الدورة العادية للعام ٢٠٠٨	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة أربع ساعات	

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البياناتز
- ستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)ز

I-(2 points)

Tell whether each of the following statements is true or false and justify your answer:

1) In an orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, if $A(2;-1;1)$; $B(4;-2;2)$ and $C(1;1;2)$ then the measure of the angle (BAC) is $\frac{\pi}{6}$ radian.

2) For all real numbers b, the equation $e^{x}=-x+b$ has a unique solution in IR.

3) If A and B are two points of affixes $a=1+i$ and $b=1+i-2e^{i\frac{\pi}{6}}$ respectively, then B belongs to the circle with center A and radius 2.

4) f is a function given by $f(x) = -x^2 + 3$.
For every function g such that $\lim\limits_{x\to+\infty} g(x)=+\infty$, we get $\lim\limits_{x\to+\infty}\frac{f(x)}{g(x)}=-1$.

II- (2 points)

The space is referred to an orthonormal system $\left(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k}\,\right)$.

Consider the two straight lines $(d)\begin{cases}x=m-1\\ y=m\\ z=m+1\end{cases}$ and $(d')\begin{cases}x=2t\\ y=t\\ z=-3t+2\end{cases}$

where m and t are two real parameters .

1) Prove that (d) and (d') are not coplanar (skew).

2) Let (P) be the plane containing (d) and cutting (d') at the point $E(0\,;\,0\,;2\,)$.
Prove that an equation of (P) is $x-z+2=0$.

3) Consider in plane (P) the circle (C) with center E and radius $R=1$.

a- Calculate the distance from E to (d) and prove that (C) cuts (d) at two points A and B.

b- Calculate the coordinates of the points A and B.

c- Calculate the area of triangle EAB.

III- (3 points)

A- Two fair dice, each having six faces numbered from 1 to 6, are rolled. Let S designate the sum of the numbers on the two appearing faces, and consider the following events:

H : « the sum S is greater than or equal to 8 »,

C : « the sum S is less than or equal to 5 »,

E : « the sum S is equal to 6 or 7 ».

Show that the probability $P(H)=\frac{5}{12}$ and calculate P(C) and P(E).

B- 1) At a " Kermes " organized at the end of the school year, a students is in charge of a " stand " at which the following game is proposed:

The player rolls two fair dice each having six faces numbered from 1 to 6.

- If he gets a sum greater than or equal to 8, then he draws randomly one ticket from a bag that contains 30 tickets out of which 20 win.
- If he gets a sum less than or equal to 5, then he draws randomly one ticket from another bag that contains 30 tickets out of which 10 win.
- But if the player gets a sum equal to 6 or 7, then he chooses randomly one of the two bags and draws a ticket at random from the chosen bag.

Designate by G the event: « the player wins a prize ».

a- Calculate the probability that the player draws a winning ticket knowing that he got a sum greater than or equal to 8. Deduce $P(H \cap G)$.

b- Show that $P(E \cap G) = \frac{11}{72}$.

c- Calculate $P(C \cap G)$; then deduce the probability P(G) that the player draws a winning ticket.

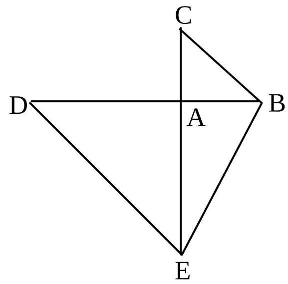
2) The school administration announced that "everybody wins". To achieve this they decided to give to every player who draws a winning ticket the amount of 5000 LL, and to every player who draws a non-winning ticket the amount of:

- 3000 LL if he realizes the event H and does not win,
- 2000 LL if he realizes the event C and does not win,
- 1000 LL if he realizes the event E and does not win.

Let X designate the random variable equal to the amount paid by the administration to a player.

Verify that $P(X = 3000) = \frac{5}{36}$ and determine the probability distribution for X.

IV- (3 points)



In the figure below, ABC and ADE are right isosceles triangles such that AB=1 ; AD=2 and $(\overrightarrow{AB};\overrightarrow{AC})=(\overrightarrow{AD};\overrightarrow{AE})=\frac{\pi}{2}\ [2\pi]$.

Let O be the midpoint of $[BE]$.

1) Let r be the rotation with center A and angle $\frac{\pi}{2}$, and h be the dilation with center E and ratio 2.
   Let $f = r \circ h$.
   a- Prove that f is a similitude whose ratio and angle are to be determined.
   b- Determine f(O) and f(A) and deduce that (AO) is perpendicular to(CD).

2) Let s be the direct plane similitude that transforms A onto C and D onto A.
   a- Precise the ratio and an angle of s.
   b- Determine the image by s of the straight line (CD) and that of the straight line (AO).
   c- Deduce the center I of s.

3) Let $g = f \circ s$ and A' be the symmetric of A with respect to D.
   a- Prove that g is the symmetry with center D.
   b- Deduce that f(C)=A'.

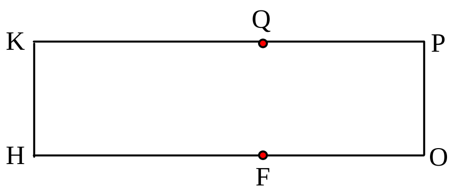
4) The complex plane is referred to a direct orthonormal system $(A;\overrightarrow{AB};\overrightarrow{AC})$.
   a- Determine the affixes of the points O, C and A'.
   b- Write the complex form of f and determine the affix of its center Ω.

V- (3 points)

In an oriented plane consider the rectangle HOPK such that HO = 10, OP = 3 and $(\overrightarrow{HO};\overrightarrow{HK})=\frac{\pi}{2}\ (\text{mod } 2\pi)$.

F and Q are two points on $[OH]$ and $[PK]$ respectively such that $OF = PQ = 4$.

Let (E) be the ellipse of foci O and F, and of directrix the line (HK) associated to F, and of eccentricity e.



A-

1) Determine the center of (E) and prove that $e=\frac{1}{2}$.
2) a- Show that the points P and Q belong to (E).
   b- Determine the vertices that are on the focal axis of (E).
3) Let B be one of the vertices belonging to the non focal axis.
   a- show that the triangle OBF is equilateral and plot B .
   b- Draw (E).

B-

The plane is referred to a direct orthonormal system $(O;\vec{i},\vec{j})$ such that $\overrightarrow{OF}=-4\vec{i}$ and $\overrightarrow{OP}=3\vec{j}$.

1) Verify that $3x^2 + 4y^2 + 12x - 36 = 0$ is an equation of (E) .
2) Consider the line (d) passing through P and having slope $-\frac{1}{2}$. Prove that (d) is tangent to (E) at P
3) Write an equation of the tangent (T) at Q to (E) and verify that H belongs to (T) .
4) The lines (d) and (T) intersect at a point R, and that (d) cuts the focal axis at a point G.
   Calculate the area of the regions bounded by the triangle RGH and the semi-ellipse

situated above the focal axis.

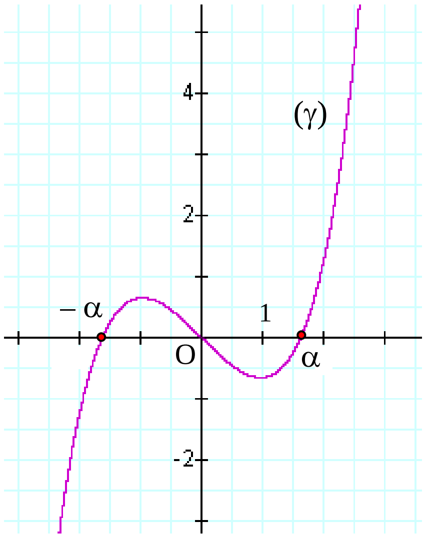
VI- (7 points)

A- Consider the differential equation (E) : $y' - y = 2e^{-x}$.

1) Determine the real number λ so that $y = \lambda e^{-x}$ is a solution of (E).
2) a- Solve the equation $y' - y = 0$.
   b- Deduce the general solution of (E).
   c- Verify that the function g defined on IR, by $g(x) = e^{x} - e^{-x}$ is a particular solution of (E).

B- Consider the function f defined on IR by $f(x) = g(x) - 2x$ and designate by (C) its representative curve in an orthonormal systems $(O ; \vec{i}, \vec{j})$.

1) a- Prove that f is odd.
   b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to +\infty} \frac{f(x)}{x}$.
   c- Verify that $f'(x) = e^{-x}(e^{x} - 1)^2$.
   d- Set up the table of variation of f.



2) The adjacent figure shows the representative curve (γ) of the function h defined on $[0 ; +\infty[$ by $h(x) = g(x) - 3x$.
   a- Prove that $1.62 < \alpha < 1.63$.
   b- Using (γ), prove that the curve (C) of f cuts the straight line (d) of equation $y = x$ at three points whose abscissas are to be determined.

3) a- Calculate $f(1)$ and $f(2)$.
   b- Draw (d) and (C).

4) a- Prove that f has an inverse function f^{-1} and determine domain
   The domain of definition of f^{-1}.
   b- Determine the values of x satisfying $1 < f^{-1}(x) < 2$.
   c- Draw the representative curve (C') of f^{-1} in the system $(O ; \vec{i}, \vec{j})$.

5) Calculate the area of the regions bounded by (C) and (C') in terms of α.

C- Consider the sequence (U_n) defined by $U_0 \in]0 ; \alpha[$ and $U_{n+1} = f(U_n)$ for every n.

1) Prove by mathematical induction on n that, for every n, $0 < U_n < \alpha$.
2) Noticing that $f(x) < x$ for every x in $]0;\alpha[$, prove that the sequence (U_n) is strictly decreasing. and deduce that it is convergent.

مشروع معيار التصحيح	الفرع : علوم عامة	الدورة العادية للعام ٢٠٠٨

Q:I	Answer	Mark
1	$\cos(B\hat{A}C) = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{AB\times AC}$, but $\overrightarrow{AB}(2;-1;1)$ and $\overrightarrow{AC}(-1;2;1)$ so $\overrightarrow{AB}\cdot\overrightarrow{AC} = -2-2+1 = -3$ and $AB = \sqrt{6}$ and $AC = \sqrt{6}$; consequently $\cos B\hat{A}C = \frac{-3}{6} = -\frac{1}{2}$ and $B\hat{A}C = \frac{2\pi}{3}$ radians . The statement 1) is **false**.	1
2	Consider the function U defined over IR by $U(x) = e^x + x - b$; $U'(x) = e^x + 1 > 0$ so U is strictly increasing over IR . Moreover, $\lim\limits_{x\to-\infty} U(x) = -\infty$ and $\lim\limits_{x\to+\infty} U(x) = +\infty$ Hence, U increases from $-\infty$ to $+\infty$ and it is continuous so the equation has a unique solution . Then the statement 2) is **true**.	1
3	AB = $\lvert b-a \rvert = \lvert -2e^{i\frac{\pi}{6}} \rvert = 2$ .The statement 4) is **true**.	1
4	Let f be the function defined by $f(x) = -x^2 + 3$ and g the function defined by $g(x) = 3x^3 - 2x + 5$; then $\lim\limits_{x\to+\infty} f(x) = -\infty$ and $\lim\limits_{x\to+\infty} g(x) = +\infty$; $\lim\limits_{x\to+\infty} \frac{f(x)}{g(x)} = \lim\limits_{x\to+\infty} \frac{-x^2}{3x^3} = 0$ .The statement 3) is **false.**	1

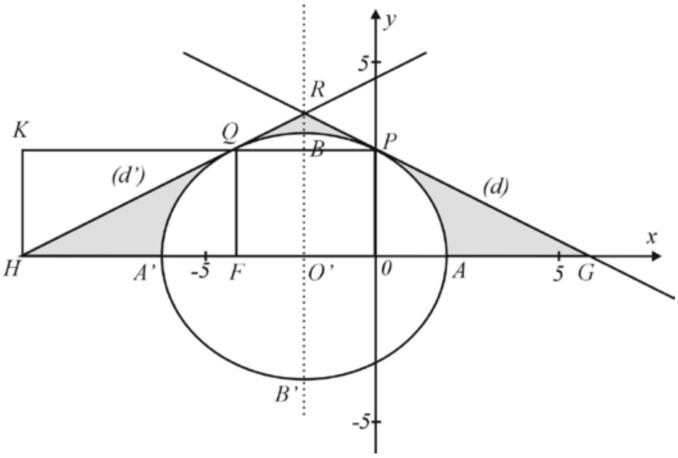
Q :II	Answer	Mark
1	Let $I(-1;0;1)$ be a point on (d) and $J(0;0;2)$ be a point on (d'). $\overrightarrow{IJ}.\left(\overrightarrow{v_d} \wedge \overrightarrow{v_{d'}}\right) = -5 \neq 0$, hence (d) and (d') are not coplanar .	0.5
2	$\overrightarrow{EI} \wedge \overrightarrow{v_d}(1,0,-1)$ is a normal vector to plane (P) and $E \in (P)$; (P) : $x - z + 2 = 0$. ***Or*** : Verify that (d) is included in (P) and that (P) cuts (d') at E.	0.5
3a	$d(E;(d)) = \frac{\left\Vert \overrightarrow{EI} \wedge \overrightarrow{V_d} \right\Vert}{\left\Vert \overrightarrow{V_d} \right\Vert} = \frac{\left\Vert \vec{i} - \vec{j} \right\Vert}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$ $d(E;(d)) < R$; then (C) cuts (d) at two points A and B .	1
3b	$EA = 1$ gives $(m-1)^2 + m^2 + (m-1)^2 = 1$, hence $m' = 1$, $m'' = \frac{1}{3}$. Therefore: $A(0\,;1\,;2)$ and $B\left(-\frac{2}{3};\frac{1}{3};\frac{4}{3}\right)$.	1
3c	Let H be the foot of the perpendicular drawn through E to (AB). $HA^2 = EA^2 - EH^2 = 1 - \frac{6}{9} = \frac{3}{9}$; The area of EAB = $\frac{EH \times 2HA}{2} = \frac{\sqrt{2}}{3}$ u^2. **Or** Area (EAB) = $\frac{\Vert \overrightarrow{EA} \wedge \overrightarrow{EB} \Vert}{2}$.	1

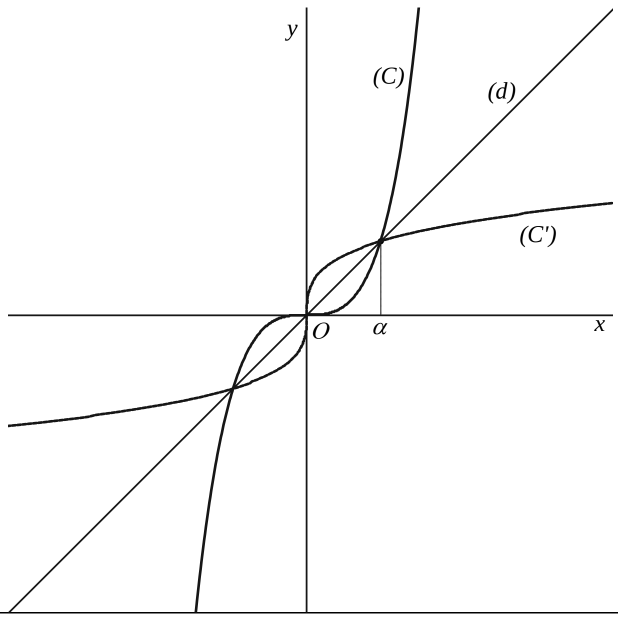
<table>
<tr><th>Q :III</th><th>Answer</th><th>Mark</th></tr>
<tr><td>A</td><td>
<table>
<tr><td>s_i</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
<tr><td>p_i</td><td>$\frac{1}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{5}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{5}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{2}{36}$</td><td>$\frac{1}{36}$</td></tr>
</table>
* $P(H) = P(S \geq 8) = \frac{15}{36} = \frac{5}{12}$; * $P(C) = P(S \leq 5) = \frac{10}{36} = \frac{5}{18}$.
* $P(E) = P(S = 6 \text{ OR } S = 7) = \frac{11}{36}$.
</td><td>1.5</td></tr>
<tr><td>B1a</td><td>$P(G/H) = \frac{20}{30} = \frac{2}{3}$; $P(G\cap H) = P(H) \times P(G/H) = \frac{5}{12} x \frac{2}{3} = \frac{10}{36} = \frac{5}{18}$.</td><td>1</td></tr>
<tr><td>B1b</td><td>$P(G\cap E) = P(E) xP(G/E) = \frac{11}{36}(\frac{1}{2}\times\frac{2}{3} + \frac{1}{2}\times\frac{1}{3}) = \frac{11}{36}\times\frac{1}{2} = \frac{11}{72}$.
$P(G/C) = \frac{10}{30} = \frac{1}{3}$; $P(G\cap C) = P(C) \times P(G/C) = \frac{5}{18} x \frac{1}{3} = \frac{5}{54}$.</td><td>1</td></tr>
<tr><td>B1c</td><td>$P(G) = P(G\cap H) + P(G\cap C) + P(G\cap E) = \frac{5}{18} + \frac{5}{54} + \frac{11}{72} = \frac{113}{216}$.</td><td>1</td></tr>
<tr><td>B2</td><td>$P(X = 3000) = P (H \cap \overline{G}) = P(H) - p(H \cap G) = \frac{5}{36}$.
$P(E\cap\overline{G}) = P(E) - P(G\cap E) = \frac{11}{36} - \frac{11}{72} = \frac{11}{72}$;
$P(C\cap\overline{G}) = P(C) - P(G\cap C) = \frac{5}{18} - \frac{5}{54} = \frac{5}{27}$
<table>
<tr><td>x_i</td><td>1 000</td><td>2 000</td><td>3 000</td><td>5000</td></tr>
<tr><td>p_i</td><td>$\frac{11}{72}$</td><td>$\frac{5}{27}$</td><td>$\frac{5}{36}$</td><td>$\frac{113}{216}$</td></tr>
</table>
</td><td>1.5</td></tr>
</table>

<table>
<tr><th>Q :IV</th><th>Answer</th><th>Mark</th></tr>
<tr><td>1a</td><td>f is the composite of a dilation h and a rotation r ; f is a similitude of ratio 2 and angle $\frac{\pi}{2}$.</td><td>0.5</td></tr>
<tr><td>1b</td><td>h(O)=B and r(B)=C so f(O)=C; f(A)= E' that is the symmetric of E with respect to A and r(E')=D, therefore f(A) = D and consequently the image of the straight line (OA) by f is (CD) . Hence, (OA) and (CD) are perpendicular.</td><td>1</td></tr>
<tr><td>2a</td><td>Ratio of s $= \frac{CA}{AD} = \frac{1}{2}$ and angle of s $= \left(\overrightarrow{AD};\overrightarrow{CA}\right) = \left(\overrightarrow{AD};\overrightarrow{AE}\right) = \frac{\pi}{2}[2\pi]$</td><td>0.5</td></tr>
<tr><td>2b</td><td>

s (D)=A then the image of the straight line (CD) is the straight line passing through A and perpendicular to (CD) ; it is then the straight line (OA).
s (A)=C then the image of the straight line (AO) is the straight line passing through C and perpendicular to (AO); it is then the straight line (CD).

</td><td>1</td></tr>
<tr><td>2c</td><td>Let I be the point of intersection of (AO) and (CD) and let I'= s(I) :
$I\in(AO)$ so $I'\in s(AO)$ =(CD) , and $I\in(CD)$ so $I'\in s(CD)$ =(AO).
So, the intersection of (AO) and (CD) is I'=I . Therefore, s(I)=I and I is the center of s.</td><td>0.5</td></tr>
<tr><td>3a</td><td>g is the composite of two similitudes, then it is the similitude of ratio $2\times\frac{1}{2}=1$ and angle
$\frac{\pi}{2}+\frac{\pi}{2}=\pi$; so g is a central symmetry.
Moreover, g(D)= f(s(D)) = f(A) =D then g is the central symmetry of center D .</td><td>1</td></tr>
<tr><td>3b</td><td>g(A) = A' then $f\circ s(A)$ =A' therefore f(s(A)) =A' but s(A) = C . Hence, f(C) =A'</td><td>0.5</td></tr>
<tr><td>4a</td><td>$z_C=i$ and $z_{A'}= -4$. Similarly, $z_B=1$ and $z_E = -2i$ .Thus $z_O = \frac{1}{2} - i$.</td><td>0.5</td></tr>
<tr><td>4b</td><td>The complex form of f is z'=az +b with $a=2e^{i\frac{\pi}{2}}=2i$; f(C)=A' then $z_{A'}=2iz_C+b \Leftrightarrow$
$-4 = 2i(i)+b$ so $b = -4 + 2 = -2$
Thus the complex form of f becomes: $z' = 2i z - 2$,
and $z_\Omega = \frac{b}{1-a} = \frac{-2}{1-2i} = \frac{-2(1+2i)}{5} = \frac{-2}{5} - \frac{4}{5}i$</td><td>0.5</td></tr>
</table>

Q :V	Answer	Mark
A1	O' is the mid point of [OF], $2c=4$ and $c=2$, O'H= $\frac{a^2}{c}$ = 8 gives $a=4$ Thus $e=\frac{c}{a}=\frac{1}{2}$.	0.5
A2a	$PF^2=PO^2+OF^2=25$, $\frac{PF}{PK}=\frac{1}{2}$ and $\frac{QF}{QK}=\frac{1}{2}$. Thus the points P and Q belong to (E).	1
A2b	The vertices of (E) on the focal axis are the points A and A' , situated on (OH) such that O'A=O'A'= 4 .	0.5
A3a	$c^2=a^2-b^2$, then $b=2\sqrt{3}$, $\tan O'\hat{F}B=\sqrt{3}$ and so $O'\hat{F}B=60^\circ$, thus BF = BO and the triangle OBF is equilateral.	0.5
A3b		0.5
B1	$\frac{(x+2)^2}{16}+\frac{y^2}{12}=1$; $3x^2+4y^2+12x-36=0$	0.5
B2	Differentiating $\frac{(x+2)^2}{16}+\frac{y^2}{12}=1$ wrt x, we get $\frac{2(x+2)}{16}+\frac{2yy'}{12}=0$. At point P(0 , 3), the slope is $y'=-\frac{1}{2}$. An equation of the tangent at P to (E) is $y=-\frac{1}{2}x+3$.	1
B3	An equation of (T) is $y=\frac{1}{2}x+5$. H(– 10 ; 0) belongs to (T).	0.5
B4	$G(6;0)$, $L(-10;0)$ and $R(-2;4)$, $L=H$. Area of RGL = $\frac{HG\times RO'}{2}=\frac{16\times 4}{2}=32$. Area of « semi-ellipse » = $\frac{\pi ab}{2}=4\pi\sqrt{3}$. A = $(32-4\pi\sqrt{3})$ u^2 .	1

Q :VI	Answer	Mark
A1	$\lambda = -1.$ — 0.5 — A2: $y = C\,e^{x}$; $C \in IR.$	0.5
A3	$y = C\,e^{x} - e^{-x}.$ — 0.5 — A4: g is the particular solution of (E) that corresponds to C = 1.	0.5
B1a	$D_f = IR$; D_f is centered at 0 and f(–x) = – f(x).	0.5
B1b	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x[\frac{e^{x}}{x} - 1] = +\infty$ and $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = +\infty$.	0.5
B1c	$f\,'(x) = e^{x} + e^{-x} - 2 = e^{-x}(e^{x}-1)^{2}.$	0.5
B1d	x: $-\infty$; 0 ; $+\infty$ f '(x): + ; 0 ; + f (x): $-\infty$ ↗ 0 ↗ $+\infty$	1
B2a	$h(x) = g(x) - 3x = f(x) - x = e^{x} - e^{-x} - 3x$. α is the non zero solution of h(x) = 0 ; h(1.62) = -0.004 and h(1.63)=0.0179 $h(1.62)\times h(1.63) < 0$; hence $1.62 < \alpha < 1.63$.	1
B2b	f (x) = x is equivalent to h(x) = 0 . Graphically the equation h(x) = 0 has three solution – α ; 0 and α. Therefore the equation f(x) = x has 3 roots : $-\alpha$; 0 ; α .	1
B3a	$f(1) = e - e^{-1} - 2 \approx 0.35$ and $f(2) = e^{2} - e^{-2} - 4 \approx 3.25$.	0.5
B3b		1.5
B4a	f is continuous and strictly increasing on IR ; hence , f has an inverse function f^{-1}. f^{-1} is defined on $f(IR)$; $D_{f^{-1}} = IR$.	1
B4b	f is strictly increasing , then $1 < f^{-1}(x) < 2$ is equivalent to f(1)< x < f(2); that is $e - e^{-1} - 2 < x < e^{2} - e^{-2} - 4$; 0.35 < x < 3.25	1
B4c	(C') is the image of (C) by the symmetry of axis the straight line (d) of equation y = x.	0.5
B5	$4\int_0^{\alpha}[x - f(x)]dx = 4\left[\frac{1}{2}x^{2} - e^{x} - e^{-x} + x^{2}\right]_0^{\alpha} = 6\alpha^{2} + 8 - 4(e^{\alpha} + e^{-\alpha})$ units of area .	1
C1	$0 < U_0 < \alpha$; if $0 < U_n < \alpha$, then $f(0) < f(U_n) < f(\alpha)$; that is $0 < U_{n+1} < \alpha$.	1
C2	For all real numbers $x \in]0 ; \alpha[$, $f(x) < x$; and $U_n \in]0 ; \alpha[$ then $f(U_n) < U_n$; therefore $U_{n+1} < U_n$. This sequence (U_n) is strictly decreasing. (U_n) is strictly decreasing and has a lower bound 0 ; then(U_n) converges to a limit L.	1