دورة سنة ٢٠٠٨ العادية	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: اربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة) .

I- (4 points)

In the complex plane (P) referred to an orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B and C of affixes $a=\sqrt{3}-i$, $b=\sqrt{3}+i$ and $c=2i$ respectively.

1) Show that the three points A, B and C are on the same circle with center O.
2) Write $\frac{c-b}{a-b}$ in the algebraic and in the exponential forms.
3) Let M be a point other than O, of affix z = x + iy, in the plane (P); (x and y are real numbers).

   Let $Z=\frac{z-b}{z}$.

   a- Determine the set (E) of points M such that $|Z|=1$.

   b- Verify that A and C belong to (E).

   c- Determine the set (F) of points M such that Z is pure imaginary.

II- (4 points)

To encourage national tourism, a tourist agency proposes week-ends of two days, and offers its customers three choices:

- Full-board week-end
- Half-board week-end
- Luxury week-end.

The agency published the following advertisement:

Choice / Destination	Full-board	Half-board	Luxury
Mountain	150 000 LL	100 000 LL	200 000 LL
Beach	100 000 LL	75 000 LL	150 000 LL

This agency estimates that 65% of its customers choose mountains, and the others choose the beach; and that out of the customers to any destination 55% choose full-board and 30% choose half-board while the others choose luxury week-ends.

A customer is chosen at random and is interviewed.

Consider the following events:

A : « the interviewed customer has chosen the mountains».

B : « the interviewed customer has chosen the beach ».

C : « the interviewed customer has chosen full-board week-end ».

D : « the interviewed customer has chosen half-board week-end ».

S : « the interviewed customer has chosen the luxury week-end ».

1) a- Calculate the following probabilities: $P(A \cap C)$, $P(B \cap C)$ and $P(C)$.

b- The interviewed customer had chosen full-board, what is the probability that he chose the beach?

2) Let X be the random variable that is equal to the amount paid to the agency by a customer.

a- Show that P(X=150 000) = 0.41 and determine the probability distribution for X.

b- Calculate the mean(expected value) E(X). What does the number thus obtained represent?

c- Estimate the sum received by this agency when it serves 200 customers.

III- (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the points A(1; 2; 0), B(2; 1; 3), C(3; 3; 1), D(5; –3; –3) and E(–3; 7; 3).

1) Find an equation of the plane (P) determined by A, B and C.
2) Find a system of parametric equations of line (DE).
3) Prove that (P) is the mediator plane of [DE].
4) Prove that (BC) is orthogonal to (DE).
5) a- Calculate the area of triangle BCD.

b- Calculate the volume of tetrahedron ABCD, and deduce the distance from A to plane BCD.

IV- (8 points)

Let f be the function defined on IR by $f(x) = (x-1)e^x + 1$ and designate by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

1) a- Calculate $\lim\limits_{x \to -\infty} f(x)$ and deduce an asymptote (d) of (C).

b- Study, according to the values of x, the relative positions of (C) and (d).

c- Calculate $\lim\limits_{x \to +\infty} f(x)$ and find f(2) in decimal form.

2) Calculate $f'(x)$ and set up the table of variations of f.
3) Prove that the curve (C) has a point of inflection W whose coordinates are to be determined.
4) a- Draw (d) and (C).

b- Discuss graphically, according to the values of the real parameter m, the number of solutions of the equation $(m-1)e^{-x} = x - 1$.

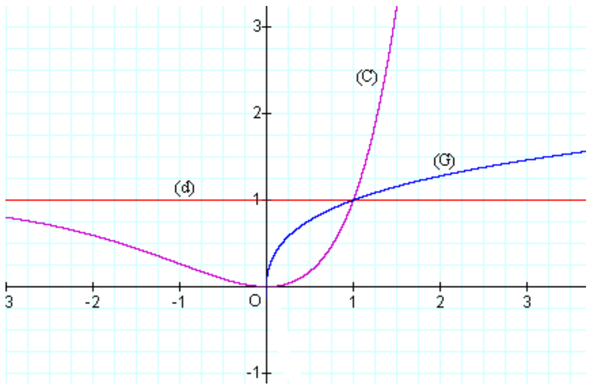
5) Calculate the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.
6) a- Show that the function f has on $[0; +\infty[$ an inverse function g and draw (G), the representative curve of g in the system $(O ; \vec{i}, \vec{j})$.

b- Find the area of the region bounded by (G), the axis of ordinates and the line (d).

Q I	Answer	Mark
1	$\lvert a\rvert=\lvert b\rvert=\lvert c\rvert=2$ so OA = OB= OC = 2.	0.5
2	$\dfrac{c-b}{a-b}=\dfrac{-\sqrt{3}+i}{-2i}=\dfrac{-1-i\sqrt{3}}{2}=e^{i\left(\pi+\frac{\pi}{3}\right)}=e^{-i\frac{2\pi}{3}}$.	1
3a	$\lvert Z\rvert=1$, iff BM = OM , so M moves on the perpendicular bisector (E) of [OB].	1
3b	AB = AO and CB = CO , so A and C are two points on (E).	0.5
3c	$Z=\dfrac{x+iy-\sqrt{3}-i}{x+iy}=\dfrac{x^2+y^2-\sqrt{3}x-y}{x^2+y^2}+\dfrac{-x+\sqrt{3}y}{x^2+y^2}i$ Z is pure imaginary iff $\begin{cases}x^2+y^2-\sqrt{3}x-y=0\\-x+\sqrt{3}y\neq 0\end{cases}$ M moves on a circle excluding O and B. **Or**: arg (Z) = $\dfrac{\pi}{2}[\pi]=(\vec{u},\overrightarrow{BM})-(\vec{u},\overrightarrow{OM})=(\overrightarrow{OM},\overrightarrow{BM})[\pi]$. So M moves on the circle (F) with diameter [OB], excluding O and B.	1

Q II	Answer	Mark
1a	$P(A\cap C)=P(A)\times P(C/A)=0.65\times 0.55=0.3575$ $P(B\cap C)=P(B)\times P(C/B)=0.35\times 0.55=0.1925$ $P(C)=P(A\cap C)+P(B\cap C)=0.3575+0.1925=0.55$ **OR** given *P(C) = 0.55.*	1
1b	$P(B/C)=\dfrac{P(B\cap C)}{P(C)}=\dfrac{0.1925}{0.55}=0.35$.	0.5
2a	P(X=150 000)= O.65×0.55 +0.15×0.35=0.41. <table><tr><td>x_i</td><td>75000</td><td>100 000</td><td>150 000</td><td>200 000</td></tr><tr><td>p_i</td><td>0.35×0.3=0.105</td><td>0.35×0.55+0.65×0.3=0.3875</td><td>0.41</td><td>0.65×0.15 = 0.0975</td></tr></table>	1.5
2b	$E(X)=\sum p_i x_i=0.105\times 75000+0.3875\times 100000+0.41\times 150000+0.0975\times 200000=127\ 625$ The average amount paid by a voyager is 127 625 LL.	0.5
2c	An estimation of the sum received is: 127625×200 = 25 525 000 LL.	0.5

QIII	Answer	Mark
1	$\vec{N}=\overrightarrow{AB}\wedge\overrightarrow{AC}$ (4;-5;-3) is normal to (P) ; (P) : $\overrightarrow{AM}.\vec{N}=0$. Hence 4x – 5y – 3z + 6 = 0.	0.5
2	(DE): x = – 8t + 5 ; y =10t –3 ; z = 6t –3.	0.5
3	A director vector of (DE) and a normal vector of (P) have the same direction; Mid point (1; 2; 0) of [DE] belongs to (P).	1
4	$\overrightarrow{DE}$ (– 8 ; 10 ; 6) . $\overrightarrow{BC}$ (1 ; 2 ; –2) = 0 . ►**OR** since (DE) is perpendicular to plane (P).	0.5
5a	$\overrightarrow{DB}\wedge\overrightarrow{DC}$ (– 20 ; 0 ; –10), area = $\dfrac{1}{2}\sqrt{500}=\sqrt{125}=5\sqrt{5}$.	0.5
5b	Volume = $\dfrac{1}{6}\left\lvert\overrightarrow{DA}.(\overrightarrow{DB}\wedge\overrightarrow{DC})\right\rvert=\dfrac{50}{6}=\dfrac{25}{3}$. V= $\dfrac{\text{base}\times h}{3}$, $\dfrac{25}{3}=\dfrac{5\sqrt{5}\,h}{3}$, so h = $\sqrt{5}$.	1

<table>
<tr><th>Q IV</th><th>Answer</th><th>Mark</th></tr>
<tr><td>1a</td><td>$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} (xe^x - e^x + 1) = 1$, so the line with equation y = 1 is asymptote to (C) .</td><td>0.5</td></tr>
<tr><td>1b</td><td>$f(x) - 1 = (x-1)e^x$.
□ (C) cuts (d) at point (1 ; 1)
□ For x > 1, (C) is above (d)
□ For x < 1, (C) is below (d).</td><td>0.5</td></tr>
<tr><td>1c</td><td>$\lim\limits_{x\to+\infty} f(x) = +\infty$ and f(2) = 8.389.</td><td>0.5</td></tr>
<tr><td>2</td><td>$f'(x) = e^x + (x-1)e^x = xe^x$.
$$\begin{array}{c|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 1 & \searrow & 0 & \nearrow & +\infty \end{array}$$</td><td>1</td></tr>
<tr><td>3</td><td>$f''(x) = (x+1)e^x$; $f''(x)$ vanishes for x = –1 and changes signs,
thus (C) has a point of inflection $W(-1, 1-\frac{2}{e})$.</td><td>0.5</td></tr>
<tr><td>4a</td><td>
</td><td>1.5</td></tr>
<tr><td>4b</td><td>$(m-1)e^{-x} = x - 1$ gives $m = (x-1)e^x + 1$.

□ For m < 0 ; no solution   □ For m = 0 ; one solution (double)
□ For 0 < m < 1 ; two solutions   □ For m ≥ 1 ; single solution.</td><td>1</td></tr>
<tr><td>5</td><td>$A = \int_0^1 [(x-1)e^x + 1]dx = \left[(x-2)e^x + x\right]_0^1 = (3-e)\,u^2$.</td><td>1</td></tr>
<tr><td>6a</td><td>f is continuous and strictly increasing on [0 ; +∞[, thus f has an inverse function g.
(G) is symmetric of (C) wrt the line of equation y=x.</td><td>1</td></tr>
<tr><td>6b</td><td>The area A’ of the region bounded by (G), the axis of ordinates and the line (d) is equal (by symmetry) to the area A of the region bounded by (C), the axis of abscissas and the two lines x = 0 and x = 1 , consequently $A' = A = (3-e)u^2$.</td><td>0.5</td></tr>
</table>