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عدد المسائل: ست	مسابقة في مادة الرياضيات المدة: أربع ساعات	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I – (2 points)

In the table below, only one among the proposed answers to each question is correct.
Write down the number of each question and give, **with justification**, the answer corresponding to it.

Nº	Questions	Answers			
		a	b	c	d
1	$z = -\sqrt{3} - i$. An argument of $\overline{z}$ is :	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{6}$
2	$\left(e^{i\frac{\pi}{4}}\right)^{12} =$	1	–1	e^3	3
3	$C_{10}^{6} - C_{9}^{6} =$	1	C_9^5	C_{19}^6	0
4	h is a function defined on IR by $h(x) = \frac{1}{4+x^2}$; A primitive H of h is given by H(x) =	$\arctan\frac{x}{2}$	$\ln(4+x^2)$	$\frac{1}{2}\arctan\frac{x}{2}$	$2\arctan x$
5	$\lim\limits_{x\to+\infty} \frac{\ln(e^x+1)}{x} =$	1	0	e	$+\infty$
6	If the affixes of points A, B and C verify the relation $\frac{z_A - z_B}{z_A - z_C} = 2$, then	C is the midpoint of [AB]	B is the midpoint of [AC]	A, B and C form a right triangle	A, B and C belong to the same circle

II–(3 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the lines (d_1) and (d_2) defined by: $(d_1): \begin{cases} x = m \\ y = m-1 \\ z = 1 \end{cases}$ and $(d_2): \begin{cases} x = -t+1 \\ y = t \\ z = -2t+4 \end{cases}$ (m and t are real numbers).

1) Prove that (d_1) and (d_2) are orthogonal and skew.

2) Verify that the vector $\vec{n}(-1; 1; 1)$ is orthogonal to (d_1) and (d_2).

3) Prove that an equation of the plane (P) containing (d_1) and parallel to $\vec{n}$ is $x - y + 2z - 3 = 0$.

4) The line (d_2) cuts the plane (P) at B. Determine the coordinates of B.

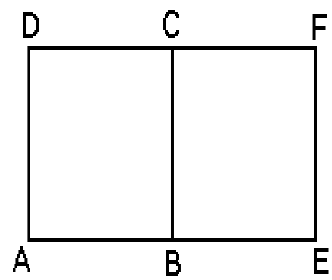
5) Prove that the line (D), passing through B and having $\vec{n}$ as a direction vector, cuts the line (d_1) at the point A (1 ; 0 ;1).

6) Let (Q) be the plane containing (d_1) and perpendicular to plane (P), and let M be a variable point on (d_2).
Prove that the distance from M to (Q) is equal to AB.

III- (3 points)

Consider, in an oriented plane, a direct rectangle AEFD such that: $(\overrightarrow{AE}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$, $AE = 2\sqrt{2}$ and $AD = 2$.

Designate by B and C the midpoints of [AE] and [FD] respectively.
Let S be the direct plane similitude that transforms A onto C and E onto B.



1) a- Determine the ratio k and an angle α of S.
b- Show that S(F) = E and deduce S(D).

2) Let W be the center of S and let h be the transformation defined by h = S o S.
a- Determine the nature and the characteristic elements of h.
b- Find h(D) and h(F) and construct the point W.

3) Designate by I the mid point of [BE].
a- Prove that W, C and I are collinear.
b- Express $\overrightarrow{WC}$ in terms of $\overrightarrow{WI}$.

4) The complex plane is referred to the orthonormal system $(A; \vec{u}, \vec{v})$ where $z_B = \sqrt{2}$ and $z_D = 2i$.
a- Find the complex form of S.
b- Determine the affix of W.

IV– (2 points)

Mr. Khalil has three sons: Sami, Farid and Zahi, all are married and have children.
The children in these three families are distributed as shown in the table below:

	Sami's Family	Farid's Family	Zahi's Family
Girls	2	1	3
Boys	2	3	1

The grandfather Khalil wants to choose randomly **one child from each family** to accompany him to their village.

1) What is the probability that he chooses three girls?

2) Consider the following events:

G: «The child chosen from Sami's family is a girl ».
B: «The child chosen from Sami's family is a boy ».
A: «The three chosen children are two girls and one boy ».

a- Prove that the probability p (A/G) is equal to $\frac{5}{8}$.

b- Calculate p (A/B) and p (A).

3) Let X be the random variable that is equal to the number of girls chosen by the grandfather. Determine the probability distribution of X.

V–(3 points)

In the plane referred to an orthonormal system $(O; \vec{i}, \vec{j})$, consider the points A(5 ; 0), F(3 ; 0) and the line (δ) of equation $x = \frac{25}{3}$.

Let (E) be the ellipse with focus F, directrix (δ), eccentricity **e** and having A as a principal vertex.

1) a- Verify that $\mathbf{e} = \frac{3}{5}$.

b- Verify that the point A' (-5 ; 0) is the other principal vertex of (E) and deduce the center of (E).

c- Write an equation of (E) and draw (E).

d- Calculate the area of the region bounded by the ellipse (E) and its auxiliary (principal) circle.

2) Let G and G' be the points on (E) with abscissa 3.

a- Write an equation of the tangent (D) to (E) at G, and an equation of the tangent (D') to (E) at G' .

b- Verify that the lines (D), (D') and (δ) intersect at the same point H on the axis of abscissas.

c- Show that $\tan \widehat{FHG} = \mathbf{e}$.

VI–(7points)

A- Consider the function f defined on IR by $f(x) = x + xe^{-x}$ and let (C) be its representative curve in orthonormal system $(O; \vec{i}, \vec{j})$ *(unit: 2 cm.)*

1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$ and show that the line (d) of equation y = x is an asymptote to (C).

b- Calculate $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to -\infty} \dfrac{f(x)}{x}$.

2) a- Calculate $f'(x)$ and $f''(x)$.

b- Set up the table of variations of f' and deduce that $f'(x) > 0$.

c- Show that the curve (C) has a point of inflection whose coordinates are to be determined.

d- Set up the table of variations of f.

3) Determine the coordinates of a point A on the curve (C) at which the tangent (T) is parallel to the line (d) of equation y = x.

4) Show that the equation $f(x) = 1$ has a unique root α and verify that $0.65 < \alpha < 0.66$.

5) Draw (d), (T) and (C).

6) Calculate, in cm^2, the area of the region bounded by the curve (C), the asymptote (d) and the two lines of equations x = 0 and x = 1.

7) Designate by g the inverse function of f , and by (G) the representative curve of g in the system $(O; \vec{i}, \vec{j})$.
Specify the asymptote and the asymptotic direction of (G), and draw (G).

B- Let f_n be the function defined on IR by $f_n(x) = x + x^n e^{-x}$ (n is a non-zero natural integer) and consider the sequence (U_n) defined by : $U_n = \int_0^1 [f_n(x) - x]dx$.

1) Determine the value of U_1.

2) Show that $0 \le x^n e^{-x} \le 1$ on [0 ; 1] , and deduce that the sequence (U_n) is bounded.

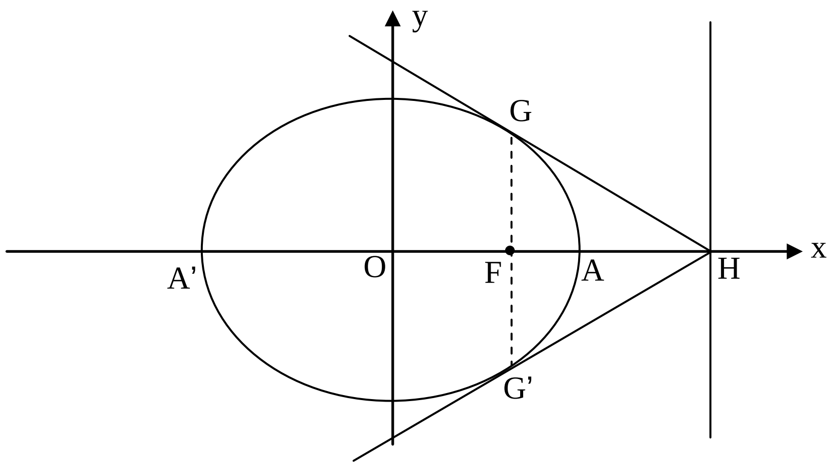
3) Prove that the sequence (U_n) is decreasing. Is the sequence (U_n) convergent? Justify.

Q1	MATH GS FIRST SESSION 2007		M
1	$\overline{z} = -\sqrt{3} + i = 2(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = 2e^{i\frac{5\pi}{6}}$.	d	4
2	$\left(e^{i\frac{\pi}{4}}\right)^{12} = e^{i(3\pi)} = -1.$	b	
3	$C_{10}^{6} - C_{9}^{6} = C_{9}^{6} + C_{9}^{5} - C_{9}^{6} = C_{9}^{5}$.	b	
4	$\int \frac{dx}{4+x^2} = \frac{1}{2}\int \frac{\frac{1}{2}dx}{1+(\frac{x}{2})^2} = \frac{1}{2}\arctan\frac{x}{2} + C$ OR : Among the given answers, the function $x \longrightarrow \frac{1}{2}\arctan\frac{x}{2}$ is the only one whose derivative is $\frac{1}{4+x^2}$.	c	
5	$\lim_{x\to+\infty} \frac{\ln(e^x+1)}{x} = \lim_{x\to+\infty} \frac{e^x}{e^x+1} = 1$	a	
6	$\frac{z_A - z_B}{z_A - z_C} = 2$; $\overrightarrow{BA} = 2\overrightarrow{CA}$; C is the mid point of [AB] .	a	

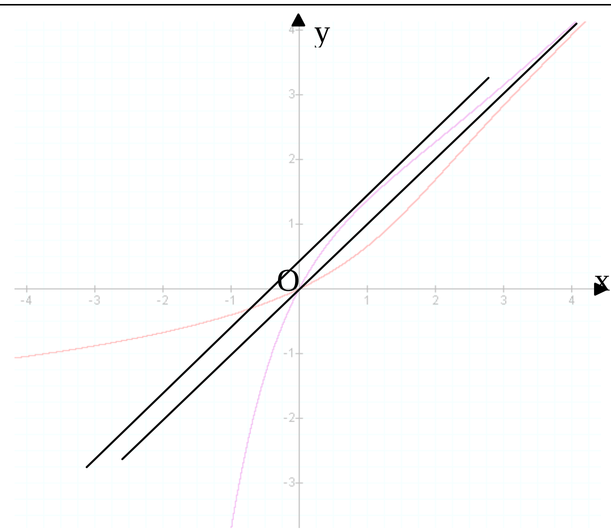
Q2	MATH GS FIRST SESSION 2007	M
1	$\overrightarrow{V_1}.\overrightarrow{V_2} = -1 + 1 = 0$, so (d_1) is orthogonal to (d_2). $(d_1)\cap(d_2)$: $\begin{cases} m = -t+1 \\ m-1 = t \\ 1 = -2t+4 \end{cases}$; $\begin{cases} m = 1 \\ t = 0 \\ 1 = 0+4 \text{ imp.} \end{cases}$ and since (d_1) is not parallel to (d_2), thus (d_1) and (d_2) are skew.	1
2	$\overrightarrow{n}.\overrightarrow{V_1} = -1 + 1 = 0$; $\overrightarrow{n}.\overrightarrow{V_2} = 1 + 1 - 2 = 0.$	½
3	$(d_1) \subset (P)$ since m – m +1 + 2 – 3 = 0 ; $\overrightarrow{N_P} \perp \overrightarrow{n}$ since $\overrightarrow{N_P}.\overrightarrow{n} = -1 - 1 + 2 = 0$. OR : I(0 ; – 1 ; 1) $\in (d_1)$; $\overrightarrow{IM}.(\overrightarrow{n} \wedge \overrightarrow{V_1}) = \begin{vmatrix} x & y+1 & z-1 \\ -1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0$; x – y +2z – 3 = 0.	1
4	$(d_2) \cap (P)$ = {B} ; – t + 1 – t – 4t +8 – 3 = 0 ; – 6t = – 6 ; t = 1, consequently B (0 ; 1 ; 2)	1
5	$\overrightarrow{n}$ and $\overrightarrow{V_1}$ are not parallel thus (D) is not confounded with (d_1). (D) $\begin{cases} x = -\lambda \\ y = \lambda + 1 \\ z = \lambda + 2 \end{cases}$; A (1 ; 0 ; 1) $\in$ (D) for $\lambda = -1$ and A(1 ; 0 ; 1) $\in (d_1)$ for m = 1 .	1
6	(Q) : $\overrightarrow{IM}.(\overrightarrow{V_1} \wedge \overrightarrow{N_P}) = 0$; $\begin{vmatrix} x & y+1 & z-1 \\ 1 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix} = 0$; 2x – 2(y + 1) + (z – 1)(– 2) = 0 ; x – y – 1 – z + 1 = 0 ; (Q) : x – y – z = 0.	1½

	$M(-t+1\,;t\,;-2t+4)$; $d(M\,;(Q)) = \dfrac{\lvert -t+1-t+2t-4 \rvert}{\sqrt{3}} = \sqrt{3}$. $\overrightarrow{AB}(-1;1\ 1)$; $AB = \sqrt{3}$. 🗣 OR: $(P) \perp (AB)$ and $(d_2) \perp (AB)$ then $(d_2) \,//\, (P)$ and all the points on (d_2) have the same distance from (P) ; so $d(B\,;(Q)) = BA$ since $(BA) \perp (Q)$.	
Q.3	MATH GS ◥ FIRST SESSION 2007	M
1-a	$S:\ A \longrightarrow C$ and $S: E \longrightarrow B$ $K = \dfrac{BC}{AE} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ and $\alpha = \left(\overrightarrow{AE}, \overrightarrow{CB}\right) = -\dfrac{\pi}{2}$.	½
1-b	$S(E) = B$; $\dfrac{BE}{EF} = \dfrac{\sqrt{2}}{2}$ and $\left(\overrightarrow{EF}, \overrightarrow{BE}\right) = -\dfrac{\pi}{2}$. Thus $S(F) = E$. $A \longrightarrow C$ thus S(D) is the 4th vertex of the direct rectangle with vertices C, B, E. because $E \longrightarrow B$ AEFD is a direct rectangle, so $S(D) = F$. $F \longrightarrow E$	1
2-a	$h = SoS$ is a similitude with center W , angle $-\pi$ and ratio $\dfrac{1}{2}$, consequently it is the homothecy $h\left(W, -\dfrac{1}{2}\right)$.	½
2-b	$h(D) = SoS(D) = S(F) = E$ and $h(F) = SoS(F) = S(E) = B$. Then $\{W\} = (ED) \cap (BF)$.	1
3-a	C midpoint of [DF] then h(C) is midpoint of [BE], thus h(C) = I and W, I, C are collinear.	1
3-b	$\overrightarrow{WI} = -\dfrac{1}{2}\overrightarrow{WC}$, $\overrightarrow{WC} = -2\overrightarrow{WI}$.	½
4-a	$z' = \dfrac{\sqrt{2}}{2} e^{-i\frac{\pi}{2}} z + b$; $z' = -\dfrac{\sqrt{2}}{2} i z + b$. $S(A) = C$; $z_C = b = \sqrt{2} + 2i$; $z' = -\dfrac{\sqrt{2}}{2} i.z + \sqrt{2} + 2i$ **1** **4-b** $z_W = \dfrac{\sqrt{2} + 2i}{1 + \dfrac{\sqrt{2}i}{2}} = \dfrac{4\sqrt{2}}{3} + \dfrac{2}{3} i$ ½	
QIV	MATH GS ◥ FIRST SESSION 2007	M
1	$P(3\text{ girls}) = \dfrac{2}{4} \times \dfrac{1}{4} \times \dfrac{3}{4} = \dfrac{6}{64} = \dfrac{3}{32}$	1
2a	A/G is the event: A girl from Farid's family with a boy from Zahi's family <u>or</u> a boy from Farid's family and a girl from that of Zahi. $P(A/G) = \dfrac{1}{4}\times\dfrac{1}{4}+\dfrac{3}{4}\times\dfrac{3}{4}=\dfrac{10}{16}=\dfrac{5}{8}$	½
2b	A/B is the event: A girl from Farid's family and a girl from Zahi's. $P(A/B) = \dfrac{1}{4}\times\dfrac{3}{4} = \dfrac{3}{16}$ and $P(A) = P(A \cap G) + P(A \cap B) = P(G) \times P(A/G) + P(B) \times P(A/B)$ $= \dfrac{1}{2}\times\dfrac{5}{8}+\dfrac{1}{2}\times\dfrac{3}{16}=\dfrac{10}{32}+\dfrac{3}{32}=\dfrac{13}{32}$	1
3	$P(X=3) = \dfrac{3}{32}$; $P(X=2) = \dfrac{13}{32}$; xi: 0, 1, 2, 3 — pi: $\dfrac{3}{32}$, $\dfrac{13}{32}$, $\dfrac{13}{32}$, $\dfrac{3}{32}$	1½

xi	0	1	2	3
pi	$\dfrac{3}{32}$	$\dfrac{13}{32}$	$\dfrac{13}{32}$	$\dfrac{3}{32}$

	$P(X=0)=\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{32}$; $P(X=1)=1-\left(\frac{3}{32}+\frac{13}{32}+\frac{3}{32}\right)=\frac{13}{32}$	
QV	MATH GS ◣ FIRST SESSION 2007	M
1a	$e=\frac{AF}{AH}=\frac{2}{\frac{25}{5}-5}=\frac{3}{5}$	½
1b	$\frac{A'F}{A'H}=\frac{5+3}{\frac{25}{3}+5}=\frac{3}{5}$; and A’ belongs to focal axis (AF),then A’ is a principal vertex of (E) . The center is O the midpoint of $[AA']$.	½
1c	$a = 5$, $c = 3$, $b^2 = a^2 - c^2 = 25 - 9$ $= 16$, thus the equation of (E) is $\frac{x^2}{25}+\frac{y^2}{16}=1.$ **1½** 	
1d	$A = \pi a^2 - \pi ab$ $= 25\,\pi - 20\,\pi = 5\pi\ u^2$ **1**	
2a	For $x=3$; $\frac{y^2}{16}=1-\frac{9}{25}=\frac{16}{25}$; $y=\frac{16}{5}$ or $y=-\frac{16}{5}$; $G\left(3;\frac{16}{5}\right)$ and $G'\left(3;-\frac{16}{5}\right)$ $(D): \frac{x_0x}{a^2}+\frac{y_0y}{b^2}=1$ gives $\frac{3x}{25}+\frac{\frac{16}{5}y}{16}=1$; $\frac{3x}{25}+\frac{y}{5}=1$; $y=-\frac{3}{5}x+5$ (D'): $y=\frac{3}{5}x-5$ by symmetry wrt the axis of abscissas.	1
2b	$\frac{3x}{5}-5=-\frac{3x}{5}+5$; $x=\frac{25}{3}$ and $y=0$ then $(D)\cap(D')=\{H\}$; $H=\left(\frac{25}{3},0\right)$	1
2c	$\tan\ F\hat{H}G=\frac{FG}{FH}=\frac{16/5}{25/3-3}=\frac{16}{5}\times\frac{3}{16}=\frac{3}{5}=e$	½

Q.6	MATH GS ◣ FIRST SESSION 2007	M
A- 1.a	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}(x + x e^{-x}) = +\infty + 0 = +\infty$ $\lim_{x\to+\infty}(f(x) - x) = \lim_{x\to+\infty}\left(xe^{-x}\right) = 0$ and so (d) is asymptote to (C).	1
1.b	$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}(x + xe^{-x}) = -\infty - \infty = -\infty$; $\lim_{x\to-\infty}\frac{f(x)}{x} = \lim_{x\to-\infty} 1 + e^{-x} = +\infty$	1
2.a	$f'(x) = 1 + e^{-x} - xe^{-x} = 1 + (1 - x)\,e^{-x}$. $f''(x) = -e^{-x} - (1 - x)e^{-x} = e^{-x}(-1 - 1 + x) = (x - 2)e^{-x}$.	1

2.b	$\begin{array}{c\|ccccc} x & -\infty & & 2 & & +\infty \\ \hline f''(x) & & - & 0 & + & \\ \hline f'(x) & +\infty & \searrow & 1-\frac{1}{e^2} & \nearrow & 1 \end{array}$ $f'(x) \geq 1-\frac{1}{e^2}$, thus $f'(x) > 0$.	1
2.c	$f''(x)$ vanishes for $x = 2$ and changing signs, so (C) has a point of inflection $I\left(2,\ 2+\frac{2}{e^2}\right)$	1
2.d	$\begin{array}{c\|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$ **1** **3-** $f'(x) = 1$; $(1-x)e^{-x} = 0$; $x = 1$; $A(1,\ 1+1/e)$ **1**	
4	f is continuous and strictly increasing from - ∞ to + ∞, so f(x) = 1 has a unique root α . f(0. 65) = 0.98 < 1 and f(0.66) = 1.,0011 > 1 thus $0.65 < \alpha < 0.66$.	1
5	$\int_0^1 xe^{-x}.dx = -\left[(x+1)e^{-x}\right]_0^1 =$ $-\left[2e^{-1}-1\right] = 1-\frac{2}{e}$; $A = 4(1-\frac{2}{e})$. $= 4-\frac{8}{e} = 1.057\,cm^2$. **1½** 	1
7	The asymptote of (G) is the line of equation y = x ; the asymptotic direction of (G) is the axis of abscissas.	1
B.1	$U_1 = 1-\frac{2}{e}$	½
2	$0 \leq x \leq 1$; $0 \leq x^n \leq 1$; $-1 \leq -x \leq 0$; $e^{-1} \leq e^{-x} \leq 1$ thus $0 \leq x^n e^{-x} \leq 1$ $0 \leq \int_0^1 x^n e^{-x} dx \leq \int_0^1 1.dx$; $0 \leq U_n \leq 1$, so (U_n) is bounded.	1
3	$U_{n+1} - U_n = \int_0^1 (x^{n+1}e^{-x} - x^n e^{-x}).dx = \int_0^1 x^n e^{-x}(x-1)dx$; but $x - 1 \leq 0$ on [0 ; 1] then $x^n e^{-x}(x-1) \leq 0$ on [0 ; 1] and $U_{n+1} - U_n \leq 0$ consequently (U_n) is decreasing. (U_n) is decreasing and has a lower bound 0 , so it converges to a limit ℓ	1