دورة سنة ٢٠٠٨ الاكمالية الاستثنائية	امتحانات الشهادة الثانوية العامة الفرع: علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل : أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the following table, only one of the proposed answers to each question is correct.
Write the number of each question and give, with justification, the corresponding answer.

Nº	Questions	Answers			
		a	b	c	d
1	If $\frac{\pi}{6}$ is an argument of z, then an argument of $\frac{i}{\bar{z}^2}$ is :	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$-\frac{5\pi}{6}$	$\frac{5\pi}{6}$
2	If $z = -\sqrt{3} + e^{i\frac{\pi}{6}}$, then the exponential form of z is:	$e^{\frac{5\pi}{6}i}$	$e^{\frac{7\pi}{6}i}$	$\sqrt{3}e^{-\frac{\pi}{6}i}$	$e^{-\frac{5\pi}{6}i}$
3	If z and z' are two complex numbers such that $\lvert z \rvert = 2$ and $z' = z - \frac{1}{\bar{z}}$, then $\lvert z' \rvert =$	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
4	If z is a complex number with $\lvert z \rvert = \sqrt{2}$, then $\lvert \bar{z} + i\bar{z} \rvert =$	$2\sqrt{2}$	2	$\sqrt{2}$	$\frac{\sqrt{2}}{2}$

II- (4 points)

In the space referred to an orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points:
A(0 ; 1 ; – 2), B(2 ; 1; 0), C(3 ; 0; – 3) and H(2 ; 2 ; – 2).

1) Show that x – 2y – z = 0 is an equation of the plane (P) determined by the points H , A and B and verify that the point C does not belong to this plane.

2) a- Show that triangle HAB is isosceles of vertex H.
b- Show that (CH) is perpendicular to (P).
c- Prove that CA = CB and determine a system of parametric equations of the interior bisector (δ) of angle ACB.

3) Let T be the orthogonal projection of H on plane (ABC).
Prove that T belongs to (δ) .

III- (4 points)

In order to prevent a certain disease, we vaccinated 40% of persons of a population.
Then we noticed that 85% of the vaccinated persons were not affected by the disease and that 75% of the persons who were not vaccinated are affected by the disease.
A person is chosen randomly from this population.
Consider the following events:

D : « the chosen person is affected by the disease».
V : « the chosen person is vaccinated ».

1) a- Verify that the probability of the event $D \cap V$ is equal to $\dfrac{6}{100}$.

b- What is the probability that the chosen person is affected by the disease and is not vaccinated?
c- Deduce the probability P(D).

2) The chosen person is not affected by the disease.
Calculate the probability that he/she is vaccinated.

3) In this part, suppose that this population is formed of 300 persons.
We choose randomly 3 persons from this population.
What is the probability that at least one, among the 3 chosen persons, is vaccinated?

IV- (8 points)

Let f be the function defined over $]1; +\infty[$ by $f(x) = x - \dfrac{1}{x \ln x}$ and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim\limits_{x \to 1} f(x)$ and deduce an asymptote to (C).

2) Calculate $\lim\limits_{x \to +\infty} f(x)$. Prove that the straight line (d) of equation y = x is an asymptote to (C) and study the position of (C) and (d).

3) Calculate f '(x) and show that f is strictly increasing.
Set up the table of variations of f.

4) Show that the equation f(x) = 0 has a unique root α and verify that $1.5 < \alpha < 1.6$.

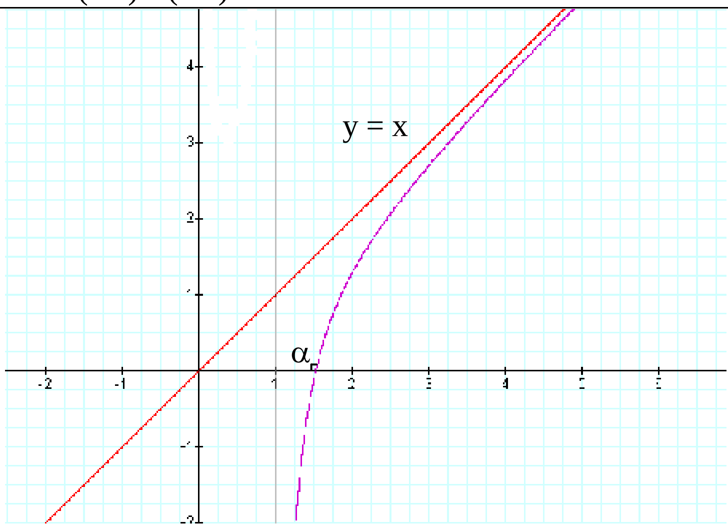
5) Draw (d) and (C).

6) a- Calculate the area A(t) of the region limited by the curve (C) , the straight line (d) and the two straight lines of equations x = e and x = t where $t > e$.
b- Show that for all $t > e$, we have $A(t) < t$.

QI	Answer		Mark
1	$\arg\left(\frac{i}{\bar{z}^2}\right) = \arg(i) - 2\arg(\bar{z})\ [2\pi] = \frac{\pi}{2} + 2\left(\frac{\pi}{6}\right)\ [2\pi] = \frac{5\pi}{6}\ [2\pi]$	**d**	1
2	$z = -\sqrt{3} + \frac{\sqrt{3}}{2} + \frac{1}{2}i = -\frac{\sqrt{3}}{2} + \frac{1}{2}i = e^{i\left(\frac{5\pi}{6}\right)}$	**a**	1
3	$z' = \frac{z\bar{z} - 1}{\bar{z}} = \frac{\|z\|^2 - 1}{\bar{z}} = \frac{3}{\bar{z}}$, so $\|z'\| = \frac{3}{\|z\|} = \frac{3}{2}$	**c**	1
4	$\|\bar{z} + i\bar{z}\| = \|\bar{z}(1+i)\| = \|\bar{z}\| \times \|1+i\| = \sqrt{2} \times \sqrt{2} = 2$	**b**	1

QII	Answer	Mark
1	$x_A - 2y_A - z_A = 0 - 2 + 2 = 0$; $x_B - 2y_B - z_B = 2 - 2 - 0 = 0$; $x_H - 2y_H - z_H = 2 - 4 + 2 = 0$, then $x - 2y - z = 0$ is an equation of the plane (P) determined by A , B and H. $x_C - 2y_C - z_C = 3 - 0 + 3 \neq 0$,then C does not belong to (P).	1
2a	$\overrightarrow{HA}(-2;-1;0)$; $\overrightarrow{HB}(0;-1;2)$ then HA = HB = $\sqrt{5}$.	0.5
2b	$\overrightarrow{HC}(1;-2;-1) = \overrightarrow{N}_{(P)}$ then (CH) is perpendicular to (P).	0.5
2c	Triangles AHC and BHC are congruent so CA = CB and triangle ABC is isosceles of vertex C (or CA = CB = $\sqrt{11}$) hence, the bisector of angle $A\hat{C}B$ is the median relative to the side [AB]. I(1 ; 1 ; – 1) is the midpoint of [AB]; $\overrightarrow{CI}(-2;1;2)$ is a direction vector of (δ) and C∈(δ) . Thus, a system of parametric equations of (δ) is : x = – 2m +3 ; y = m and z = 2m – 3.	1
3	(CH) is perpendicular to plane (P) then (CH) is orthogonal to the straight line (AB) in (P) ; the straight line (AB) being orthogonal to (CI) and (CH) , then (AB) is perpendicular to plane (CHI) , consequently planes (ABC) and (CHI) are perpendicular , Therefore T the foot of the perpendicular through H to plane (ABC) belongs to the straight line (CI) = (δ) , intersection of the two planes. •OR: $\overrightarrow{AB} \times \overrightarrow{AC} = 2\vec{i} + 8\vec{j} - 2\vec{k}$ Then, plane (ABC) has an equation: $2x + 8y - 2z - 12 = 0$ $(HT): \begin{cases} x = 2t + 2 \\ y = 8t + 2 \\ z = -2t - 2 \end{cases}$ $(HT) \cap (ABC) = \{T\}$ then $T\left(\frac{5}{3}, \frac{2}{3}, -\frac{5}{3}\right)$ T belongs to (δ) for $m = \frac{2}{3}$.	1

QIII	Answer	Mark
1a	$P(M \cap V) = P(V)\times P(M/V) = \frac{40}{100}\times\frac{15}{100} = \frac{6}{100}$.	0.5
1b	$P(M \cap \overline{V}) = P(\overline{V})\times P(M/\overline{V}) = \frac{60}{100}\times\frac{75}{100} = \frac{45}{100}$.	0.5
1c	$P(M) = P(M \cap V) + P(M \cap \overline{V}) = \frac{6}{100} + \frac{45}{100} = \frac{51}{100}$.	1
2	$P(V/\overline{M}) = \frac{P(V \cap \overline{M})}{P(\overline{M})} = \frac{\frac{40}{100}\times\frac{85}{100}}{1-\frac{51}{100}} = \frac{34}{49}$.	1
3	Let A be the event : « at least one is vaccinated among the three persons » $P(A) = 1 - P(\overline{A}) = 1 - \frac{C_{180}^3}{C_{300}^3} = 0.785$.	1

QIV	Answer	Mark
1	$\lim_{x\to 1^+} f(x) = 1 - \infty = -\infty$: the straight line of equation $x = 1$ is an asymptote to (C).	0.5
2	$\lim_{x\to+\infty} f(x) = +\infty - 0 = +\infty$; $\lim_{x\to+\infty} [f(x) - x] = 0$, then the straight line (d) of equation $y = x$ is an asymptote to (C). $f(x) - x = -\frac{1}{x\ln x} < 0$, so (C) is below (d).	1
3	$f'(x) = 1 + \frac{\ln x + 1}{x^2 \ln^2 x} > 0$ for $x > 1$, then f is strictly increasing . x: 1 → $+\infty$ f '(x): + f (x): $-\infty$ ↗ $+\infty$	1.5
4	f is continuous and strictly increasing and f(x) increases from $-\infty$ to $+\infty$ then the equation f(x) = 0 has a unique root α. and $f(1.5)\times f(1.6) = -0.14\times 0.27 < 0$ then $1.5 < \alpha < 1.6$.	1
5		1.5

6a	A(t) = $\int_e^t [x - f(x)].dx = \int_e^t \frac{1}{x\ln x}.dx = \int_e^t \frac{(\ln x)'}{\ln x}.dx = [\ln(\ln x)]_e^t$ = ln(lnt) – ln(lne) = ln(lnt).	1.5
6b	A(t) < t if ln(lnt) < t ; lnt < e^t which is true since the representative curve of the ln function is below that of the exponential function.	1