| الدورة الإستثنائية للعام ٢٠٠٨ | امتحانات الشهادة الثانوية العامة<br>الفرع : علوم عامة | وزارة التربية والتعليم العالي<br>المديرية العامة للتربية<br>دائرة الامتحانات |
|---|---|---|
| الاسم:<br>الرقم: | مسابقة في مادة الفيزياء<br>المدة ثلاث ساعات | |

**This exam is formed of four exercises in four pages numbered from 1 to 4.**
**The use of non-programmable calculator is allowed.**

## First exercise (7.5 points)

### Response of an electric component submitted to a DC voltage

In order to study the response of the current in an electric component when submitted to a DC voltage, we use a coil of inductance L = 40 mH and of resistance r = 18 Ω, a capacitor of capacitance C = 100 μF, a resistor of resistance R = 2 Ω, a switch K and a DC generator delivering across its terminals a constant voltage E = 8 V.

**A – Response of the electric component (R, L)**

We connect the coil in series with the resistor across the terminals of the generator (Fig. 1).

At the instant $t_0 = 0$, we close K. The circuit thus carries a current i. With an oscilloscope, we display the variation of the voltage $u_{AM}$ across the terminals of the resistor as a function of time (Fig. 2).

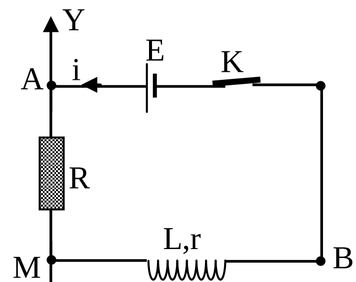


**Figure 1**

**1)** Express the voltage $u_{AM}$ across the resistor and the voltage $u_{MB}$ across the coil in terms of R, L, r, i and $\frac{di}{dt}$.

**2)** Derive the differential equation in i.

**3)** The solution of this differential equation is of the form:

$i = I_0(1 - e^{-\frac{t}{\tau}})$.

**a)** Show that $I_0 = \frac{E}{R+r}$ and $\tau = \frac{L}{R+r}$.

**b)** Calculate the values of $I_0$ and τ.

**4)** Using figure 2, determine the values of $I_0$ and that of τ.

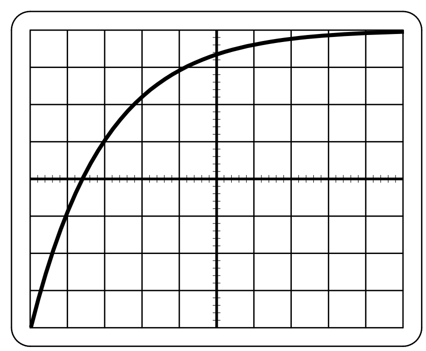


**Figure 2**

**Horizontal sensitivity : 1 ms/div**
**Vertical sensitivity: 0.1 V/div**

**B – Response of the electric component (R,C)**

We replace, in the previous circuit, the coil by the capacitor (Fig. 3).

At $t_0 = 0$, we close K. The circuit thus carries a current i. With the oscilloscope, we display the variation of the voltage $u_{AM}$ as a function of time (Fig. 4).

**1)** Express the current i in terms of C and $\frac{du_C}{dt}$, where $u_C$ is the voltage $u_{MB}$ across the terminals of the capacitor.

**2)** Using the law of addition of voltages, show that the differential equation in i is of the form: $RC\frac{di}{dt} + i = 0$.

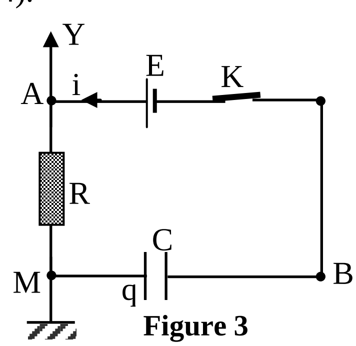


**Figure 3**

**3)** The solution of this differential equation is of the form:

$i = I_1 e^{-\frac{t}{\tau_1}}$. Determine, in terms of E, R and C, the expressions of the two constants $I_1$ and $\tau_1$ and calculate their values.

**4)** Referring to figure 4, determine the value of $I_1$ and that of $\tau_1$.

**C –** In each of the two previous circuits, we replace the resistor by a lamp. Explain the variation of the brightness of the lamp in each circuit.

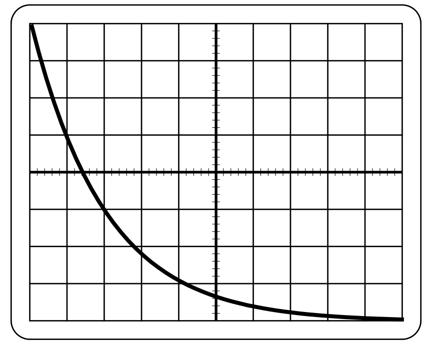


**Figure 4**

**Horizontal sensitivity : 0.1 ms/div**
**Vertical sensitivity: 1 V/div**

## Second exercise (7.5 points)

### (R,L,C) series circuit

Consider a capacitor of capacitance C = 5 μF, a resistor of resistance R = 40 Ω and a coil of inductance L and of resistance r, connected in series across the secondary of an ideal transformer.

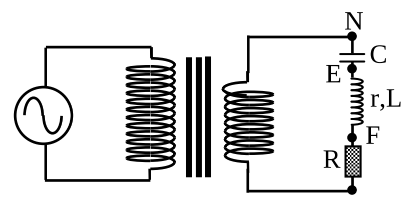



Figure 1

**A – Physical quantities of the transformer**

The primary coil of the transformer is connected to the mains (220 V; 50 Hz) (Fig.1). The secondary of the transformer delivers across its terminals a voltage: $u_{NM} = 3\cos\omega t$ (u in V ; t in s).

The circuit thus carries an alternating sinusoidal current $i = I_m\cos(\omega t - \varphi)$.

The secondary coil has 15 turns and cannot withstand a current of effective value greater than 10 A.

**1)** Give the value of the frequency of the alternating sinusoidal voltage across the secondary coil.

**2)** Determine the number of turns of the primary coil. Take $\sqrt{2} = 1.4$.

**3)** Calculate the maximum effective value of the current that the primary coil can withstand.

**B – Determination of L and r**

An oscilloscope, connected in the previous circuit, allows us to display on the channel $Y_1$ the voltage $u_1 = u_{NM}$ and on the channel $Y_2$ the voltage $u_2 = u_{FM}$ across the terminals of the resistor.

**1)** Redraw the circuit of figure 1 and show the connections of the oscilloscope.

**2)** The sensitivities of the oscilloscope are:
**Horizontal sensitivity: 4 ms/div**
**Vertical sensitivity on both channels $Y_1$ and $Y_2$: 1 V/div.**
Using the waveforms of figure 2,
show that $i = 0.05\cos(100\pi t – 0.2\pi)$; ( i in A, t in s).

**3)** Calculate the average power consumed by the component NM.

**4)** Deduce the value of the resistance r of the coil.

**5)** Knowing that $u_{NM} = u_{NE} + u_{EF} + u_{FM}$ is verified for any value of time t, determine the value of L.

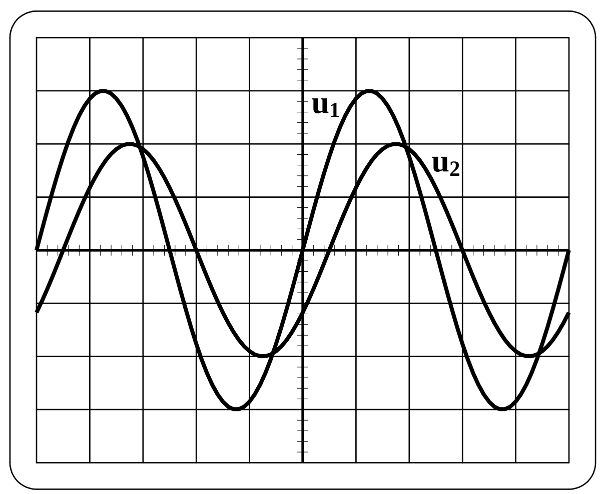
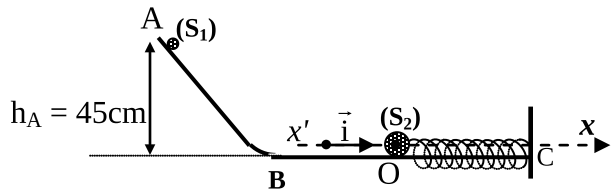



**Figure 2**

**<u>Third exercise</u> (7.5 points)**

**Determination of the stiffness constant of a spring**

To determine the stiffness constant k of a spring we attach to its extremity a solid ($S_2$), of mass $m_2 = 200$ g, which can slide without friction on the horizontal part BC of a track ABC situated in a vertical plane, the other extremity of the spring is fixed at C.




Another solid ($S_1$), of mass $m_1 = 50$ g, is released without initial velocity from a point A of the curved part of the track.

Point A is situated at a height $h_A = 45$ cm from the horizontal part of the track.

($S_2$), initially at rest at point O, is thus hit by ($S_1$). ($S_1$) and ($S_2$) are supposed to be point masses.

The horizontal plane passing through BC is taken as a gravitational potential energy reference.

Take: $g = 10\ ms^{-2}$ , $0.32\pi = 1$. Neglect all frictional forces.

**1)** Determine the value $V_1$ of the velocity $\overrightarrow{V_1}$ of ($S_1$) just before colliding ($S_2$).

**2)** After collision, ($S_1$) remains in contact with ($S_2$) and the two solids form a solid (S) of center of inertia G and of mass $M = m_1 + m_2$. Thus G performs oscillations around O with amplitude 3 cm on the axis x'Ox of origin O and unit vector $\vec{i}$.

**a)** Show that the value of the velocity $\overrightarrow{V_0}$ of G just after the collision is equal to 0.6 m/s.

**b)** Let x and v be respectively the abscissa and the algebraic value of the velocity of G at an instant t after

the collision. The instant of collision at O is considered as an origin of time $t_0 = 0$.

**i)** Write down, at an instant t, the expression of the mechanical energy of the system (S, spring, Earth) in terms of k, x , M and v.

**ii)** Deduce the second order differential equation in x that describes the motion of G.

**iii)** The time equation of oscillation of (S) is given by: $x = X_m \sin(\omega_0 t + \varphi)$. Determine the value of $\varphi$ and the expressions of the constants $X_m$ and $\omega_0$ in terms of k, M and $V_0$.

**iv)** Deduce the value of the stiffness constant k of the spring.

**3)** In reality friction is not neglected. To ensure the value of k, the extremity C of the spring is attached to a vibrator of adjustable frequency f and which can vibrate in the same direction of the spring. We notice that the amplitude of the oscillations of (S) varies with f and attains a maximum value for f = 3.2 Hz.

**a)** Name the physical phenomenon that takes place when f = 3.2 Hz.

**b)** Calculate the value of k.

**<u>Fourth exercise</u> (7.5 points)**

**The radionuclide Potassium 40**

The isotope of potassium $^{40}_{19}K$, is radioactive and is $\beta^+$ emitter; it decays to give the daughter nucleus argon $^{A}_{Z}Ar$. The object of this exercise is to study the decay of potassium 40.

**Given:**

masses of nuclei: m ($^{40}_{19}K$) = 39.95355 u ; m($^{A}_{Z}Ar$) = 39.95250 u ;

masses of particles: m ($^{0}_{1}e$) = $5.5\times10^{-4}$ u; m (neutrino) ≈ 0 ;

Avogadro's number: N = $6.02\times10^{23}$ $mol^{-1}$; 1 u = 931.5 $MeV/c^2$ ;

Radioactive period of $^{40}_{19}K$: T = $1.5\times10^9$ years; molar mass of $^{40}_{19}K$ = 40 g $mol^{-1}$.

1 MeV = $1.6\times10^{-13}$ J.

## A – Energetic study of the decay of potassium 40

### 1) Energy liberated by one decay

- **a)** Write down the equation of the decay of one potassium 40 nucleus and determine Z and A.
- **b)** Calculate, in MeV, the energy $E_1$ liberated by this decay.
- **c)** The daughter nucleus is supposed to be at rest. The energy carried by $\beta^+$ is, in general, smaller than $E_1$. Why?

### 2) Energy received by a person

The mass, of potassium 40 at an instant t, in the body of an adult is, on the average, equal to $2.6\times10^{-3}$ % of its mass.

An adult person has a mass M = 80 kg.

- **a) i)** Calculate the mass m of potassium 40 contained in the body of that person at the instant t.
  **ii)** Deduce the number of potassium 40 nuclei in the mass m at the instant t.
- **b) i)** Calculate the radioactive constant λ of potassium 40.
  **ii)** Deduce the value of the activity A of the mass m at the instant t.
- **c)** Deduce, in J, the energy E liberated by the mass m per second.

## B – Dating by potassium 40

Certain volcanic rocks contain potassium and part of it, is potassium 40. At the instant of its formation ($t_0 = 0$), the number of nuclei of potassium 40 is $N_0$ in the volcanic rock and that of argon is zero. At the instant t, the rock contains respectively $N_K$ and $N_{Ar}$ nuclei of potassium 40 and of argon 40.

**1) a)** Write down the expression of $N_K$ **,** that explains the law of radioactive decay, as a function of time.

**b)** Deduce the expression of $N_{Ar}$ as a function of time.

**2)** A geologist analyzes a volcanic rock. He notices that the number of argon 40 nuclei is twice less than the number of potassium 40 nuclei in this rock. Determine the age of this rock.

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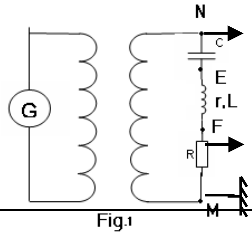
**First exercise (7.5 points)**

| Part of the Q | Answer | Mark |
|---|---|---|
| A.1 | $u_{AM} = Ri$ and $u_{MB} = L\frac{di}{dt} + ri$. | 0.5 |
| A.2 | We have $E = Ri + L\frac{di}{dt} + ri \Rightarrow i + \frac{L}{R+r}\frac{di}{dt} = \frac{E}{R+r}$. | 0.75 |
| A.3.a | $\frac{di}{dt} = \frac{I_0}{\tau}e^{-\frac{t}{\tau}}$; $I_0 - I_0 e^{-\frac{t}{\tau}} + \frac{L}{R+r}\frac{I_0}{\tau}e^{-\frac{t}{\tau}} = \frac{E}{R+r}$<br>$\Rightarrow I_0 = \frac{E}{R+r}$ and $\frac{L}{R+r}\frac{I_0}{\tau} - I_0 = 0$ ; let $\tau = \frac{L}{R+r}$. | 1.25 |
| A.3.b | $I_0 = \frac{8}{18+2} = 0.4$ A and $\tau = \frac{0.04}{18+2} = 2\times10^{-3}$ s = 2 ms. | 0.5 |
| A.4 | From graph 2: $u_R(max) = 0.1\times8 = 0.8$ V and $u_R(max) = R\times I_0$<br>$\Rightarrow I_0 = \frac{u_R(max)}{R} = 0.4$ A.<br>Also, for $t = \tau$, $u_R = 0.63\, u_R(max) = 0.5$ V which corresponds to<br>$\tau$ = 2 divisions, $\tau$ = 2 ms. | 1.00 |
| B.1 | $i = \frac{dq}{dt} = C\frac{du_C}{dt}$. | 0.25 |
| B.2 | $E = u_{AM} + u_{MB} \Rightarrow E = u_C + Ri$ . By deriving with respect to time:<br>$0 = \frac{du_C}{dt} + R\frac{di}{dt} \Rightarrow \frac{i}{C} + R\frac{di}{dt} = 0$<br>Thus : $RC\frac{di}{dt} + i = 0$ | 0.75 |
| B.3 | $i = I_1 e^{-\frac{t}{\tau_1}}$.<br>For $t_0 = 0$, $u_C = 0$ and $i = I_1 \Rightarrow E = 0 + RI_1$<br>$\Rightarrow I_1 = \frac{E}{R} = \frac{8}{2} = 4$ A.<br>$\frac{di}{dt} = -\frac{I_1}{\tau_1}e^{-\frac{t}{\tau_1}}$ ; by replacing: $-RC\frac{I_1}{\tau_1}e^{-\frac{t}{\tau_1}} + I_1 e^{-\frac{t}{\tau_1}} = 0$<br>$\Rightarrow -RC\frac{I_1}{\tau_1} + I_1 = 0 \Rightarrow \tau_1 = RC = 2\times10^{0}\times10^{-6} = 2\times10^{-4} = 0.2$ ms. | 1 |
| B.4 | $u_R(max) = 8$ V $= RI_1 \Rightarrow I_1 = 8/2 = 4$ A and for $t = \tau_1$,<br>$u_R = 0.37\, u_R(max) = 3$ V $\Rightarrow \tau_1 = 0.2$ ms. | 0.5 |
| C | In A: after closing the switch the brightness of the lamp increases and reaches after a very short time a stable brightness.<br>In B : at the instant of closing the switch the lamp shines then the brightness decreases and vanishes after a short time | 1 |

**Second exercise (7.5 points)**

| Part of the Q | Answer | Mark |
|---|---|---|
| A.1 | f = 50 Hz | 0.5 |
| A.2 | $\frac{U_2}{U_1} = \frac{N_2}{N_1} \Rightarrow \frac{3/\sqrt{2}}{220} = \frac{15}{N_1} \Rightarrow$ $N_1$ = 1540 turns. | 0.75 |
| A.3 | $\frac{I_2}{I_1} = \frac{N_1}{N_2} \Rightarrow \frac{10}{I_1} = \frac{1540}{15} \Rightarrow I_1$ = 97 mA | 0.75 |
| B.1 | <br>Fig.1 | 0.25 |
| B.2 | T = 5 div × 4 ms/div = 20 ms = 0.02 s ⇒ ω = $\frac{2\pi}{0,02} = 100\pi$ rad/s.<br>$(U_R)_{max}$ = $RI_{max}$ ⇒ $I_{max}$ = $\frac{2}{40}$ = 0.05 A. φ = 0.5 × 2π/5 = 0.2 π rad.<br>i is in lag on $u_{NM}$ ⇒ i = 0.05 cos(100πt – 0.2 π) | 1.5 |
| B.3 | P = UIcos φ = $\frac{3}{\sqrt{2}} \times \frac{0.05}{\sqrt{2}} \times \cos 0.2\pi$ = 0.061 W. | 0.75 |
| B.4 | P = $R_{total}$ $I^2$ => $R_{totale}$ = $\frac{0.061}{(0.05/\sqrt{2})^2}$ = 48.8 Ω = R + r = 40 + r<br>=> r = 8.8 Ω | 1 |
| B.5 | $u_{NE}$ = $u_C$ = 1/C primitive (i) = 100/ π sin(1ooπt – 0.2 π )<br>$u_{EF}$ = ri + Ldi/dt<br>$u_{EF}$ = 8,8 × 0.,05cos(100πt – 0.2 π) - L × 5 π sin(100πt – 0,.2 π).<br>$u_{FM}$ = Ri = 2 cos(100πt – 0.2 π).<br>3cosωt = 100/ π sin(ωt – 0.2 π) + 8,8 × 0.05cos(100πt – 0.2 π)<br>- L × 5 π sin(100πt– 0.2 π) + 2 cos(100πt – 0.2 π).<br>For t = 0, we obtain L = 2.15 H. | 2 |

**Third exercise (7.5 points)**

| Part of the Q | Answer | Mark |
|---|---|---|
| 1 | Conservation of mechanical energy between A and B: $m_1gh_A+0 = 0+ ½\ m_1 V_1^2$ ; $V_1 = \sqrt{2gh_A} = \sqrt{2\times10\times0.45}$ = 3 m/s . | 1.25 |
| 2.a | Conservation of linear momentum:$m_1 \vec{V}_1 + \vec{0} = (m_1 + m_2)\vec{V}_0$ ; projection : $V_0 = \frac{m_1}{m_1+m_2}V_1 = \frac{0.05}{0.05+0.2}3$ = 0.6 m/s | 1.00 |
| 2.b. i | $ME = ½\ M\ v_G^2 + ½\ kx^2$; $(M = m_1 + m_2)$. | 0.50 |
| 2.b.ii | ME is conserved: Derivative w.r.t time $\frac{d(ME)}{dt} = 0$ $\Rightarrow Mv\dot{v} + kx\dot{x} = 0 \Rightarrow \ddot{x} + \frac{k}{M}x = 0$ | 1.00 |
| 2.b.iii | $x' = \omega_0 X_m\cos(\omega_0t + \varphi)$ and $\ddot{x} = -\omega_0^2.X_m\sin(\omega_0t + \varphi)$. By replacing : $-\omega_0^2X_m\sin(\omega_0t + \varphi) + \frac{k}{M}X_m\sin(\omega_0t + \varphi) \Rightarrow \omega_0^2 = \frac{k}{M} \Rightarrow \omega_0 = \sqrt{\frac{k}{M}}$ ; At t = 0: $x = 0 \Rightarrow X_m\sin\varphi = 0 \Rightarrow \varphi = 0$ or $\pi$. At t = 0: $v = V_0 \Rightarrow \omega_0 X_m\cos\varphi = V_0 > 0 \Rightarrow \varphi = 0$, $X_m = \frac{V_0}{\omega_0} = V_0\sqrt{\frac{M}{k}}$ | 2.00 |
| 2.b.iv | $X_m = X_0 = V_0\sqrt{\frac{M}{k}} \Rightarrow k = \frac{V_0^2M}{X_m^2} = \frac{0.36\times0.25}{0.03^2}$ = 100 N/m. | 0.75 |
| 3.a | Resonance. | 0.25 |
| 3.b | $\omega_0 = \omega = 2\pi f = \sqrt{\frac{k}{M}}$ ; $4\pi^2f^2 = \frac{k}{M} \Rightarrow k = 4\pi^2f^2M$ = 100 N/m | 0.75 |

**Fourth exercise (7.5 points)**

| Part of the Q | Answer | Mark |
|---|---|---|
| A.1.a | ${}^{40}_{19}K \rightarrow {}^{A}_{Z}Ar + {}^{0}_{1}e + {}^{0}_{0}\nu$. Z = 18; A = 40. | z0.75 |
| A.1.b | $\Delta m = 39.95355 - 39.95250 - 5.5\times 10^{-4} = 5\times10^{-4}$ u.<br>$E_1 = mc^2 = 5\times10^{-4}\times 931.5$ MeV/$c^2 \times c^2 = 0.47$ MeV. | 1.00 |
| A.1.c | Because $E_1 = E(\beta^+) + E({}^{0}_{0}\nu) + E(\gamma)$ | 0.50 |
| A.2.a.i | $m = \frac{80\times2.6\times10^{-3}}{100} = 2.1\times10^{-3}$ kg = 2,1 g | 0.50 |
| A.2.a.ii | $N = \frac{m}{M}$ **N =** $3.16\times10^{22}$ nuclei. | 0.50 |
| A.2.b.i | $\lambda = \frac{0.693}{1.5\times10^{9}\times365\times24\times3600} = 1.46\times10^{-17}\ s^{-1}$ | 0.5 |
| A.2.b.ii | $A = \lambda N = 1.46\times10^{-17}\times3.16\times10^{22} = 4.61\times10^{5}$ Bq | 0.75 |
| A.2.c | The energy received in each second:<br>$E = 4.16\times10^{5}\times0.47 = 2.17\times10^{5}$ MeV $= 3.47\times10^{-8}$ J. | 07.5 |
| B.1.a | $N_K = N_0 e^{-\lambda t}$ | 0.50 |
| B.1.b | $N_{Ar} = N_0 - N_K = N_0(1-e^{-\lambda t})$ | 0.50 |
| B.2 | $\frac{N_{Ar}}{N_K} = \frac{1}{2} \Rightarrow \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} = \frac{1}{2} \Rightarrow e^{\lambda t} = \frac{3}{2}$<br>$\Rightarrow t = \frac{T}{0.693}\ln\frac{3}{2} \Rightarrow t = 8.8\times10^{8}$ years | 1.25 |