المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: الثانوية العامة الفرع: العلوم العامة نموذج رقم -٢- المدّة : أربع ساعات

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (2 points)

Choose the true answer and justify.

Questions	Answers		
	A	**B**	**C**
1) $\arccos\left(\sin\frac{18\pi}{5}\right) =$	$\frac{18\pi}{5}$	$\frac{9\pi}{10}$	$\frac{-13\pi}{5}$
2) If θ is an argument of Z, then the argument of $\frac{1-i}{(\overline{Z})^2}$ is	$2\theta-\frac{\pi}{4}$	$\frac{\pi}{4}-2\theta$	$2\theta+\frac{\pi}{4}$
3) $\lim\limits_{x\to1}\frac{\int_1^{x^2}\sqrt{1+t^2}\,dt}{x^2-1}$	$\frac{\sqrt{2}}{2}$	$\sqrt{2}$	$2\sqrt{2}$
4) A solution of the differential equation $y'-xy=0$ is	$y=e^{x^2}$	$y=e^{\frac{x^2}{2}}$	$y=\frac{x^2}{2}$
5) $\int_1^2\frac{2x}{x^2-2x+2}dx$	$\ln2+\frac{\pi}{2}$	$\ln2+\frac{\pi}{4}$	$2\ln2+\frac{\pi}{4}$

II- (2,5 points).

The space is referred to an orthonormal system $(O,\vec{i},\vec{j},\vec{k})$.

Consider the points : A (0 ; 0 ; 2), B (0 ; 4 ; 0) and C (2 ; 0 ; 0).

1)

a) Verify that an equation of the plane (ABC) is : 2x + y + 2z = 4.

b) Determine the nature of the triangle ABC.

2) Consider (Δ), the altitude to [BC] in the triangle ABC, and let $\vec{N}$ be the vecteur with coordinates (2,1,2).

a) Show that $\vec{N}\wedge\overrightarrow{BC}$ is a direction vector of (Δ).

b) Determine a system of parametric equations for the line (Δ).

3) Let (Δ') be the bissector the angle B in the triangle ABC.

Show that the parametric equations of (Δ') are : $\begin{cases} x = t \\ y = 4 - 4t \quad t \in \mathbb{R} \\ z = t \end{cases}$

4) Let H be the point of intersection of the lines (Δ) and (Δ').

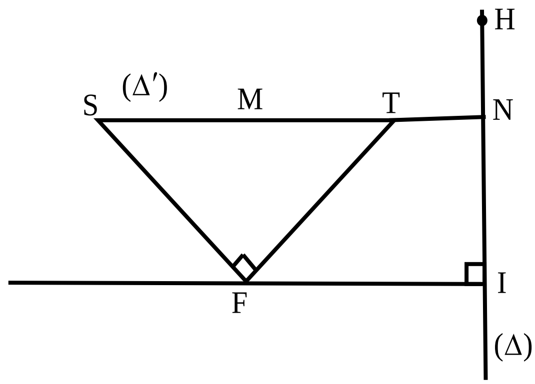
a) Show that the point H is the orthogonal projection of the point O on the plane (ABC).

b) Deduce the volume of the tetrahedron OABC.

III- (2,5 points)

In the next figure:

- The triangle FIH is a right isosceles triangle at I .
- IF = IH = 3.
- N is a variable point on (HI).
- $(\acute{\Delta})$ is the parallel through N to (IF).
- T, M and S are three points on $(\acute{\Delta})$ so that NT = TM = MS.
- The triangle SFT is right angled at F.



Partie A

1) Show that M moves on an ellipse (E) with focus F, directrice (Δ) and excentricity $e = \frac{1}{2}$.

2) A is a point on [IF] so that IA = 2 and A' is the symmetric of I with respect to F.

a) Show that A et A' are two vertices of (E).

b) Determine the center O of (E) and Show that F is the midpoint of [OA]. Determine the second focus F' .

3) The circle with center F and radius OA intersects the non focal axis of (E) at B and B' .

Show that B and B' are two vertices of (E).

4) Let L a point so that $\overrightarrow{FL} = \frac{1}{2}\overrightarrow{IH}$.

a) Show that L is a point on (E).

b) Calculate LF + LF’. Deduce LF' .

Partie B

The plane is referred to an orthonormal system $(O; \vec{i}, \vec{j})$ so that $\vec{i} = \overrightarrow{OF}$.

1) Write an equation of (E).

2) The line (LI) intersects the y- axis at G.

a) Show that (LI) is tangent to (E).

b) Calculate the area of the domain bounded by region inside of the triangle OGI and outside (E).

3) (C) is the hyperbola with center $J(\sqrt{3},0)$ and asymptotes (JB) and (JB’) and having A one of its vertices.

Write an equation of (C).

IV- (3 points)

In the oriented plane, consider the right isosceles triangle ABC so that AB = AC = 4 cm and $(\overrightarrow{AB},\overrightarrow{AC}) = \frac{\pi}{2} + 2k\pi$ Denote by D the symmetric point of A with respect to B, O the midpoint of [CD].and (T) the circle with diameter [CD].
Denote by S the similitude that maps D onto B and B onto C.

1) Determine the ratio k and the angle α of S.
2) Let I be the center of S and h the transformation defined by h = $S o S$.
   a) Show that h(I) = I and h(D) = C.
   b) Deduce that I is a point on a circle (T) and that IC = 2ID.
   c) Show that ID = 4.
   d) Deduce that I is the 4th vertex of a rectangle and plot I.
3) Consider the orthonormal system (A, $\vec{U}$, $\vec{V}$), so that $\vec{U} = \frac{1}{4}\overrightarrow{AB}$, $\vec{V} = \frac{1}{4}\overrightarrow{AC}$.

   Determine the complex form of S.
4) For all n ∈ N consider the sequence of points (Dn) defined by
   $D_0=D$ and $D_{n+1}= S(D_n)$.
   (U_n) is the sequence defined by U_n= Area of the triangle I D_nD_{n+1}
   a) Calculate U_n in terms of n.
   b) Calculate in terms of n, the product $P=U_0 \times U_1 \times \dots\dots\dots \times U_n$.

V- (3 points)

y is a real number belongs to the interval [0 ; 80].
A box U contains 100 small wooden cubes out of which 60 are blue and the others are red .

- The blue cubes 40 % are labeled by a circle, 20 % are labeled by a rhombus and the others are labeled by a star.
- The red cubes 20 % are labeled by a circle, y % are labeled by a rhombus and the others are labeled by a star.

<u>Part A:</u> experiment 1.

One cube is selected randomly from the box U.

1) Show that the probability of selecting one cube labeled by a rhomb is equal to 0.12 + 0.004y.
2) Determine y so that the probability of selecting a cube labeled by a rhombus and the probability of selecting a cube that is labeled by a star are the same.
3) Determine y so that the events « select one blue cube» and « select one cube labeled by a rhombus » are independent.

<u>Part B:</u> experiment 2

In this part we select randomly one cube of the box U.
If the cube is blue, we place back it in U and we select another.
Otherwise, we put it aside, we select simultaneously 2 other cubes from the box.

1) Calculate the probability of selecting only one red cube.
2) Calculate the probability so that the selected cubes have the same color.

VI- (7 points)

Part A

1) Let f be the function defined , over $]0;+\infty[$, as $f(x) = \frac{1+2\ln x}{x^2}$ and (C) its representative curve.
   h is the function defined , over $]0;+\infty[$, as $h(x) = \frac{1}{x}$ and (C') its representative curve.
   a) Show that the line with equation x = 0 is a vertical asymptote to (C).

b) Calculate $\lim_{x \to +\infty} f(x)$. Deduce that the x-axis is a horizontal asymptote to (C).

c) Set up the table of variations of f over $]0;+\infty[$.

2) Let g be the function defined over $]0;+\infty[$ as $g(x) = 1 - x + 2\ln(x)$.

a) Discuss the variations of g over $]0;+\infty[$.

b) Show that the equation g(x) = 0 has 2 roots α and β so that $3{,}51 < \alpha < 3{,}52$.

c) Prove that $f(x) - h(x) = \frac{g(x)}{x^2}$.

d) Deduce the relative position of (C) and (C').

e) Plot (C) and (C').

3) a) Calculate the derivative of $d(x) = \frac{1+\ln(x)}{x}$. Deduce the antiderivative F(x) of f verifying that $F(1) = -3$.

b) Show that $F(\alpha) - F(1) = 2 - \frac{2}{\alpha}$, Give a geometric interpretation to the result.

Part B

For all $n \geq 1$ Denote by **(**I_n) the sequence defined by $I_n = F(n+1) - F(n)$

1) Prove that $I_1 + I_2 < 2 - \frac{2}{\alpha}$.

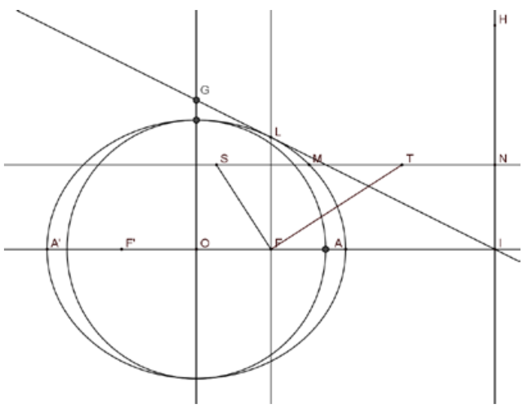
2) Calculate $S_n = I_1 + \ldots\ldots + I_n$ in terms of n and calculate its limit.

المادة: الرياضيات الشهادة: الثانوية العامة الفرع: العلوم العامة نموذج رقم -٢- المدّة : أربع ساعات	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

	Answers key.	
Q1		Note
1	B : $\arccos\left(\sin\frac{18\pi}{5}\right)=\arccos\left(\cos\frac{9\pi}{10}\right)=\frac{9\pi}{10}$.	0,25
2	A : $\arg\frac{1-i}{(\bar{z})^2}=\frac{-\pi}{4}+2\theta$.	0,25
3	B :Hospital rule $\lim\limits_{x\to1}\frac{\sqrt{1+x^4}.2x}{2x}=\sqrt{2}$	0,5
4	B : $\frac{y'}{y}=x$, then $\int\frac{y'}{y}=\int x$, then $\ln\|y\|=\frac{x^2}{2}+c'$, then $y=ce^{\frac{x^2}{2}}$ or verification	0,5
5	A : $\int_1^2\frac{2x-2}{x^2-2x+2}dx+\int_1^2\frac{2}{x^2-2x+2}dx=\left[\ln\left(x^2-2x+2\right)\right]_1^2+2\left[\arctan(x-1)\right]_1^2=\ln 2+\frac{\pi}{2}$	0,5

Q2		Note
1.a	$2x_A+y_A+2z_A=4$ et $2x_B+y_B+2z_B=4$ et $2x_C+y_C+2z_C=4$ Or $\det\left(\overrightarrow{AM},\overrightarrow{AB},\overrightarrow{AC}\right)=0$	0,25
1.b	AB=BC=$\sqrt{20}$	0,25
2.a	$\vec{N}$ is a normal vector to the plane (ABC) $\vec{N}\wedge\overrightarrow{BC}$ is a direction vector to (Δ). Since (Δ) is orthogonal to $\vec{n}$ and (BC).	0,25
2.b	(Δ) is an altitude through A ; then its parametric equations : $\begin{cases}x=4m\\y=2m\\z=-5m+2\end{cases}$	0,5
3	(Δ’) = (BI) so that I is the midpoint of [AC].	0,25
4.a	$H\in\Delta'$ for $t=\frac{8}{9}$ and $H\in\Delta$ for $m=\frac{2}{9}$ and H is the orthocenter of ABC . The coordinates of the point H are $\left(\frac{8}{9};\frac{4}{9};\frac{8}{9}\right)$. $\overrightarrow{OH}=\frac{4}{9}\vec{n}$	0,5
4.b	Area of the triangle ABC= 6 and OH = $\frac{4}{3}$ The volume of the tetrahedron = $\frac{8}{3}$	0,25

Q3		Note
		0,25
1	$\frac{MF}{MN}=\frac{1}{2}$,([FM] is a mediane in the triangle STF equal half of ST. Then M moves on the ellipse with focus F and directrice (Δ) and excentricity ½..	0,25
2.a	A is a point on the focal axis and AF=$\frac{1}{2}$ AI then A is a vertex on the ellipse. Same A'F=$\frac{1}{2}$ A'I then A'is the second vertex .	0,25
2.b	O is the midpoint of [AA'],OF=1 and OA=2 then F midpoint of [OA] and F' midpoint of [OA'].	0,25
3	Pythagorus .OB = $\sqrt{3}$ = b because b=$\sqrt{a^2-c^2}$ = $\sqrt{3}$.	0,25
4.a	LF=$\frac{1}{2}$ IF=$\frac{1}{2}$ LH' so that H' orthogonal projection of L on (Δ).Then L is on the ellipse.	0,25
4.b	LF +LF'=2a=4 but LF=1,5 then LF'=2.5.	0,25
	$\frac{x^2}{4}+\frac{y^2}{3}=1$	
1	The equation of the tangent (T) at the point L(1,1.5) to (E) is y=-0,5x+2 that is the same of the line (IL).	0,25
2.a	Area of the domain = Area of triangle OIG - $\frac{\pi ab}{4}$ -	0,25
2.b	(JB) is perpendicular to (JB') then the hyperbola is rectangular so that a=2-√3 Its equation is $(x-\sqrt{3})^2-(y)^2=(2-\sqrt{3})^2$	0,25

Q4		Note
1	$\frac{BC}{DB}=\frac{4\sqrt{2}}{4}=\sqrt{2}=k$ and $\left(\overrightarrow{DB},\overrightarrow{BC}\right)=-\frac{\pi}{4}+2k\pi$.	0,25
2.a	$h(I)=SoS(I)=S(I)=I$ and $h(D)=SoS(D)=S(B)=C$ and $\alpha+\alpha=-\frac{\pi}{2}; k\times k=2$, then h is a similitude with ratio 2 and angle $-\frac{\pi}{2}$.	0,25
2.b	$\left(\overrightarrow{ID},\overrightarrow{IC}\right)=-\frac{\pi}{2}+2k\pi$, then $I\in$ to the circle (T) with diameter [CD] . IC = 2ID.	0,25
2.c	CD^2= 5 ID^2 ; then ID = 4 .	0,25
2.d	It is a parallelogram with an angle of 90 degree then it is a rectangle .	0,25
3	$z'=(1-i)z$ -4+8i.	0,5

4.a	$\frac{Un+1}{Un}=2$ then the sequence is geometric with ratio 2 and the first term U0 = 8	0,75
4.b	$P=U_0 \text{ x} U_1 \text{x} \ldots\ldots\ldots \text{x} U_n = 8^{n+1}.\ 2^{1+2+\ldots+n} = 8^{n+1}.2^{\frac{n(n+1)}{2}}$	0,5

Q5		Note
	Part A	
1	P(L) = P(L/R) × P(R) + P(L/B) × P(B) = $\frac{y}{100}\times\frac{40}{100}+\frac{20}{100}\times\frac{60}{100}=0.004y+0.12$	0,5
2	P(E) = P(E/B) × P(B) + P(E/R) × P(R) = $\frac{80-y}{100}\times\frac{40}{100}+\frac{40}{100}\times\frac{60}{100}$ P(E) = P(L) *then* $0{,}008y = 0{,}44$, *then* $y = 55$.	0,5
3	P(L/B) = P(L) = P(L/R), then y = 20.	0,5
	Part B	
1	P(R)= p(B)x p(R) +P(R)x P(BB)=$\frac{6}{10}$x$\frac{4}{10}$ + $\frac{4}{10}$ x$\frac{C_{60}^2}{C_{99}^2}$	0,75
2	P(same color)= p(B)x p(B) +P(R)x P(RR)=$\frac{6}{10}$x$\frac{6}{10}$ + $\frac{4}{10}$ x$\frac{C_{39}^2}{C_{99}^2}$	0,75

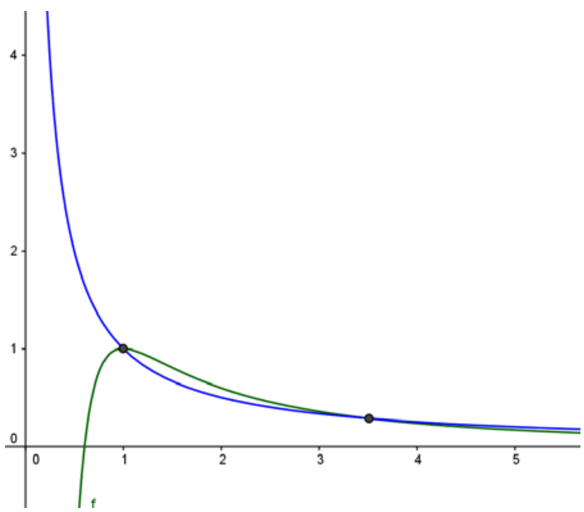
Q6		Note
	Part A	
1.a	$\lim_{x\to 0} f(x) = -\frac{\infty}{0} = -\infty$, *then* $x = 0$ A.V.	0,25
1.b	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} + \frac{1}{x^2} + \frac{2\ln x}{x^2} = 0+0=0$. then y = 0 AH.	0,5
1.c	$f'(x) = \frac{-4x\ln x}{x^4}$	0,5

x	0		1		$+\infty$
f '(x)		+	0	-	
f(x)	$-\infty$	↗	1	↘	0

2.a	(table below)	0,5

x	0		2		$+\infty$
$g'(x)$		$+$	0	$-$	
$g(x) = 1 - x + 2\ ln(x)$	$-\infty$	↗	$-1+2ln2$	↘	$-\infty$

2.b	g(1) = 0, g is continuous with g(3.51) x g(3.52)< 0 ,then the equation g(x) = 0 has 2 roots α and 1 so that 3.51 < α < 3.52 .	0,25

<table>
<tr><td>2.c</td><td>f(x) –h(x)= $\frac{g(x)}{x^2}$</td><td>0,25</td></tr>
<tr><td>2.d</td><td>
<table>
<tr><td>x</td><td>0</td><td>1</td><td>α</td><td>$+\infty$</td></tr>
<tr><td>f(x) –h(x)</td><td>-</td><td>0 +</td><td>0 -</td><td></td></tr>
<tr><td>position</td><td>(C’) above (C)</td><td>(C’) below (C)</td><td>(C’)above (C)</td><td></td></tr>
</table>
(C) et (C’) have 2 common points with abscissas 1 and α .</td><td>0,5</td></tr>
<tr><td>2.e</td><td></td><td>0,75</td></tr>
<tr><td>3.a</td><td>d’(x)= -ln(x)/x^2 then f(x) = $\frac{1}{x^2}$ -2 d’(x) then F(x)= $\frac{-1}{x}$ -2d(x) +K= $\frac{-3-2\ln x}{x}$ +K.
but F(1)=−3 then k= 0 and therefore F(x)= $\frac{-3-2\ln x}{x}$.</td><td>0,75</td></tr>
<tr><td>3.b</td><td>F(α)-F(1)= $\frac{-3-2\ln\alpha+3\alpha}{\alpha}$ but g(α)=0 then –2ln(α)=1-α and therefore
F(α) – F(1) = $2-\frac{2}{\alpha}$.
$2-\frac{2}{\alpha}$ = Area of the domain bounded by (C), the x-axis and the two lines x=1 and x=α</td><td>0.75</td></tr>
<tr><td></td><td>Part B</td><td></td></tr>
<tr><td>3.a</td><td>$I_1 + I_2$=F(3)-F(1)= Area of the domain bounded by (C) , the x-axis and the two lines x=1 and x= 3 or 3 < α then F(3)-F(1) < F(α) – F(1)= Area of the domain bounded by (C), the x-axis and the two lines x=1 and x=α .</td><td>1</td></tr>
<tr><td>3.b</td><td>Sn= I_1+…….+I_n = F(2)-F(1) +F(3)-F(2) +………..+F(n+1)-F(n) =
= F(n+1)-F(1)= $\frac{-3-2\ln(n+1)}{n+1}$ +3.
Lim Sn= 3 when n→ $+\infty$.</td><td>1</td></tr>
</table>