^2(1+\alpha^2)$.</td></tr>
</table>

3	3	g(1) = 5 g(e)=e^2+1. 
4.a	1	[x(lnx-1)]'= lnx
4.b	2	z = x $(2-\ln x)^2$. z'=$(2-\ln x)^2$-2+lnx. $\int g(x)dx = \frac{x^3}{3} + \int (z'+2-\ln x)dx = \frac{x^3}{3} + z + 2x + x(\ln x - 1) = \frac{x^3}{3} + z + x + x\ln x$
5.a	2	for x ≤ α, g is continuous and strictly decreasing , then it has an inverse function h ; $D_h = [\alpha^2(1+\alpha^2) , +\infty[$ R_h=]0,α] ; (C_h) is the symmetric of (C') wrt (y=x)(see the graph (Ch).
5.b	3	Area = area bounded by (C'), x = α and y = 5 = 5(α – 1) - $\int_1^{\alpha} g(x)dx$
5.c	2	h' (x) = $\frac{-1}{2}$; g'(x) = -2 or f(x) = -x , then x = 1. (1,5) is on (C') ; (5,1) is on C_h.