وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع الاجتماع والاقتصاد

دورة سنة 2007 العادية

عدد المسائل : اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– (4points)

An employee deposits an amount of 10 000 000 LL in a bank at 9.6 % annual interest rate, compounded **monthly**.
At the **end of every month**, he adds 200 000 LL to the amount in his account.
Designate by S_0 the initial amount deposited by this employee ($S_0 = 10\ 000\ 000$), and by S_n the amount in his account at the end of the nth month.

1) Verify that $S_1 = 10\ 280\ 000$.
2) Justify that $S_{n+1} = 1.008\ S_n + 200\ 000$.
3) Consider the sequence (U_n) defined by $U_n = S_n + 25\ 000\ 000$.
   a- Prove that the sequence (U_n) is a geometric sequence of common ratio 1.008 .
   b- Express U_n in terms of n, and deduce S_n in terms of n.
   c- In how many months would the amount of money in this employee's account exceed 40 000 000 LL for the first time?

II– (4 points).

A library has **100** calculators distributed according to type and year of manufacture as shown in the following table:

	Type P	Type G	Type O
Manufactured in 2007	20	15	25
Manufactured in 2006	10	12	18

A- A customer chooses at random **one** of these calculators.
1) Knowing that the chosen calculator was manufactured in 2007, show that the probability that it is of type G is equal to 0.25 .
2) What is the probability that the chosen calculator is of the type O and manufactured in 2007?
3) The prices of the calculators are given in the following table:

	Type P	Type G	Type O
Manufactured in 2007	100 000 LL	80 000 LL	60 000 LL
Manufactured in 2006	50 000 LL	40 000 LL	30 000 LL

What is the probability that the price of the chosen calculator does not exceed 70 000 LL?

B- In this part, the customer chooses randomly and simultaneously **two** out of these **100** calculators.

1) What is the probability that the two chosen calculators are manufactured in 2007?

2) What is the probability that the price of the two chosen calculators is 180 000 LL?

III – (4points)

The development in the number of monitors in a sports club during the last 6 years is as shown in the following table :

Year	2001	2002	2003	2004	2005	2006
Rank of the year: x_i	1	2	3	4	5	6
Number of monitors: y_i	15	20	25	28	30	32

1) Draw, in a rectangular system, the scatter plot of the points associated to the distribution $(x_i ; y_i)$.

2) Calculate the coordinates of the center of gravity G and plot this point in the preceding system.

3) Determine an equation of $D_{y/x}$, the regression line of y in terms of x, and draw this line in the same system.

4) Suppose that the above pattern remains valid till the year 2015.

a- Estimate the number of monitors in this club in the year 2010.

b- During which year would the number of monitors in this club exceed 50 for the first time ?

IV**– (8points).**

Let f be the function defined, on $[0 ; +\infty[$, by : $f(x) = x + 1 + e^{-x+1}$ and designate by (C) its representative curve in an orthonormal system $(O ; \vec{i} , \vec{j})$.

A-1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$.

b- Prove that the line (d) of equation y = x + 1 is an asymptote to (C).

2) Calculate $f'(x)$ and set up the table of variations of f.

3) Draw (d) and (C).

4) Show that the equation f(x) = 4 has a unique root α and verify that: $2.84 < \alpha < 2.86$.

5) Calculate the area of the region bounded by the curve (C), its asymptote (d) and the two lines of equations x = 0 and x = 1.

B- In all what follows, let $\alpha = 2.85$.

A factory produces x thousands of toys; $(1 \leq x \leq 5)$.

The cost of production, in millions of LL, is given by : $C(x) = x+1+e^{-x+1}$.

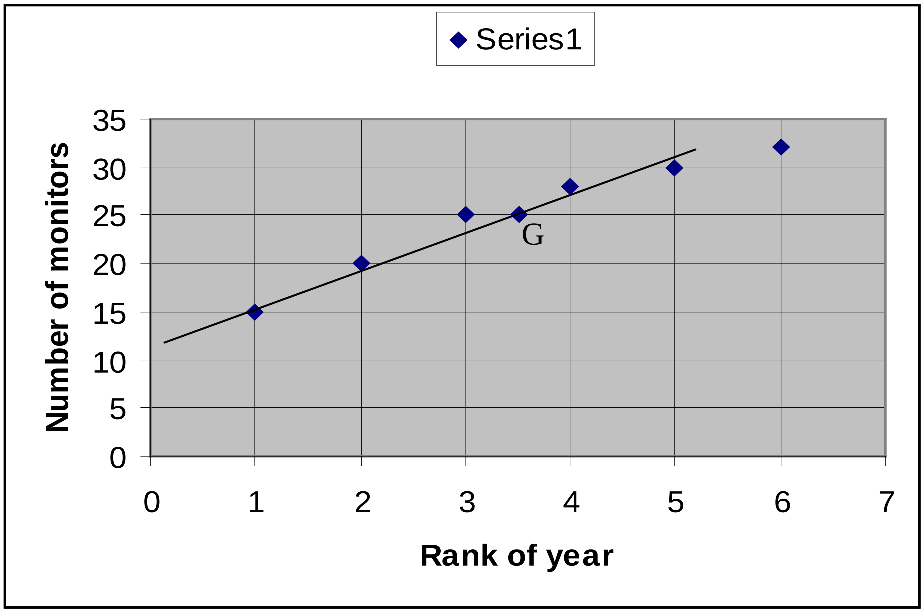
1) Calculate the cost of production of 2000 toys.

In this case, what is the cost of production of one toy?

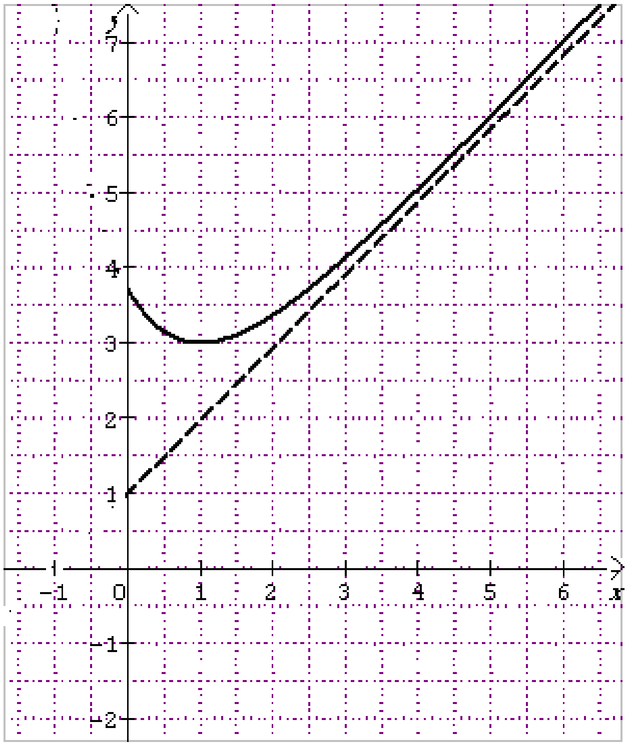
2) Find the number of toys that should be produced for which the cost of production would be 4 million LL.

Q1	SHORT ANSWERS	MARKS
1	$S_1 = 10\,000\,000 + 10\,000\,000 \times \frac{0.096}{12} + 200\,000 = 10\,280\,000.$	½
2	$S_{n+1} = S_n + S_n(0.008) + 200\,000 = 1.008S_n + 200\,000.$	1
3.a	$U_{n+1} = S_{n+1} + 25\,000\,000 = 1.008S_n + 200\,000 + 25\,000\,000$ $= 1.008S_n + 25\,200\,000 = 1.008(S_n + 25\,000\,000) = 1.008U_n$ Thus (U_n) is a geometric sequence of common ratio 1.008.	2
3.b	$U_n = U_0(1.008)^n$ where $U_0 = S_0 + 25\,000\,000 = 35\,000\,000$ $U_n = 35\,000\,000(1.008)^n$ and $S_n = 35\,000\,000(1.008)^n - 25\,000\,000.$	1 ½
3.c	$S_n > 40\,000\,000$; $35\,000\,000(1.008)^n - 25\,000\,000 > 40\,000\,000$ $(1.008)^n > \frac{65}{35}$; $n > \frac{\ln(\frac{13}{7})}{\ln(1.008)}$; $n > 77.689$ In 78 months the amount exceeds 40 000 000 LL.	2

Q2	SHORT ANSWERS	MARKS
A1	Let A be the event : "the chosen calculator was manufactured in 2007" $P(G/A) = \frac{15}{60} = 0.25.$	1
A2	$P(O \cap A) = \frac{25}{100} = 0.25.$	1
A3	P(price < 70 000) = P(50 000) + P(40 000) + P(60 000) + P(30 000) $= \frac{10+12+18+25}{100} = \frac{65}{100} = 0.65.$	1 ½
B1	$P(\text{2 calculators manufactured in 2007}) = \frac{C_{60}^2}{C_{100}^2}$ $= \frac{1770}{4950} = 0.357.$	1 ½
B2	$P(100\,000 \text{ and } 80\,000) = \frac{C_{20}^1 \times C_{15}^1}{C_{100}^2}$ $= \frac{20 \times 15}{4950} = \frac{300}{4950} = 0.06.$	2

Q3	SHORT ANSWERS	MARKS
1		1
2	$\overline{X} = 3.5$, $\overline{Y} = 25.$	1 ½
3	y = 3.371x + 13.2.	1 ½
4a	For x = 10; y = 33.71 + 13.2 = 46.91 thus 47 monitors.	1 ½
4b	3.371x + 13.5 > 50 ; 3.371x > 36.5 ; x > 10.827. In 2011 the number of monitors exceeds 50 for the first time.	1 ½

Q4	SHORT ANSWERS	MARKS
A1a	$\lim_{x\to+\infty} f(x) = +\infty + 0 = +\infty.$	½
A1b	$\lim_{x\to+\infty} [f(x) - (x+1)] = \lim_{x\to+\infty} e^{-x+1} = 0.$	1
A2	$f'(x) = 1 - e^{-x+1}$, $f'(x) \geq 0$ for $1 \geq e^{-x+1}$, $0 \geq -x+1$, $x \geq 1$. $\begin{array}{c\|ccccc} x & 0 & & 1 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 1+e & \searrow & 3 & \nearrow & +\infty \end{array}$	2

A3		2
A4	The line of equation y = 4 cuts (C) in a unique point of abscissa α , so the equation f(x) = 4 has a unique root α. f(2.84) = 3.99 < 4 ; f(2.86) = 4.01 > 4 , thus 3.99 < α < 4.01.	2
A5	$A = \int_0^1 e^{-x+1}dx = -\left[e^{-x+1}\right]_0^1$ $= -(1-e) = (e-1)u^2.$	2
B1	$C(2) = 3 + e^{-1} = 3.367$ meaning 3 367 000 LL The cost of production of a toy is : $\frac{3367000}{2000}$ =1 683.5LL.	2 ½
B2	C(x) = 4 for x = α meaning 2850 toys.	2