وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة المتوسطة	دورة سنة ٢٠٠٧ العادية

عدد المسائل : ستة	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

إرشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات .
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- *(2 points)*

Questions 1) and 2) of this exercise are independent.

1) Given the two numbers A and B defined by :

$$A = \frac{13}{7} - \frac{3}{7} \times \frac{14}{9} \quad , \quad B = 2\sqrt{36} + 5\sqrt{12} - 9\sqrt{75} + 4\sqrt{27} .$$

Show all the steps of the following calculations :

a) Calculate A and give the result in the form of an irreducible fraction.

b) Write B in the form $a + b\sqrt{3}$ where a and b are two integers.

2) x is any acute angle, establish the following equalities :

a) $(1 + \tan^2 x)\cos^2 x = 1$.

b) $(\cos x + \sin x)^2 - 2\cos x \sin x = 1$.

II- *(2points)*

A statistical series is given in the opposite table where a, b, c and d are integers.

1) Calculate the numerical value of each of the numbers a, b, c and d.

2) Calculate the mean of this statistical series.

Values	5	7	8	12	**Total**
Frequencies	12	18	a	15	75
Relative frequencies in %	16	c	d	20	b

III- *(2points)*

In what follows, designate by x the price of a pen in L L and y the price of a copybook in L L.
To buy one pen and one copybook we pay 2500 L L. If the price of a pen is decreased by 30% and the price of a copybook is decreased by 20% the amount we pay becomes 1900 L L.

1) Prove that the preceding information is translated into the following system : $\begin{cases} x + y = 2500 \\ 7x + 8y = 19000 \end{cases}$

2) Solve the preceding system, showing the followed steps in detail, and find the price of one pen and the price of one copybook.

IV- *(3 points)*

Part A

1) Verify the equality: $2(x - 3)(x + 7) = 2x^2 + 8x - 42$.

2) Solve the equation : $2x^2 + 8x - 42 = 0$.

Part B

In this part, the unit of length is the centimeter.

ABC is a triangle such that AB = x , AC = x + 4 and BC = $\sqrt{58}$, **where x is an integer strictly greater than 1.**

1) Can we find a value for x such that triangle ABC is right angled at C ? Justify.

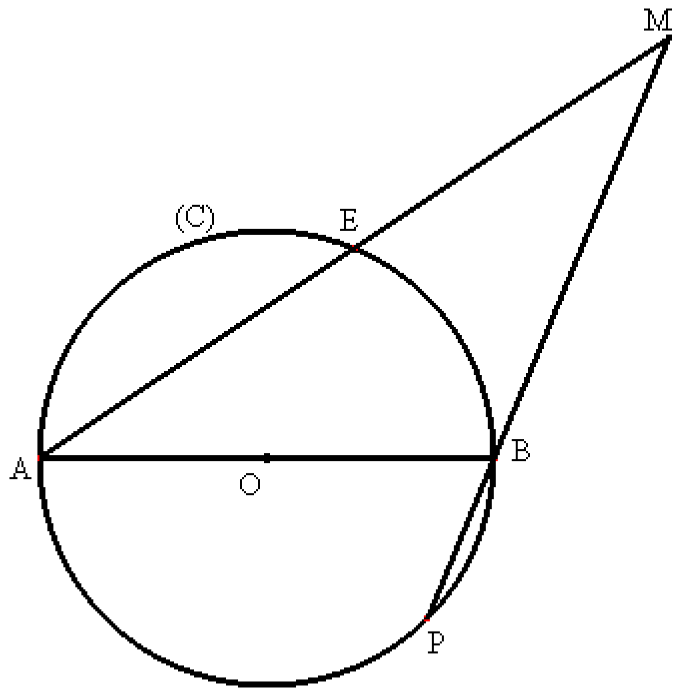
2) Calculate x so that triangle ABC is right angled at A. (You can use the results of **part A**).

3) Calculate x so that the perimeter of triangle ABC is less than or equal 18. (**In this question,** you can consider 7.6 as an approximate value of $\sqrt{58}$).

V- ***(5 points)***

Consider a circle (C) of center O and diameter [AB] such that AB = 6 cm. E is a variable point of (C) and M is the symmetric of A with respect to E.
The straight line (BM) cuts circle (C) in a second point P (see the figure below).
Designate by J the point of intersection of (BE) with (AP), T the point of intersection of (AB) with (MJ) and S the midpoint of [MB].

1) Draw a figure.
2) Prove that triangle ABE is right.
3) **a)** Prove that triangle ABM is isosceles of principal vertex B.
b) On what line does S move when E describes the circle (C) ?
4) Prove that triangle ABM is an enlargement of triangle OBS and precise the scale factor of this enlargement.
5) **a)** Prove that (AT) is perpendicular to (MJ).
b) Prove that the points E, B, T and M belong to the same circle. Determine a diameter of this circle.



VI- ***(6 points)***

In the plane of an orthonormal system x′ O x, y′ O y, where the unit of length is the centimeter, consider the straight line (d) of equation $y = -\frac{3}{2}x - 1$ and the points A(– 4 ; 5), B(6 ; 3) and G(0, –1).

1) Plot the points A, B and G.

2) Verify by calculation, that A and G are two points of (d), then draw (d).

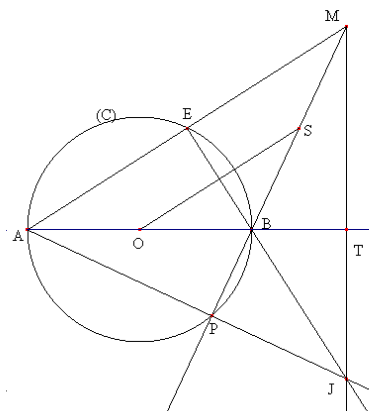
3) Write an equation of the straight line (BG) and deduce that the straight lines (d) and (BG) are perpendicular.

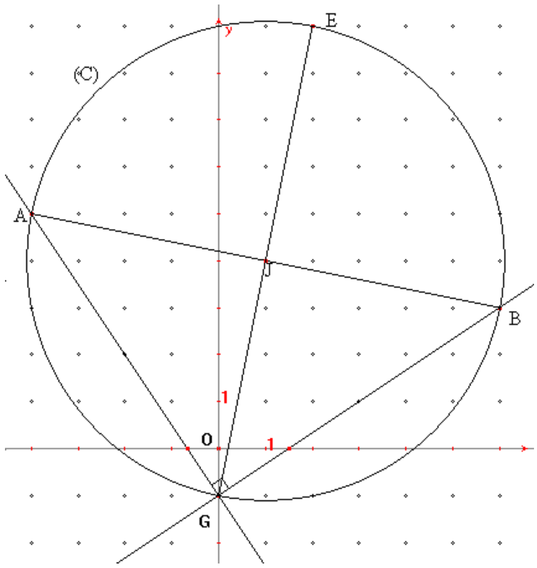
4) Knowing that $AG = 2\sqrt{13}$. Calculate BG and deduce that AGB is an isosceles right triangle.

5) Let (C) be the circle circumscribed about triangle ABG. Calculate the radius of (C) and the coordinates of its center J.

6) Designate by E the point defined by $\overrightarrow{GE} = \overrightarrow{GA} + \overrightarrow{GB}$.
a) Prove that AGBE is a square.
b) Calculate the coordinates of E.
c) Prove that E is a point of (C).

توزيع علامات مسابقة الرياضيات

I	**1.a)**	$A=\frac{13}{7}-\frac{2}{3}=\frac{25}{21}$.	½
	1.b)	$B=12+10\sqrt{3}-45\sqrt{3}+12\sqrt{3}=12-23\sqrt{3}$.	½
	2.a)	$(1+\tan^2 x)\cos^2 x=\cos^2 x+\frac{\sin^2 x}{\cos^2 x}\cos^2 x=\cos^2 x+\sin^2 x=1$.	½
	2.b)	$(\cos x+\sin x)^2-2\cos x\sin x=\cos^2 x+\sin^2 x+2\cos x\sin x-2\cos x\sin x=1$.	½
II	**1**	a = 30 ; b = 100 ; c = 24 ; d = 40.	**1 ¼**
	2	$\bar{x}=\frac{606}{75}=8{,}08$.	¾
III	**1**	$\begin{cases}x+y=2500\\0{,}7x+0{,}8y=1900\end{cases}$ then $\begin{cases}x+y=2500\\7x+8y=19000.\end{cases}$	¼, ½, ¼
	2	x = 1000 and y = 1500 The price of one pen is 1000LL ; The price of one copybook is 1500LL.	¾ ¼
IV	**A.1)**	$2(x-3)(x+7)=2(x^2+4x-21)=2x^2+8x-42$	½
	A.2)	x = 3 ; x = -7	½
	B.1)	No, because $x < x+4$, so [AB] can not be a hypotenuse. or : $x^2=(x+4)^2+58$ gives $x=-\frac{74}{8}$. This value of x is rejected because x is negative. …	½
	B.2)	$BC^2=AB^2+AC^2$ so $2x^2+8x-42=0$, then x = 3 or x = -7 ; -7 rejected.	¾
	B.3)	$x+x+4+7{,}6\leq 18$; $x\leq 3{,}2$ so x = 2 or x = 3.	¾
V	**1**	Figure 	¼

V	**2**	[AB] is a diameter E is a point of the circle so triangle AEB is right at E.	½
	3.a)	In triangle ABM, [BE] is a median and a height so the triangle ABM is isosceles of principal vertex B.	¾
	3.b)	S moves on the circle of center B and radius BS = 3cm.	¾
	4	(OS) // (AM) ; $\frac{BM}{BS}=\frac{BA}{BO}=\frac{AM}{OS}=2$ because … ; The scale factor is 2.	¾
	5.a)	In triangle AMJ, B is the orthocenter so (JE) is perpendicular to (AM).	**1**
	5.b)	MEB and MTB are two right triangles of the same hypotenuse [BM]. So, they are inscribed in the circle of diameter [BM].	**1**
VI	**1**	A , B and G.	½
	2	$-\frac{3}{2}x_A-1=5=y_A$; A is a point of (d). $-\frac{3}{2}x_G-1=-1=y_G$; G is a point of (d). Draw (d).	¾
	3	Equation of (BG) : $y=\frac{2}{3}x-1$ $a_{(BG)}\times a_{(d)}=-1$; (BG) ⊥ to (d).  	¾ ¼
	4	$BG=2\sqrt{13}$ BG = AG and (BG) ⊥ (AG) so AGB is a right isosceles triangle.	½ ½
	5	$r=\frac{AB}{2}=\sqrt{26}$ because [AB] is a diameter of (C). J (1 ; 4) (J midpoint of [AB]).	½ ½
	6.a)	$\overrightarrow{GE}=\overrightarrow{GA}+\overrightarrow{GB}$ so AGBE is a parallelogram. AGB is an isosceles right triangle so AGBE is a square.	¾
	6.b)	E (2 ; 9).	¾
	6.c)	E is the 4th vertex of the square ; so E is a point of (C).	¼