

عدد المسائل : ثلاث	مسابقة في مادة الرياضيات	الاسم:
	المدة: ساعة واحدة	الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات  
يستطيع المرشح الاجابة بالترتيب الذي يناسبه ( دون الالتزام بترتيب المسائل الوارد في المسابقة )

### I - (5 points)

The 40 employees (technicians and workers) in a factory are distributed according to their ages as shown in the table below :

Age in years	[20 ; 30[	[30 ; 40[	[40 ; 50[	[50 ; 60]
Technicians	4	2	3	1
Workers	10	12	6	2

- 1) a- Determine the average age of the technicians.  
b- Determine the average age of the workers.
- 2) An employee is chosen at random from this factory.  
Consider the following events :  
T : « the chosen employee is a technician ».  
A : « the chosen employee is less than 40 years old ».  
a- Verify that the probability of A is equal to 0.7 .  
b- Calculate the following probabilities :  
P(T), P(A / T), P(A∩T) and P(T / A).

### II - (5points)

A florist has 260 flowers (roses and tulips) distributed into 40 bouquets of two kinds:

- Bouquets of roses, each containing 8 roses.
- Bouquets of tulips, each containing 6 tulips.

- 1) a- What is the number of bouquets of each kind?  
b- Determine the number of roses and the number of tulips?
- 2) The florist sold all of the flowers for a sum of 440 000 LL.  
Calculate the selling price of a tulip knowing that the selling price of a rose is 1000 LL?

### III - (10 points)

Consider the function  $f$  defined, on  $] - \infty ; 1 [ \cup ] 1 ; + \infty [$ , by:

$$f(x) = \frac{x^2 - 2x + 2}{x - 1} .$$

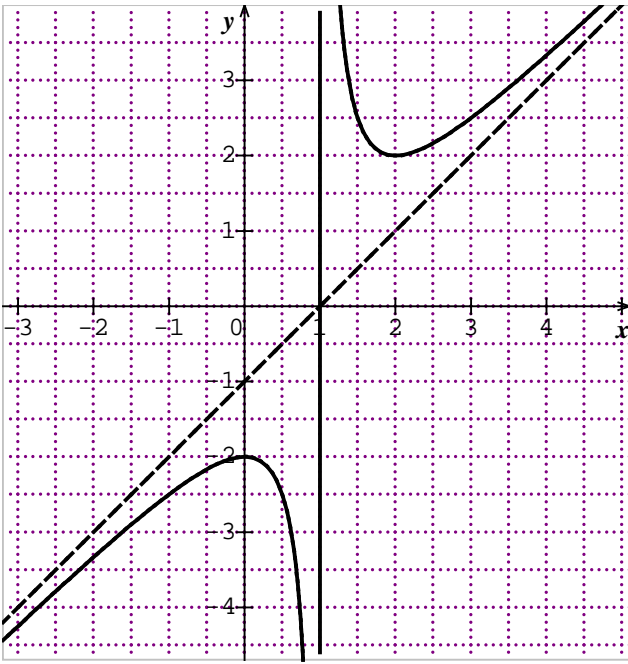
Designate by (C) the representative curve of  $f$  in an orthonormal system  $(O; \vec{i}, \vec{j})$ .

- 1) Calculate  $\lim_{\substack{x \rightarrow 1 \\ x < 1}} f(x)$  and  $\lim_{\substack{x \rightarrow 1 \\ x > 1}} f(x)$ . Deduce an asymptote (D) of (C).
- 2) a- Calculate  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow +\infty} f(x)$ .  
b- Show that the line (d) of equation  $y = x - 1$  is an asymptote to (C).
- 3) Verify that  $f'(x) = \frac{x(x - 2)}{(x - 1)^2}$ .
- 4) Set up the table of variations of  $f$ .
- 5) Draw the lines (D), (d) and the curve (C).
- 6) Discuss graphically, according to the values of the real number  $m$ , the number of solutions of the equation  $\frac{x^2 - 2x + 2}{x - 1} = m$ .

Q1	SHORT ANSWERS	MARKS
1.a	$\bar{T} = 36$ . The average age of the technicians is 36 years.	1
1.b	$\bar{O} = 35$ . The average age of the workers is 35 years	$\frac{1}{2}$
2.a	$P(A) = \frac{28}{40} = 0.7$	1
2.b	$P(T) = \frac{10}{40} = 0,25$ ; $P(A/T) = \frac{6}{10} = 0.6$ ; $P(A \cap T) = \frac{6}{40} = \frac{3}{20}$ ; $P(T/A) = \frac{6}{28} = \frac{3}{14}$ .	$2 \frac{1}{2}$

Q2	SHORT ANSWERS	MARKS
1.a	Let x be the number of bouquets of roses, and y be that of bouquets of tulips $\begin{cases} x + y = 40 \\ 8x + 6y = 260 \end{cases} ; x=10 \text{ and } y=30$	$2 \frac{1}{2}$
1.b	Number of roses: $10 \times 8 = 80$ Number of tulips: $30 \times 6 = 180$	1
2	Price of the roses is $80 \times 1000 = 80\,000$ LL Price of the tulips is $440\,000 - 80\,000 = 360\,000$ LL. Price of a tulip is $360\,000 \div 180 = 2\,000$ LL	$1 \frac{1}{2}$

Q3	SHORT ANSWERS	MARKS
1	$\lim_{\substack{x \rightarrow 1 \\ x < 1}} f(x) = \frac{1}{0^-} = -\infty$ ; $\lim_{\substack{x \rightarrow 1 \\ x > 1}} f(x) = \frac{1}{0^+} = +\infty$ . The line (D) of equation $x = 1$ is an asymptote of (C).	$1 \frac{1}{2}$
2a	$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2}{x} = \lim_{x \rightarrow -\infty} (x) = -\infty,$ similarly: $\lim_{x \rightarrow +\infty} f(x) = +\infty$ .	1
2b	$\lim_{x \rightarrow +\infty} [f(x) - (x - 1)] = \lim_{x \rightarrow +\infty} \frac{1}{x - 1} = 0$ and $\lim_{x \rightarrow -\infty} [f(x) - (x - 1)] = 0$ . The line (d) of equation $y = x - 1$ is an asymptote of (C).	1

3	$f'(x) = \frac{(2x-2)(x-1) - x^2 + 2x - 2}{(x-1)^2}$ $= \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$	1 ½																		
4	<table style="border-collapse: collapse; margin: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">x</td> <td style="padding: 5px; text-align: center;">-∞</td> <td style="padding: 5px; text-align: center;">0</td> <td style="border-left: 1px solid black; padding: 5px; text-align: center;">1</td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">+∞</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">f'(x)</td> <td style="padding: 5px; text-align: center;">+</td> <td style="padding: 5px; text-align: center;">0</td> <td style="border-left: 1px solid black; padding: 5px; text-align: center;">-</td> <td style="padding: 5px; text-align: center;">-</td> <td style="padding: 5px; text-align: center;">0</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px; text-align: center;">f(x)</td> <td style="padding: 5px; text-align: center;">-∞</td> <td style="padding: 5px; text-align: center;">-2</td> <td style="border-left: 1px solid black; padding: 5px; text-align: center;">+∞</td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">+∞</td> </tr> </table>	x	-∞	0	1	2	+∞	f'(x)	+	0	-	-	0	f(x)	-∞	-2	+∞	2	+∞	1 ½
x	-∞	0	1	2	+∞															
f'(x)	+	0	-	-	0															
f(x)	-∞	-2	+∞	2	+∞															
5		2																		
6	<p>For <math>m &lt; -2</math> : 2 solutions.  For <math>m = -2</math> : 1 solution (double).  For <math>-2 &lt; m &lt; 2</math> : no solution.  For <math>m = 2</math> : 1 solution (double).  For <math>m &gt; 2</math> : 2 solutions.</p>	1 ½																		