وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : إجتماع و إقتصاد	الدورة العادية للعام ٢٠٠٨
عدد المسائل : اربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

The table below shows the blood pressure y_i, according to the weight x_i, of a group of women .

Weight in kg x_i	55	58	60	64	65	70
Blood pressure y_i	13.2	13.5	13.8	14.6	15.2	15.8

1) Calculate the averages $\bar{x}$ and $\bar{y}$ of the two statistical variables x_i and y_i respectively.
2) Represent graphically the scatter plot as well as the center of gravity G ($\bar{x}$; $\bar{y}$) of the points (x_i ; y_i) in a rectangular system.

3) Write an equation of the regression line $D_{y/x}$ of y in terms of x and draw this line in the preceding system.

4) Suppose that the above pattern remains valid for weights of women between 45 and 75 kg, estimate the blood pressure of a woman weighing 72 kg.

5) Doctors assume that a normal blood pressure for a woman should belong to the interval [12 ;13]. Estimate a corresponding interval to which the weights of women having normal blood pressure should belong.

II-(4 points)

A person rented an apartment at the beginning of the year 2000.
The annual rent in the year 2000 was 4 000 000 LL to be increased by 10% every year.
Let $U_0 = 4\ 000\ 000$ and designate by U_n the annual rent in LL in the year (2000 + n).

1) Calculate U_1 and U_2.

2) a- Prove that (U_n) is a geometric sequence, and determine its common ratio.
   b- Calculate U_n in terms of n, and deduce U_{n+1} in terms of n.

3) Let $S_n = U_0 + U_1 + \ldots\ldots. + U_n$.
   a- Show that $1.1\times S_n = U_1 + U_2 + \ldots\ldots + U_{n+1}$.
   b- Deduce that $1.1\times S_n = S_n + U_{n+1} - U_0$, and that $S_n = 40\ 000\ 000\ [(1.1)^{n+1} - 1]$.

4) This person rented the apartment for 6 consecutive years starting from the beginning of the year 2000 till the end of 2005.
   Calculate the total sum of money paid by this person for renting this apartment during this period.

III- (4 points)

The 40 students in the basketball club of a school are distributed as shown in the table below:

	Boys	Girls
1^{st} secondary year	8	6
2^{nd} secondary year	7	4
3^{rd} secondary year	8	7

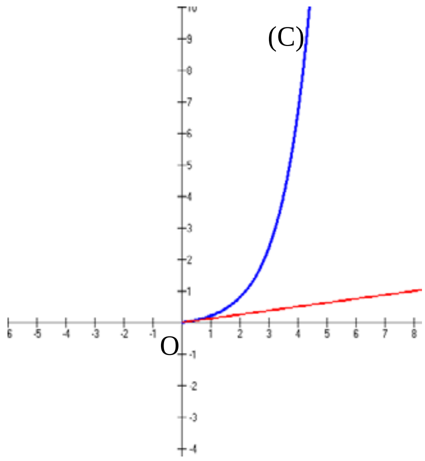
A group of 3 students is chosen simultaneously and randomly from this club.

1) Consider the following events:
 A: « The three chosen students are all in the 3^{rd} secondary year ».
 B: « The three chosen students are all girls, each one from a different year ».
 C: « The three chosen students are all in the same secondary year ».

 Verify that the probability p (A) is equal to $\frac{7}{152}$ and calculate p (B) and p (C).

2) The chosen group is formed of three boys, what is the probability that they are all from the same secondary year?

3) In this part we choose randomly and successively three students from this club.
 a- What is the probability that the first is in the 1^{st} year, the second is in the 2^{nd} year and the third is in the 3^{rd} year.
 b- What is the probability that at least one of them is in the 1^{st} year.



IV- (8 points)

A- The curve (C) to the right is the graphical representation, in an orthonormal system, of the function f defined over $[0\,;+\infty[$ by $f(x) = a\,e^x + b$, where a and b are real numbers.

The curve (C) has at O a tangent (T) of equation $y = \frac{x}{8}$.

Show that $f(x) = \frac{e^x - 1}{8}$.

B- Let g be the function defined over $[0\,;+\infty[$ by $g(x) = \frac{120}{e^x + 15}$, and let (G) be its representative curve.

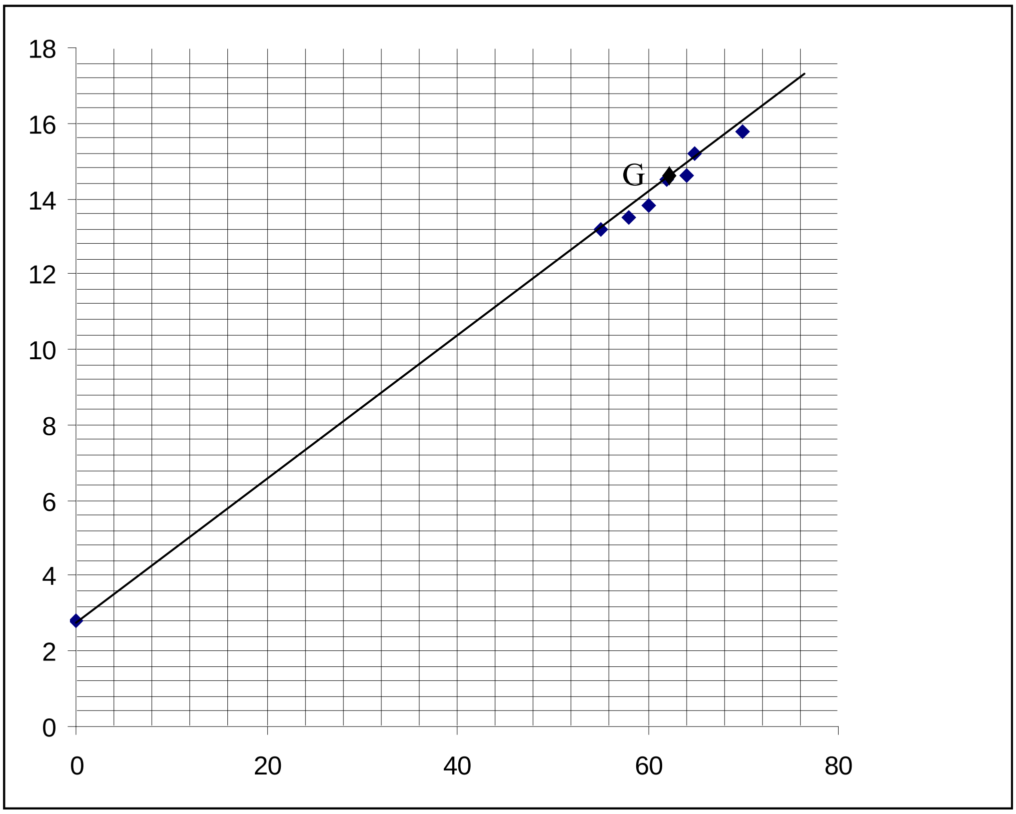
1) Determine $\lim\limits_{x \to +\infty} g(x)$ and deduce an asymptote to (G).

2) Show that $g'(x) < 0$, for every $x \geq 0$.

3) Set up the table of variations of g.

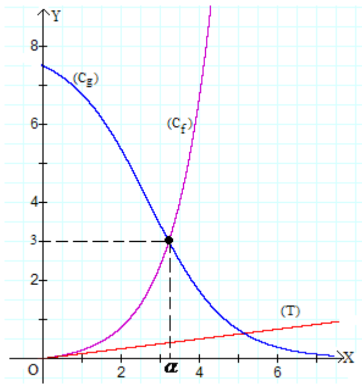
4) Copy (C) and draw (G) in the same orthonormal system .

5) Suppose that f and g are respectively the supply and the demand functions of a certain object, in terms of the unit price x. (x is expressed in millions LL, f(x) and g(x) in hundreds of objects).
 a- Estimate the number of objects demanded when the unit price is 5 000 000 LL.
 b- Show that the equation $f(x) = g(x)$ has a unique solution α and verify that $\alpha = \ln(25)$.
 c- Calculate $g(\alpha)$ and give an economical interpretation for the values of α and $g(\alpha)$.

QI	Answer	Mark
1	$\overline{x} = 62$, $\overline{y} = 14.35$. (calculator)	1
2		2
3	$y = 0.186x+2.799$. (Calculator)	1
4	For x =72; $y = 0.186\times72+2.799 = 16.191$. So for a weight of 72 kg, the pressure is estimated to be 16.2 .	1
5	$12 \le y \le 13$ is equivalent to $12 \le 0.186x + 2.799 \le 13$ and $49.46 \le x \le 54.84$. The weights of such women belong to the interval [49.5 ; 54.8].	2

QII	Answer	Mark
1	$U_1 = U_0 + 0.1\ U_0 = 1.1\ U_0 = 1.1 \times 4\ 000\ 000 = 4\ 400\ 000$. $U_2 = 1.1\ U_1 = 1.1\times4\ 400\ 000 = 4\ 840\ 000$.	1
2a	$U_{n+1} = U_n + 0.1\ U_n = 1.1\ U_n$. Thus (U_n) is a geometric sequence, common ratio is $q = 1.1$, first term is $U_0 = 4\ 000\ 000$.	1
2b	$U_n = U_0\ q^n = 4\ 000\ 000\ (1.1)^n$; $U_{n+1} = U_0\ q^{n+1} = 4\ 000\ 000\ (1.1)^{n+1}$.	1
3a	$S_n = U_0 + U_1 + U_2 + \ldots + U_n$. $1.1S_n = 1.1\ U_0 + 1.1\ U_1 + 1.1\ U_2 + \ldots + 1.1\ U_n$ $= U_1 + U_2 + U_3 + \ldots + U_{n+1}$.	1.5
3b	$1.1\ S_n + U_0 = U_0 + U_1 + U_2 + \ldots + U_n + U_{n+1} = S_n + U_{n+1}$. Thus $1.1\ S_n = S_n + U_{n+1} - U_0$ $1.1\ S_n - S_n = U_{n+1} - U_0$ $0.1\ S_n = 4\ 000\ 000\ (1.1)^{n+1} - 4\ 000\ 000$ $0.1\ S_n = 4\ 000\ 000\ [(1.1)^{n+1} - 1]$ $S_n = 40\ 000\ 000\ [(1.1)^{n+1} - 1]$.	1.5
4	$S_5 = 40\ 000\ 000\ [(1.1)^6 - 1] = 30\ 862\ 440$.	1

QIII	Answer	Mark
1	•P(A)= $\frac{C_{15}^{3}}{C_{40}^{3}}=\frac{7}{152}$ •P(B)= $\frac{C_{6}^{1}\times C_{4}^{1}\times C_{7}^{1}}{C_{40}^{3}}=0.017$ •P(C)= $\frac{C_{14}^{3}+C_{11}^{3}+C_{15}^{3}}{C_{40}^{3}}=0.099$.	3
2	P(same section/3 boys)= $\frac{C_{8}^{3}+C_{7}^{3}+C_{8}^{3}}{C_{23}^{3}}=0.083$.	1.5
3a	P(1^{st} sec, 2^{nd} sec, 3^{rd} sec) $=\frac{14}{40}\times\frac{11}{39}\times\frac{15}{38}=0.389$.	1.5
3b	P(at least one is from the 1^{st} year) =1 – (the 3 are from the other years) $=1-\frac{26}{40}\times\frac{25}{39}\times\frac{24}{38}=0.736$.	1

QIV	Answer	Mark
A	$f(x) = a\,e^{x} + b$ and $f(0) = 0$, then : $a + b = 0$. $f'(x) = a\,e^{x}$ and $f'(0)=\frac{1}{8}$, then : $a=\frac{1}{8}$, consequently $b = -\frac{1}{8}$. Hence, $f(x)=\frac{e^{x}-1}{8}$.	2.5
B1	$\lim\limits_{x\to+\infty} g(x) = 0$, then $y = 0$ is an asymptote.	1
B2	$g'(x) = -\frac{120\,e^{x}}{(e^{x}+15)^{2}}$ and $g(0) = 7.5$.	1.5
B3	$\begin{array}{c\|ccc} x & 0 & & +\infty \\ \hline g'(x) & & - & \\ \hline g(x) & 7.5 & \searrow & 0 \end{array}$	1.5
B4		2.5
B5a	$g(5)=\frac{120}{e^{5}+15}=0.73$; 73 objects are demanded.	1
B5b	$f(x) = g(x) \Rightarrow e^{2x} + 14\,e^{x} - 975 = 0 \Rightarrow e^{x} = 25 \Rightarrow x = \ln(25) \approx 3.21887 = \alpha$.	2
B5c	$g(\alpha) = g(\ln 25) = \frac{120}{e^{\ln 25}+15}=\frac{120}{25+15}=3$. α = Equilibrium price = 1 000 000 × 3.218 875 = 3 218 875 LL. $g(\alpha)$ = Equilibrium quantity = 100 × 3 = 300 objects.	2