وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة 2007 العادية

عدد المسائل: اربع

مسابقة في مادة تايضايرلا
المدة: ساعتان

الاسم:
الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the points A (1 ; 1 ; 0), B (2; 0 ; 0), C (1; 3; –1), E (2 ; 2 ; 2) and the plane (P) of equation $x + y + 2z - 2 = 0$.

1) a- Verify that (P) is the plane determined by A, B and C.
   b- Show that the line (AE) is perpendicular to the plane (P).
   c- Calculate the area of triangle ABC and the volume of tetrahedron EABC.

2) Designate by L the midpoint of [AB] and by (Q) the plane passing through L and parallel to the two lines (AE) and (BC).
   a- Write an equation of plane (Q).
   b- Prove that the planes (P) and (Q) are perpendicular.
   c- Prove that line (d), the intersection of the planes (P) and (Q), is parallel to (BC).

II– (4 points)

The 20 employees in a factory are distributed into two departments as shown in the table below:

	Technical Department	Administrative Department
Women	3	5
Men	10	2

1) The manager of this factory wants to offer a gift to one of the employees. To do this, he chooses randomly an employee of this factory.
   Consider the following events:
   W : « the chosen employee is a woman ».
   M : « the chosen employee is a man ».
   T : « the chosen employee is from the technical department ».
   A : « the chosen employee is from the administrative department ».
   a- Calculate the following probabilities:
   $P(W / T)$, $P(W / A)$, $P(W \cap T)$ and $P(W)$.
   b- Knowing that the chosen employee is a man, what is the probability that he is from the technical department ?

2) On a different occasion, the factory manager chooses **two** employees randomly and simultaneously from the technical department and also chooses **one** employee randomly from the administrative department.
   Designate by X the random variable that is equal to the number of women chosen.
   a- Verify that $P(X=1) = \frac{95}{182}$.
   b- Determine the probability distribution of X.

III– (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points E, F, G of respective affixes $z_E = 2i$, $z_F = -2i$, $z_G = -1+i$ and let M be a point of affix z.

1) a- Find the set (T) of points M such that $|z-2i| = \sqrt{2}$.
   b- Show that the point G belongs to (T).

2) a- Find the line (L) on which point M moves when $\left|\frac{z-2i}{z+2i}\right| = 1$.
   b- Determine the affix z_0 of a point W on (L) such that $|z_0 - 2i| = 3$.

3) Let A and B be the points of respective affixes z_A and z_B such that: $z_A = z_F + z_G$ and $z_B = z_F \times z_G$.
   a- Write the complex numbers z_A and z_B in the exponential form.
   b- Prove that the points O, A and B are collinear.

IV– (8 points)

Consider the function f defined over $]-\infty, 0[\cup]0, +\infty[$ by $f(x) = x - 1 - \frac{4}{e^x - 1}$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Show that the axis of ordinates is an asymptote to (C).
   b- Calculate $\lim\limits_{x\to+\infty} f(x)$ and prove that the line (d) of equation y = x – 1 is an asymptote to the curve (C).
   c- Prove that the line (D) of equation y = x + 3 is an asymptote to (C) at $-\infty$.

2) Prove that the point S(0 ; 1) is a center of symmetry of (C).

3) a- Calculate $f'(x)$ and set up the table of variations of f.
   b- Show that the equation f(x) = 0 has two roots α and β and verify that: $1.7 < \alpha < 1.8$ and $-3.2 < \beta < -3.1$.

4) Draw (d), (D) and (C).

5) a- Prove that $f(x) = x + 3 - \frac{4e^x}{e^x - 1}$.
   b- Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations x = 2 and x = 3.

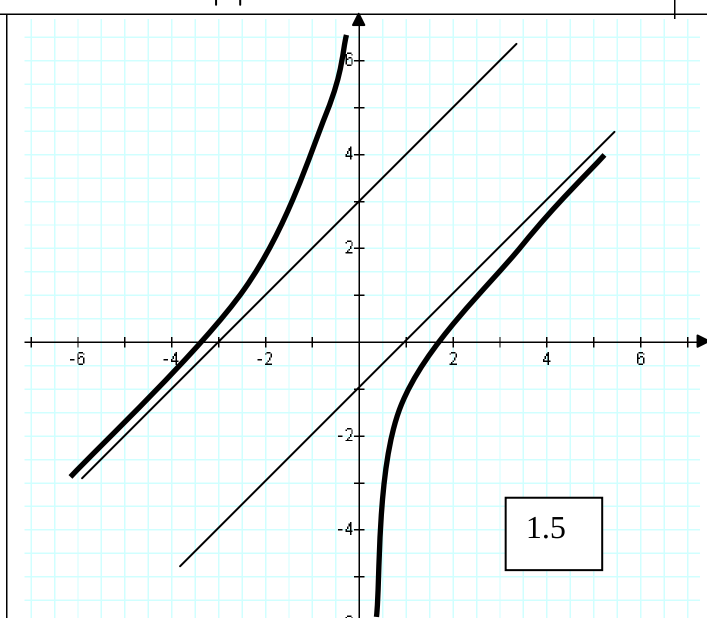
6) Let g be the inverse function of f on $]0, +\infty[$.
   Prove that the equation f(x) = g(x) has no roots.

Q1	MATH LS FIRST SESSION-2007	Marks
1-a	◇ $1 + 1 + 0 - 2 = 0$; $A \in (P)$ ◇ $2 + 0 + 0 - 2 = 0$; $B \in (P)$ ◇ $1 + 3 - 2 - 2 = 0$; $C \in (P)$ OR $\vec{AM}.(\vec{AB} \wedge \vec{AC}) = 0$; $\begin{vmatrix} x-1 & y-1 & z \\ 1 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix} = 0$; $x + y + 2z - 2 = 0$	1/2
1-b	$\vec{AE}(1;1;2)$ and $\vec{N_P}(1;1;2)$; (AE) is perpendicular to plane (P).	1/2
1-c	$S = \frac{1}{2} \lVert \vec{AB} \wedge \vec{AC} \rVert = \frac{\sqrt{1+1+4}}{2} = \frac{\sqrt{6}}{2}$. $V = \frac{1}{3} S \times AE = \frac{1}{3} \times \frac{\sqrt{6}}{2} \times \sqrt{6} = 1$ OR : $V = \frac{1}{6} \lvert \vec{AE}.(\vec{AB} \wedge \vec{AC}) \rvert = \frac{1}{6} \lvert 1+1+4 \rvert = 1$	1
2-a	$\vec{IM}.(\vec{AE} \wedge \vec{BC}) = 0$ where $I(\frac{3}{2};\frac{1}{2};0)$; So $\begin{vmatrix} x-\frac{3}{2} & y-\frac{1}{2} & z \\ 1 & 1 & 2 \\ -1 & 3 & -1 \end{vmatrix} = 0$ $(Q): 7x + y - 4z - 11 = 0$.	1
2-b	$\vec{N_P}.\vec{N_Q} = -7 - 1 + 8 = 0$; (P) and (Q) are perpendicular.	1/2

2-c	(BC) // (Q) and (BC) is a line in (P) , so (BC) is parallel to the line of intersection of (P) and (Q).
	OR : $(d) = (P) \cap (Q)$: $\begin{cases} x + y + 2z - 2 = 0 \\ 7x + y - 4z - 11 = 0 \end{cases}$; $(d): \begin{cases} x = t + \frac{3}{2} \\ y = -3t + \frac{1}{2} \\ z = t \end{cases}$
	$\vec{BC}(-1 ; 3 ; -1)$ and $\vec{V_d}(1 ; -3 ; 1)$.
	So (BC) //(d).
		1/2

<table>
<tr><th>Q2</th><th colspan="2">MATH LS FIRST SESSION-2007</th><th>Marks</th></tr>
<tr><td>1-a</td><td colspan="2">$P(W/T) = \frac{3}{13}$; $P(W/A) = \frac{5}{7}$;
$P(W \cap T) = \frac{3}{20}$; $P(W) = \frac{8}{20}$</td><td>1</td></tr>
<tr><td>1-b</td><td colspan="2">$P(T/M) = \frac{10}{12} = \frac{5}{6}$</td><td>1/2</td></tr>
<tr><td>2-a</td><td>$P(X = 1) = \frac{3\times 10}{C_{13}^{2}} \times \frac{2}{7} + \frac{C_{10}^{2}}{C_{13}^{2}} \times \frac{5}{7}$
$= \frac{285}{546}$
$= \frac{95}{182}$</td><td>
<table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>Or</td><td>0</td><td>2</td><td>1</td><td>0</td></tr>
</table></td><td>1</td></tr>
<tr><td rowspan="4">2-b</td><td>$P(X= 0) = \frac{C_{10}^{2}}{C_{13}^{2}} \times \frac{2}{7}$
$= \frac{90}{546}$
$= \frac{15}{91}$</td><td>
<table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>0</td><td>2</td><td>0</td><td>1</td></tr>
</table></td><td rowspan="4">1 1/2</td></tr>
<tr><td>$P(X= 2) = \frac{C_{3}^{2}}{C_{13}^{2}} \times \frac{2}{7} + \frac{3\times 10}{C_{13}^{2}} \times \frac{5}{7}$
$= \frac{156}{546}$
$= \frac{26}{91}$</td><td>
<table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>2</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>Or</td><td>1</td><td>1</td><td>1</td><td>0</td></tr>
</table></td></tr>
<tr><td>$P(X= 3) = \frac{C_{3}^{2}}{C_{13}^{2}} \times \frac{5}{7}$
$= \frac{15}{546}$
$= \frac{5}{182}$</td><td>
<table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>2</td><td>0</td><td>1</td><td>0</td></tr>
</table></td></tr>
<tr><td colspan="2">
<table>
<tr><td>x_i</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P_i</td><td>$\frac{15}{91}$</td><td>$\frac{95}{182}$</td><td>$\frac{26}{91}$</td><td>$\frac{5}{182}$</td></tr>
</table></td></tr>
</table>

Q3	MATH LS FIRST SESSION-2007	Marks
1-a	$\|z-2i\|=\sqrt{2}$ is equivalent to EM=$\sqrt{2}$ Thus (T) is the circle of center E and radius $\sqrt{2}$.	1
1-b	EG=$\|z_G - z_E\| = \|-1-i\| = \sqrt{2}$,thus G$\in$(T).	1/2
2-a	$\left\|\frac{z-2i}{z+2i}\right\| = 1$ is equivalent to $\|z-2i\| = \|z+2i\|$ so ME=MF and (L) is the perpendicular bisector of [EF], which is the axis of abscissas.	1/2
2-b	W$\in$(L) so $\|z_0-2i\| = \|z_0+2i\| = 3$; Let $z_0 = x+iy$. $\|x+iy-2i\| = \|x+iy+2i\|$ is equivalent to $x^2+(y-2)^2 = x^2+(y+2)^2$ y = 0 gives $x^2+4=9$; $x=\sqrt{5}$ or x=$-\sqrt{5}$, consequently $z_0=\sqrt{5}$ or $z_0=-\sqrt{5}$ OR : W$\in$ x'x and EW=3 so $OW^2 = EW^2 - OE^2 = 9-4=5$; OW=$\sqrt{5}$ thus $z_0=\sqrt{5}$ or $z_0=-\sqrt{5}$.	1/2
3-a	$z_A = -1-i = \sqrt{2}e^{5\frac{\pi}{4}i}$, $z_B = 2+2i = 2\sqrt{2}e^{\frac{\pi}{4}i}$.	1
3-b	$\arg z_A = 5\frac{\pi}{4}$, $\arg z_B = \frac{\pi}{4}$; $\arg z_A = \arg z_B + \pi$ so O, A and B are collinear OR : $z_B = -2z_A$ or $\overrightarrow{OB} = -2\overrightarrow{OA}$.	1/2

Q4	MATH LS FIRST SESSION-2007	Marks
1-a	$\lim\limits_{\substack{x\to 0\\ x>0}} f(x) = -1-\frac{4}{0^+} = -\infty$; $\lim\limits_{\substack{x\to 0\\ x<0}} f(x) = -1-\frac{4}{0^-} = +\infty$ So the axis of ordinates of equation x = 0 is an asymptote of (C) .	1/2
1-b	$\lim\limits_{x\to+\infty} f(x) = +\infty - 0 = +\infty$; $\lim\limits_{x\to+\infty} [f(x)-(x-1)] = \lim\limits_{x\to+\infty} \frac{4}{e^x-1} = 0$	1
1-c	$\lim\limits_{x\to-\infty} [f(x)-(x+3)] = \lim\limits_{x\to-\infty} (x-1-\frac{4}{e^x-1}-x-3) = \lim\limits_{x\to-\infty} (-4-\frac{4}{e^x-1})$ $= -4+4 = 0.$	1/2
2	The domain of f is centered at O.. $f(-x)+f(x) = -x-1-\frac{4}{e^{-x}-1}+x-1-\frac{4}{e^x-1} = -2+\frac{4e^x}{e^x-1}+\frac{4}{e^x-1}$ $= -2+4 = 2$, thus S(0 ; 1) is a center symmetry of (C).	1/2
3-a	$f'(x) = 1+\frac{4e^x}{(e^x-1)^2} > 0.$ x: $-\infty$; 0 ; $+\infty$ f '(x): + ‖ + f (x): $-\infty$ ↗ $+\infty$ ‖ $-\infty$ ↗ $+\infty$	1
3-b	f is continuous and strictly increasing from $-\infty$ to $+\infty$ on $]-\infty\,;0[$;the equation f(x) = 0 has over this interval a unique negative root β ; $f(-3.2) = -0.03 < 0$ and $f(-3.1) = 0.088 > 0$ so $-3.2 < \beta < -3.1$. Similarly : f(x) = 0 has over $]0\,;+\infty[$ a unique positive root α ; $f(1.7) = -0.154 < 0$ and $f(1.8) = +0.0078 > 0$ so $1.7 < \alpha < 1.8$. 1 point  	2.5
5-a	$x+3-\frac{4e^x}{e^x-1} = x-1+4-\frac{4e^x}{e^x-1} = x-1+\frac{4e^x-4-4e^x}{e^x-1} = x-1-\frac{4}{e^x-1}.$	1/2
5-b	$A = \int_2^3 (x+3-\frac{4e^x}{e^x-1})dx = \left[\frac{x^2}{2}+3x-4\ln(e^x-1)\right]_2^3 = \frac{11}{2}+4\ln\left(\frac{e^2-1}{e^3-1}\right) = 1.122\ u^2$	1
6	f(x) = g(x) equivalent to f(x) = x ; $x-1-\frac{4}{e^x-1} = x$; $-1 = \frac{4}{e^x-1}$; $e^x = -3$ is impossible since ($e^x > 0$) .So the equation f(x) = g(x) has no roots.	1/2